Dark matter from brane oscillations

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Introduction:

We consider the effective theory describing the Standard Model on a brane in an embedding space.

The effective theory generically contains additional scalar fields (global case) or massive vector fields (local case) associated with the spontaneous breaking of translation symmetries.

These degrees of freedom can be stable and have weak scale interactions and masses. Hence they are dark matter candidates.

We consider the bounds on the parameters of the effective theory based on the observed dark matter relic abundance (WMAP) and results of direct detection experiments (CDMSII).

Brane dynamics

The action describing the motion of a thin brane embedded in a higher dimensional space can be constructed in terms of the induced brane metric. For simplicity, we consider the case with co-dimension equal to one.

The ISO(4,1) invariant interval is

$$ds^2 = dx^M \, \eta_{MN} \, dx^N \qquad with \;\; M,N = 0..4$$

Consider a p=3 brane oriented perpendicular to the x^4 direction. The induced metric on the brane is

$$x^4 = \varphi(x^\mu)$$

$$ds^2 = dx^\mu \, \eta_{\mu\nu} \, dx^\nu \; - \; dx^\mu \, \partial_\mu\varphi \, \partial_\nu\varphi \, dx^\nu = \; dx^\mu \, g_{\mu\nu} \, dx^\nu \qquad with \;\; \mu,\nu = 0..3$$

$$g_{\mu\nu} = \eta_{\mu\nu} - \partial_\mu\varphi \, \partial_\nu\varphi$$

The ISO(4,1) invariant action describing the motion of the brane into the extra dimension is

$$S = - \underset{\text{brane tension}}{\sigma} \int dx^4 \sqrt{- \det(g)} = \underbrace{- \sigma \int dx^4 \sqrt{1 - \partial^\mu\varphi \, \partial_\mu\varphi}}_{\text{Nambu Goto action in static gauge}}$$

Coupling to Matter

(Standard Model on a brane)

The vierbein and the connection follow from the induced metric g. The ISO(4,1) invariant extension of the Standard Model is obtained as

$$S = -\sigma \int dx^4 \sqrt{-g} + \int dx^4 \sqrt{-g}\, L_{SM}(\partial_\mu \rightarrow D_\mu)$$

Expanding up to quadratic power in terms of the scalar field yields the Lagrangian density

$$L = -\sigma + \frac{1}{2}\sigma\, \partial^\mu\varphi\, \partial_\mu\varphi + L_{SM} + \frac{1}{2}\partial^\mu\varphi\, \partial^\nu\varphi\, T^{SM}_{\mu\nu}$$

Rescaling the scalar field so that it obtains its usual engineering dimension and canonical kinetic term gives

$$L = -\sigma + \frac{1}{2}\underbrace{\partial^\mu S\, \partial_\mu S}_{\text{Massless scalar field}} + L_{SM} + \frac{1}{2\sigma}\partial^\mu S\, \partial^\nu S\, \underbrace{T^{SM}_{\mu\nu}}_{\text{Energy momentum tensor}} \qquad S = \frac{\varphi}{\sqrt{\sigma}}$$

The massless scalar field is the Goldstone boson associated with the spontaneous breaking of the translational symmetry.

When the extrinsic curvature is considered, additional ISO(4,1) invariant terms linear in the Goldstone field can also be constructed. Such terms break the discrete symmetry that is manifest here. Sherwin Love discussed the phenomenology (in the local case) when linear terms are allowed in a talk earlier today.

Gauging translations

If all translational symmetries are gauged, then regular gravity is also included in the model. Here we focus on gauging the translational symmetry perpendicular to the brane. The locally invariant interval is obtained by the introduction of a vector gauge field and a covariant derivative.

$$ds^2 = dx^\mu \, \eta_{\mu\nu} \, dx^\nu - dx^\mu \, D_\mu\varphi \, D_\nu\varphi \, dx^\nu$$

with

$$D_\mu\varphi = \partial_\mu\varphi + g \, X_\mu$$

and we define the gauge coupling as $\quad g = \dfrac{M_X}{\sqrt{\sigma}}$

In unitary gauge, $\partial_\mu\varphi = 0$ and therfore $D_\mu\varphi = \dfrac{M_X}{\sqrt{\sigma}} \, X_\mu$.

To obtain the locally invariant action, replace $\partial_\mu\varphi \longrightarrow \dfrac{M_X}{\sqrt{\sigma}} \, X_\mu$

in the globally invariant action and add a kinetic term for the vector field. The leading part of the invariant action thus obtained is

$$L = -\sigma + \frac{1}{2} \, \underbrace{M_X^2}_{\text{Mass through Higgs mechanism}} \, X^\mu \, X_\mu + L_{SM} + \underbrace{\frac{M_X^2}{2\,\sigma}}_{\frac{1}{2\,F_X^2}} X^\mu \, X^\nu \, T^{SM}_{\mu\nu} - \frac{1}{4} \, F^{\mu\nu} \, F_{\mu\nu}$$

Chi Xiong discussed extra-dimensional models that yield this type of effective action in the low energy limit earlier today.

The model

$$L_{SMX} = \frac{1}{2\,F_X^2}\, X_i^{\mu}\, X_i^{\nu}\, T_{\mu\nu}^{SM} + \frac{1}{2}\, M_X^2\, X_i^{\mu}\, X_{i\mu} - \frac{1}{4} F_i^{\mu\nu}\, F_{i\mu\nu}$$

Parameters:

Mass of the vectors M_X

Interaction scale F_X

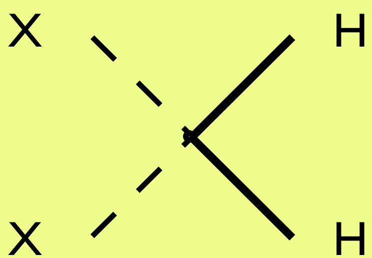
Number of flavors $i = 1 \,..\, N$

The X vector is stable due to a discrete symmetry. It is therefore a dark matter candidate.

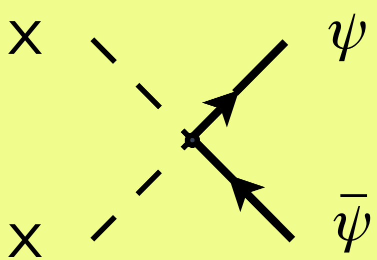
Objective: study how the parameters are constrained by the WMAP dark matter relic abundance and CDMSII direct dark matter detection results.

Relic abundance

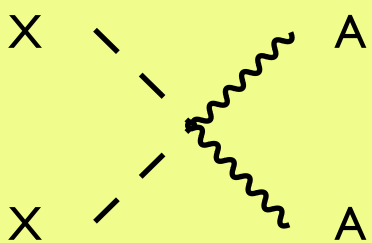
Annihilation cross-sections: (non-relativistic limit shown)



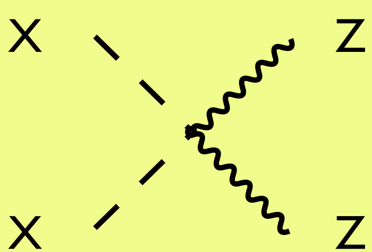
$$\sigma_{HH} = \frac{1}{144\,\pi}\,\frac{2\,M_X^4 + M_H^4}{F_X^4\,M_X^2}\,\frac{\sqrt{M_X^2 - M_H^2}}{\sqrt{s - 4\,M_X^2}}$$



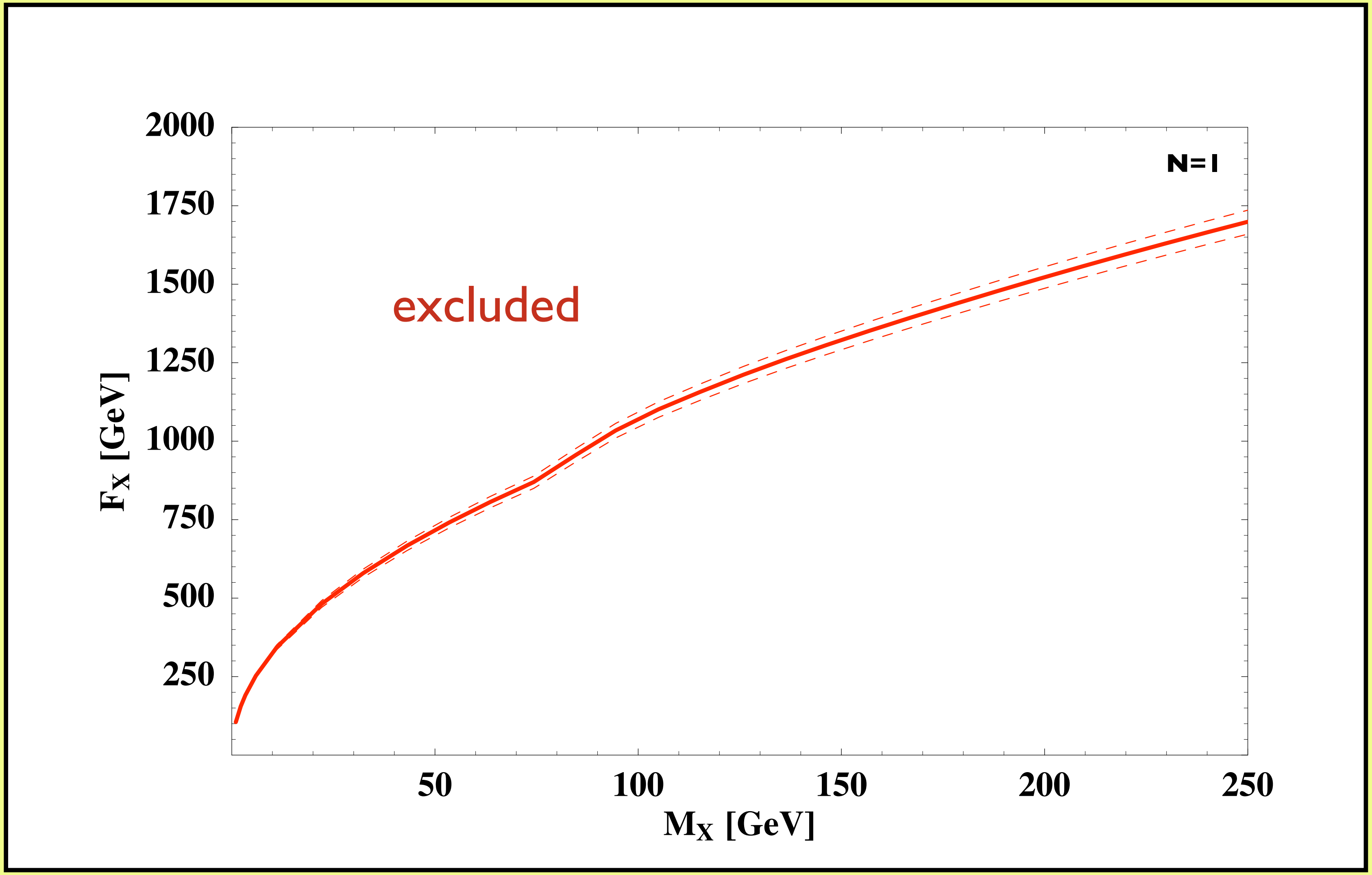
$$\sigma_{\bar{\psi}\psi} = \frac{1}{72\,\pi}\,\frac{M_X^4 - M_\psi^4}{F_X^4\,M_X^2}\,\frac{\sqrt{M_X^2 - M_\psi^2}}{\sqrt{s - 4\,M_X^2}}$$



$$\sigma_{AA} = \frac{1}{18\,\pi}\,\frac{M_X^3}{F_X^4\sqrt{s - 4\,M_X^2}}$$



$$\sigma_{ZZ} = \frac{1}{144\,\pi}\,\frac{10\,M_X^4 + 8\,M_X^2\,M_Z^2 + 3\,M_Z^4}{F_X^4\,M_X^2}\,\frac{\sqrt{M_X^2 - M_Z^2}}{\sqrt{s - 4\,M_X^2}}$$

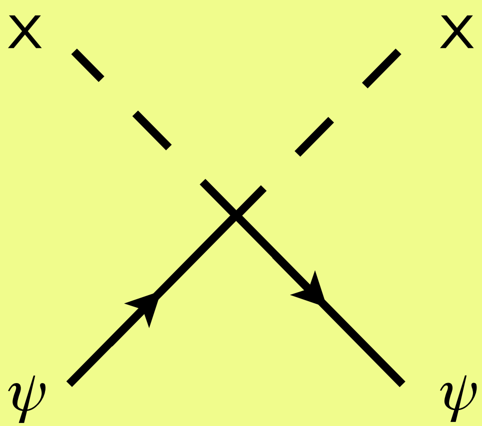


Red line: $\Omega_c h^2 = 0.105 \pm 0.009$ WMAP Collaboration, astro-ph/0603449

Below the red line the X vector only gives a partial contribution to the observed dark matter density.

Direct detection

Elastic X vector - target cross-section:



Proportional to square of target mass

$$\sigma = \frac{2}{\pi}\,\underbrace{\frac{M_X^2 M_\psi^2}{(M_X + M_\psi)^2}}\,\frac{M_\psi^2}{M_X^2 F_X^4}$$

square of reduced mass

Relation between scattering cross-sections X - nucleus and X - nucleon:

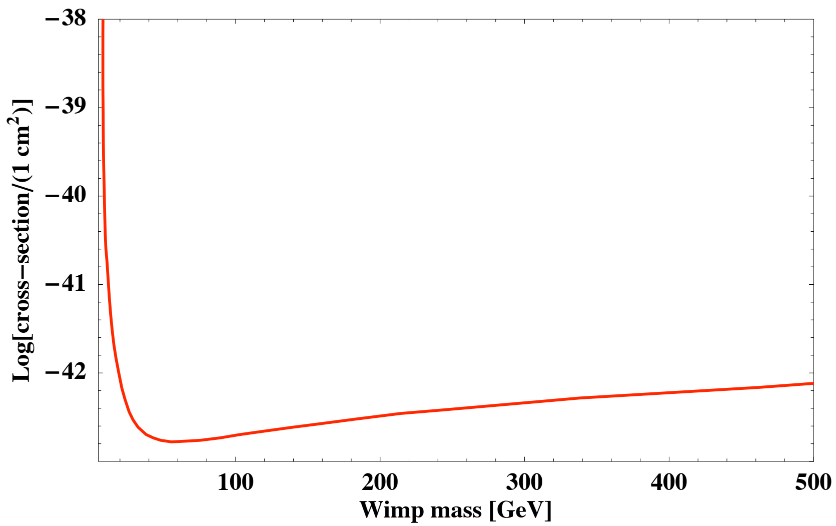
$$\sigma_{Nucleus} = N_n^2\,\frac{\mu(M_X, M_{Nucleus})^2}{\mu(M_X, M_n)^2}\,\sigma_n$$

Nucleon mass M_n

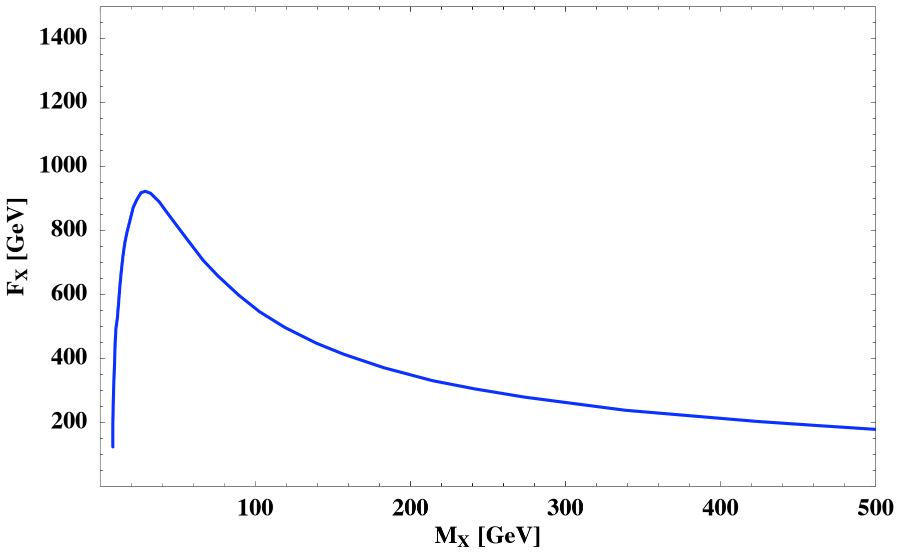
Number of nucleons in Nucleus N_n

CDMSII bound

spin independent cross-section

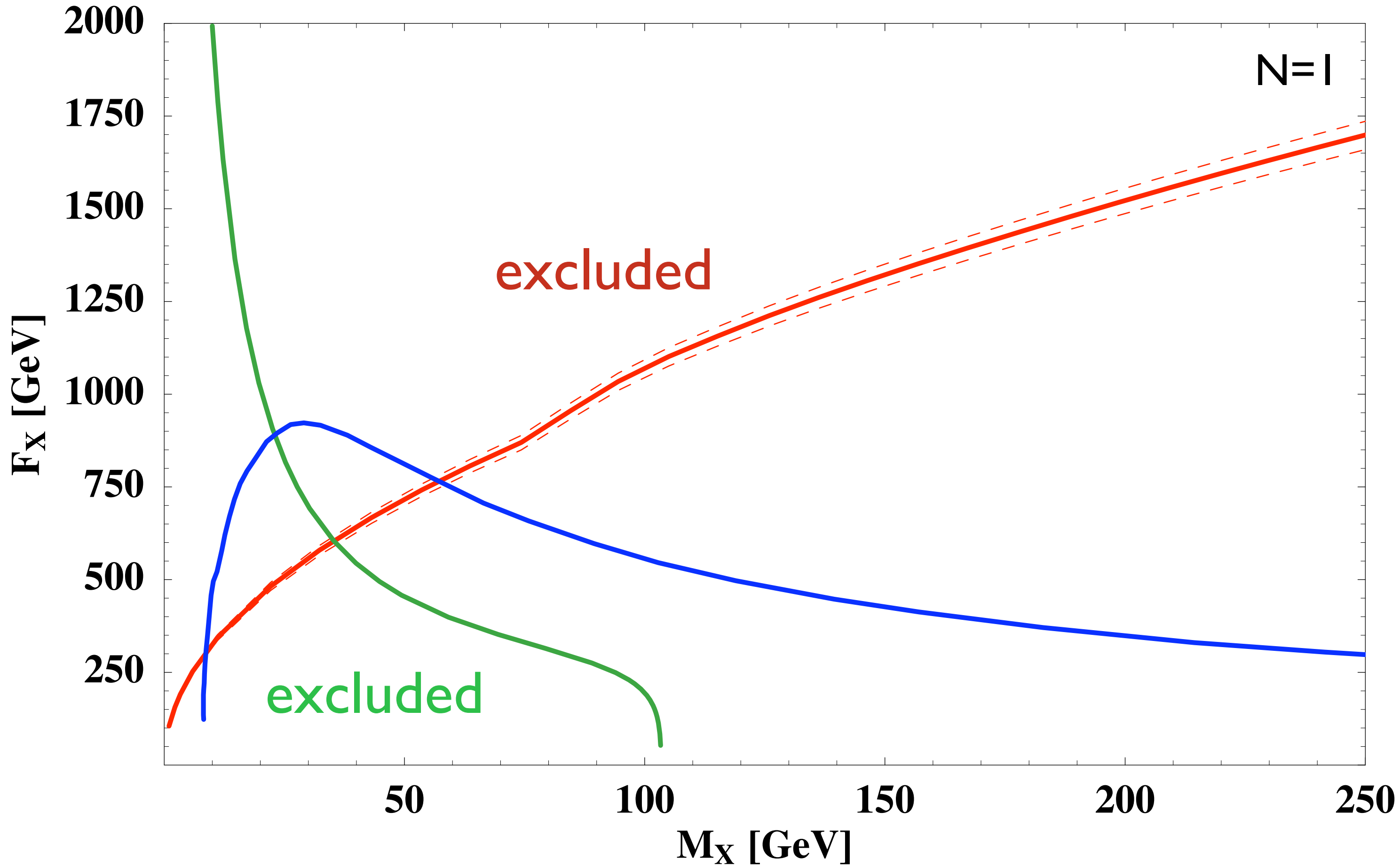


CDMS Collaboration; **Phys.Rev.Lett.96:011302,2006**.



Bound assumes Wimp candidate accounts for all observed dark matter.

Combined bounds



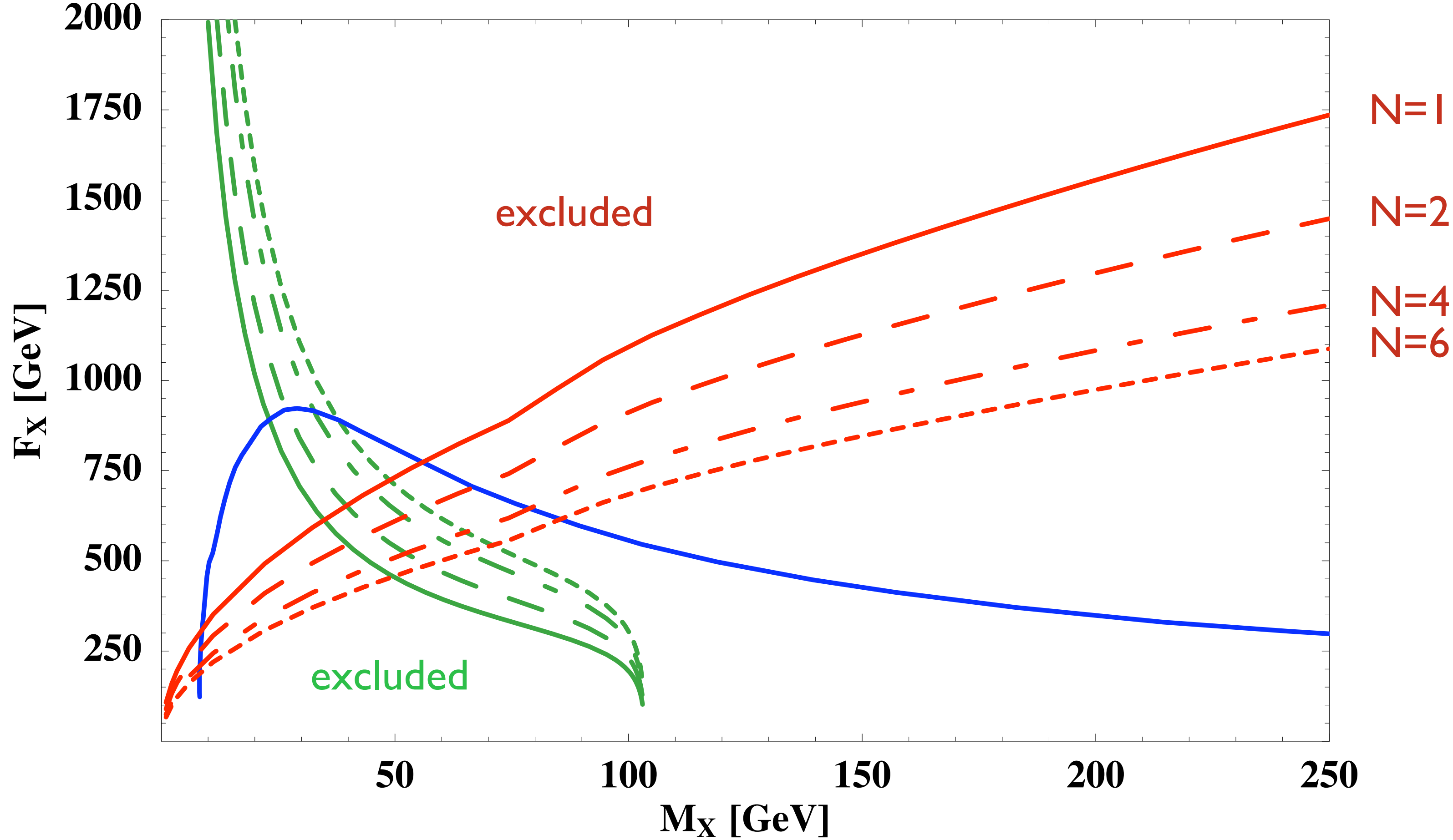
WMAP

CDMSII

LEPII (Thomas Clark's talk, earlier today.)

Conclusion: $F_X > 750\ GeV$ *and* $M_X > 60\ GeV$
if X provides all WMAP observed dark matter.

N flavors of X vectors



WMAP Each flavor contributes only 1/N fraction of observed relic density.

CDMSII Bound does not depend on N

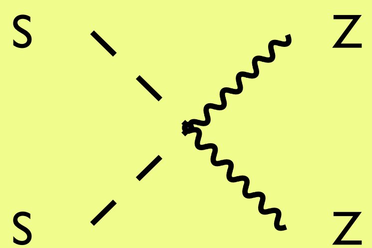
LEPII (Thomas Clark's talk, earlier today)

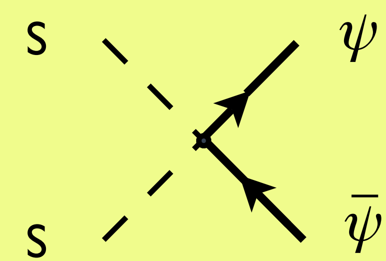
Vector versus Scalar

$$L_{SMX} = \frac{1}{2F_X^2} X_i^\mu X_i^\nu T^{SM}_{\mu\nu} + \frac{1}{2} M_X^2 X_i^\mu X_{i\mu} - \frac{1}{4} F_i^{\mu\nu} F_{i\mu\nu}$$

$$L_{SMS} = \frac{1}{2F^4} \partial^\mu S_i \partial^\nu S_i T^{SM}_{\mu\nu} + \frac{1}{2} M_S^2 S_i S_i + \frac{1}{2} \partial^\mu S_i \partial_\mu S_i$$

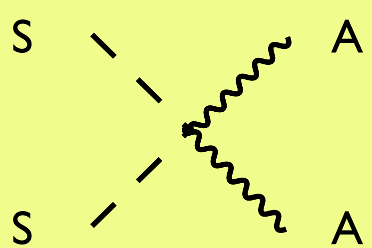
Annihilation cross-sections: (non-relativistic limit shown)

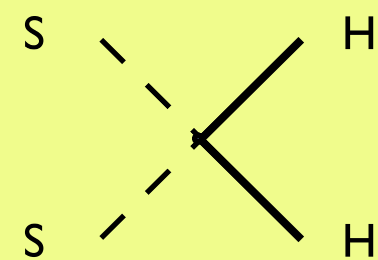




$$\sigma_{ZZ} = \frac{1}{5120\pi} \frac{\left(3M_Z^4 + 4M_Z^2 M_X^2 + 8M_X^4\right)}{F^8 M_X^2} \sqrt{M_X^2 - M_Z^2}\, \underline{\left(s - 4M_S^2\right)}^{\frac{3}{2}}$$

$$\sigma_{\psi\psi} = \frac{1}{7680\pi} \frac{\left(3M_\psi^2 + 2M_X^2\right)}{F^8 M_X^2} \left(M_X^2 - M_\psi^2\right)^{\frac{3}{2}} \underline{\left(s - 4M_S^2\right)}^{\frac{3}{2}}$$

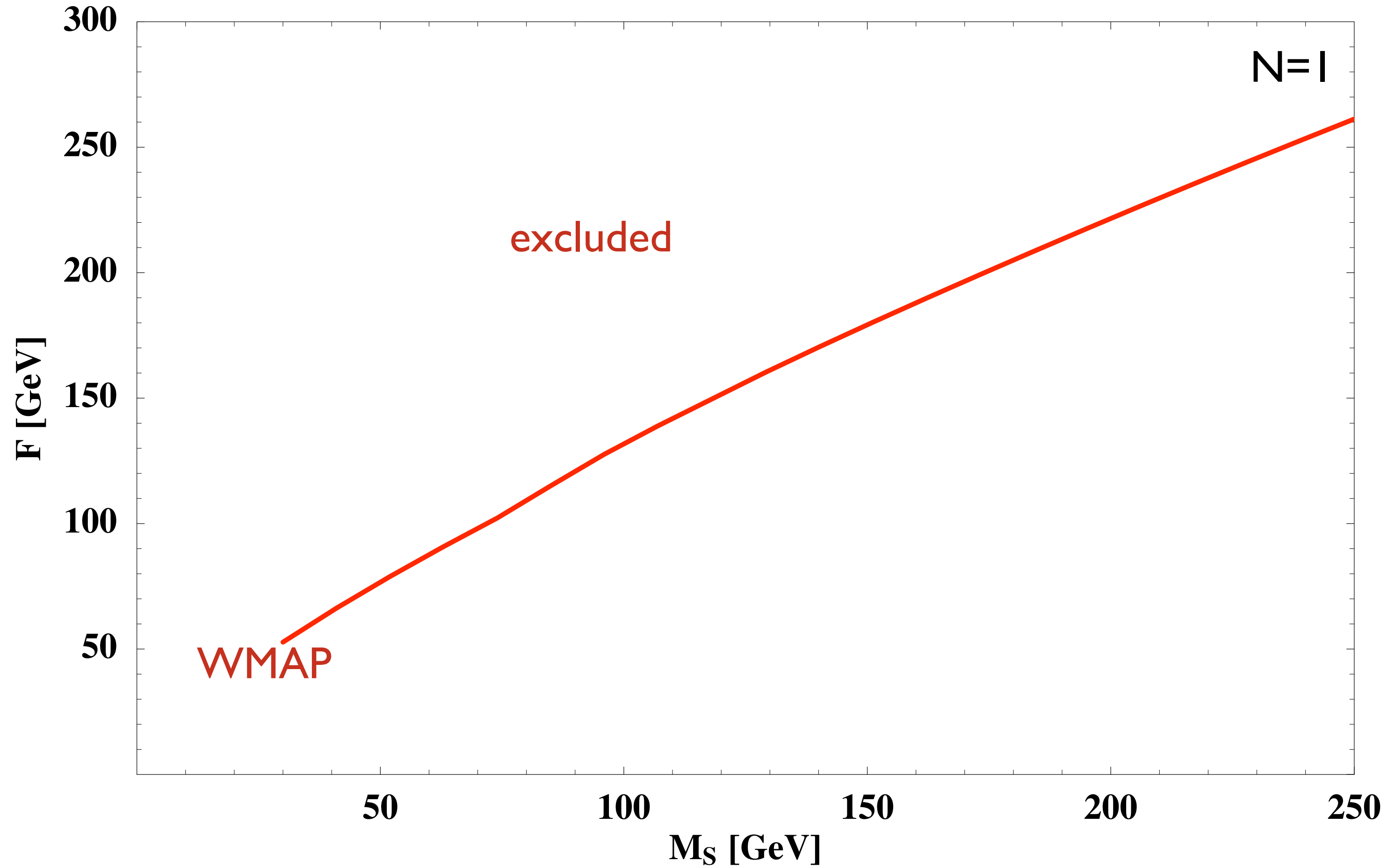




$$\sigma_{AA} = \frac{1}{960\pi} \frac{M_S^3}{F^8} \underline{\left(s - 4M_S^2\right)}^{\frac{3}{2}}$$

$$\sigma_{HH} = \frac{1}{15360\pi} \frac{\left(3M_H^4 + 4M_H^2 M_X^2 + 8M_X^4\right)}{F^8 M_X^2} \sqrt{M_X^2 - M_H^2}\, \underline{\left(s - 4M_S^2\right)}^{\frac{3}{2}}$$

scalar relic abundance

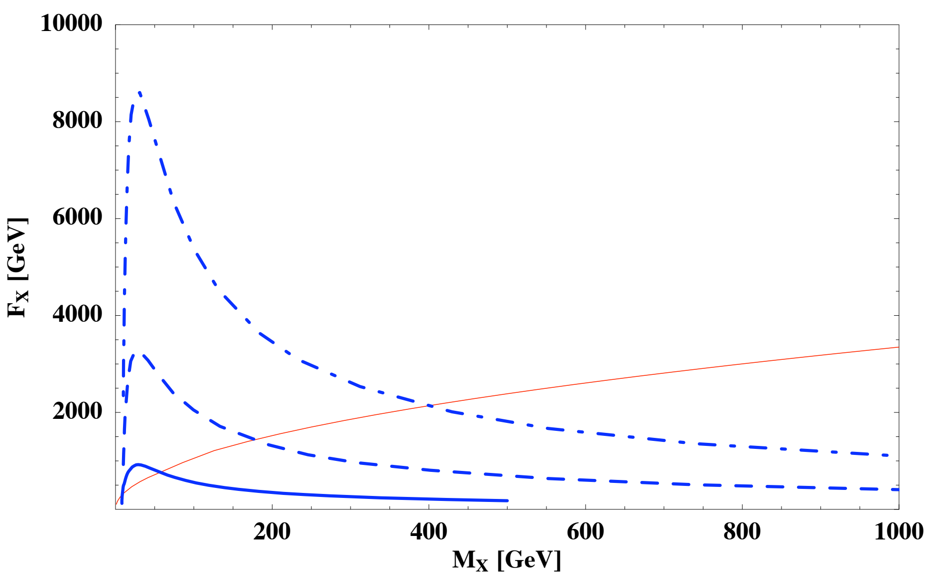


- Scalar annihilation cross-section is proportional to v^3 versus $1/v$ for vector.
- To obtain observed relic density, F is only slightly larger than M_X.
- Situation is different with interaction $L = \frac{1}{8F^4} M_S^2 S_i S_i \eta^{\mu\nu} T_{\mu\nu}$

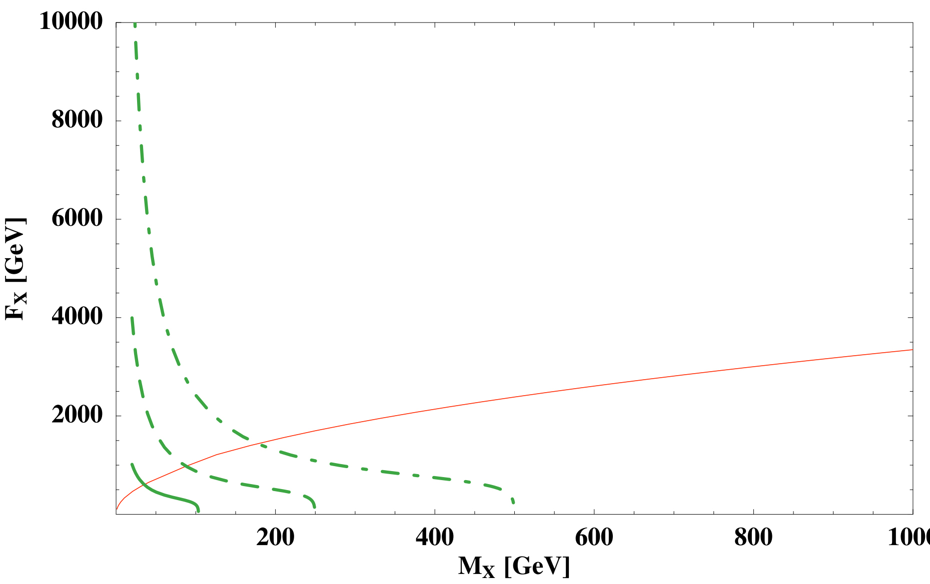
Cembranos, Dobado, and Maroto; Phys. Rev. D68:103505,2003.

Future reach

SuperCDMS



International Linear Collider



Plan A: 25 kg of Ge

Plan C: 1000 kg of Ge, 500 day ton exposure.

astro-ph/0502435, SuperCDMS Collaboration

ILC-1: 500 GeV, 200 fb^{-1}.

ILC-2: 1000 GeV, 1000 fb^{-1}.

Thomas Clark's talk, earlier today

Discussion

- Our approach: investigate experimental and observational consequences of the existence of extra dimensions through universal features that are only related to symmetry breaking patterns and that therefore are not dependent on the details of short distance physics.

- These features may give hints about extra dimensions without the necessity to produce the heavy degrees of freedom that propagate into the bulk.

- The parameters of the effective theory that describes the Standard Model on a brane can be constrained by a combination of accelerator experiments and astrophysical observations.