TESTING
LORENTZ SYMMETRY VIOLATING NEUTRINO OSCILLATIONS†

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PHENO '07

Madison, WI

7May2007

- The Standard Model Extension (SME)
- Lorentz invariance and CPT violation in neutrinos
- The bicycle model
- Conclusions

† With V. Barger and D. Marfatia, in preparation

Sources for Lorentz invariance violation

- Minimal $SU(3) \times SU(2) \times U(1)$ SM is likely the low-energy limit of a more fundamental theory that includes some form of quantum gravity
- Natural scale for such a theory is the Planck mass, M_P
- Vestiges of underlying theory might remain at low energies, suppressed by M_P
- E.g., in a covariant string theory Lorentz symmetry can be spontaneously broken if tensor fields achieve a negative vev (Kostelecky & Samuel)
- Because Lorentz symmetry is broken spontaneously, the effective low-energy theory remains invariant under *observer* Lorentz transformations
- Likewise, 4-momentum conservation and gauge symmetry are also preserved
- Besides string theory, other possible sources include

    Non-string quantum gravity

    Non-commutative field theory

Standard Model Extension (SME) (Colladay & Kostelecky)

- *Particle* Lorentz transformations that leave background vevs unchanged may be affected
- Introduces new terms in equations of motion for a free particle

$$
\begin{aligned}
(i\Gamma^{\nu}_{ab}\partial_{\nu} &- M_{ab})\nu_b = 0 \qquad\qquad \text{a, b} \;=\; \text{flavor indices} \\
\Gamma^{\nu}_{ab} &= \gamma^{\nu}\delta_{ab} + (c_L^{\mu\nu})_{ab}\frac{1}{2}\gamma_{\mu}(1-\gamma_5) + (c_R^{\mu\nu})_{ab}\frac{1}{2}\gamma_{\mu}(1+\gamma_5) + e^{\nu}_{ab} + i\; f_{ab}\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}_{ab}\sigma_{\lambda\mu} \\
M_{ab} &= m_{ab} + im_{5ab}\gamma_5 + (a^{\mu}_L)_{ab}\frac{1}{2}\gamma_{\mu}(1-\gamma_5) + (a^{\mu}_R)_{ab}\frac{1}{2}\gamma_{\mu}(1+\gamma_5) + \frac{1}{2}H^{\mu\nu}_{ab}\sigma_{\mu\nu}
\end{aligned}
$$

- SME: all Lorentz symmetry-breaking terms that preserve $SU(3) \times SU(2) \times U(1)$
- Corresponding Hamiltonian for ν_L propagation in SME:

$$
(h_{eff})_{ab} = |\vec{p}|\;\delta_{ab} + \frac{(m^2)_{ab}}{2|\vec{p}|} + \frac{1}{|\vec{p}|}\left[(a_L)^{\mu}p_{\mu} - (c_L)^{\mu\nu}p_{\mu}p_{\nu}\right]_{ab}
$$

- For $\bar{\nu}$, $a_L \rightarrow -a_L$ (CPT violation)
- CPT violation requires Lorentz invariance violation, but not vice versa (Greenberg)

- For relativistic ν's, $|\vec{p}| \simeq E$ and $p^\mu = (E, -E\ \hat{p})$

$$(h_{eff})_{ab} = |\vec{p}|\ \delta_{ab} + \frac{(m^2)_{ab}}{2E} + \left[a_L^T + \vec{a}_L \cdot \hat{p} + E(c_L^{TT} + 2\vec{c}_L \cdot \hat{p} + \hat{p} \cdot \mathbf{c}_L \cdot \hat{p})\right]_{ab}$$

where

$$c_L^{\mu\nu} = \begin{pmatrix} c^{TT} & \vec{c}_L \\ \vec{c}_L^{\ t} & \mathbf{c}_L \end{pmatrix}$$

- a_L terms are energy-independent (dimensions of E), c_L terms proportional to E (dimensionless)

  a_L^T, c_L^{TT} are direction independent

  $\vec{a}_L, \vec{c}_L, \mathbf{c}_L$ are direction dependent

- Direction-dependent coefficients imply a preferred direction

$$\begin{aligned} \vec{a}_L &= |\vec{a}_L|\ \hat{n} \\ \vec{a}_L \cdot \hat{p} &= |\vec{a}_L|\ \hat{n} \cdot \hat{p} \\ \hat{n} \cdot \hat{p} &\equiv \cos\Theta \end{aligned}$$

Bicycle Model (Kostelecky & Mewes)

- Simple two-parameter model without ν mass
- Attempts to explain $1/E$ behavior in atmos ν osc. *and* E dependence in solar ν osc.
- Non-zero coefficients $(c_L^{TT})_{ee} = 2c$, $(\vec{a}_L)_{e\mu} = (\vec{a}_L)_{e\tau} = \frac{1}{\sqrt{2}} a\ \hat{n}$

$$h_{eff} = \begin{pmatrix} -2cE & \frac{1}{\sqrt{2}} a \cos\Theta & \frac{1}{\sqrt{2}} a \cos\Theta \\ \frac{1}{\sqrt{2}} a \cos\Theta & 0 & 0 \\ \frac{1}{\sqrt{2}} a \cos\Theta & 0 & 0 \end{pmatrix}$$

- Eigenvalues $\lambda_i = 0, -cE \pm \sqrt{(cE)^2 + a^2 \cos^2\Theta}$ $\qquad \lambda_i - \lambda_j = \Delta_{ij} = \frac{1}{2E}(\delta m^2_{eff})_{ij}$
- Mixing matrix

$$U = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ -\frac{1}{\sqrt{2}}\sin\theta & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\cos\theta \\ -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\cos\theta \end{pmatrix}, \qquad \sin\theta = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{cE}{\sqrt{(cE)^2 + a^2\cos^2\Theta}}}$$

- $\theta_{13} = \theta, \theta_{23} = \frac{\pi}{4}, \theta_{12} = 0$

Atmospheric and Long-Baseline (LBL) Neutrinos

- In large E limit ($cE \gg a$) there is a see-saw mechanism

$$\Delta_{32} \simeq a^2 \cos^2\Theta/(2cE)$$

$$\Delta_{31} \simeq \Delta_{21} \simeq 2cE$$

$$\sin^2\theta \simeq a^2\cos^2\Theta/(4c^2E^2) \lesssim 5\times 10^{-5} \text{ for } E = 1 \text{ GeV}$$

- Oscillation probabilities

$$P(\nu_\mu \to \nu_\tau) = \sin^2(\Delta_{32}\tfrac{L}{2}) \qquad \Longrightarrow \qquad \text{maximal mixing!}$$

$$P(\nu_e \to \nu_\mu) = P(\nu_e \to \nu_\tau) = \sin^2\theta\,\sin^2(\Delta_{31}\tfrac{L}{2}) \qquad \Longrightarrow \qquad \text{very small mixing}$$

- Define $a^2/c \equiv m_0^2$

$\Longrightarrow \qquad (\delta m^2_{eff})_{32} = m_0^2\cos^2\Theta$ (independent of E, but not $\hat{n}$ and $\hat{p}$)

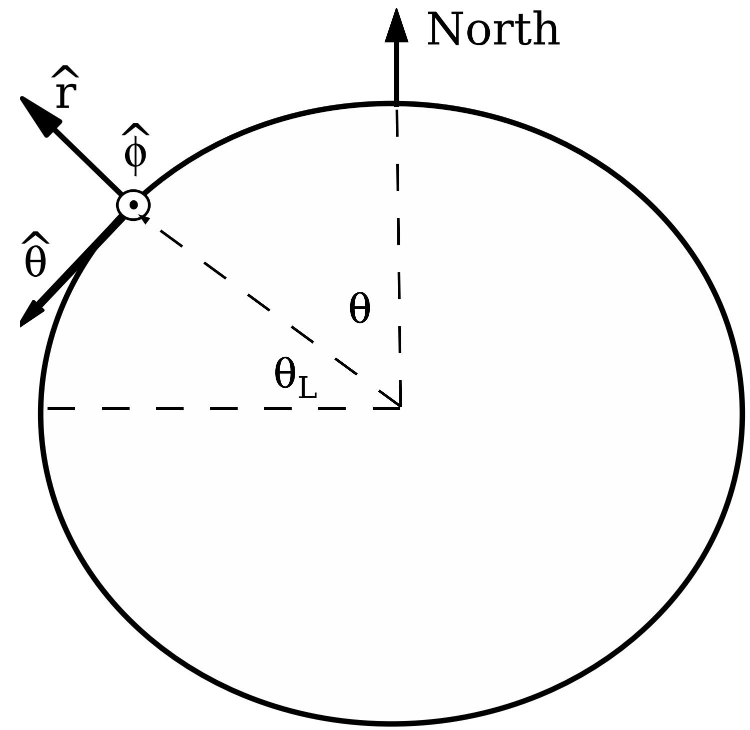
- Must determine $\cos\Theta$

- Detector position on Earth defined by (θ, ϕ) w.r.t equatorial celestial coordinates

  $\hat{r}$: upward

  $\hat{\theta}$: south

  $\hat{\phi}$: east



  $\theta_L = \frac{\pi}{2} - \theta =$ latitude of detector

  ϕ depends on time of *sidereal* day
  (w.r.t. celestial coord.)

- In these coordinates, ν direction is

  $\hat{p} = -\cos\beta\ \hat{r} + \sin\beta(-\sin\alpha\ \hat{\theta} + \cos\alpha\ \hat{\phi})$

  $\beta =$ zenith angle of ν ($\beta = 0$ for downward event)

  $\alpha =$ compass direction of ν velocity projected onto plane tangent to Earth's surface
  ($\alpha = 0$ for eastward event)

- Preferred direction: $\hat{n} = \sin\xi\cos\chi\ \hat{X} + \sin\xi\sin\chi\ \hat{Y} + \cos\xi\ \hat{Z}$, $\hat{Z}$ is North

- Choose $\phi = 0$ reference point in preferred direction

- Angular dependence: preferred direction, neutrino direction, detector position

$$\hat{n}\cdot\hat{p} = \cos\Theta \;=\; \cos\xi(\sin\beta\sin\alpha\cos\theta_L - \cos\beta\sin\theta_L)$$
$$- \sin\xi(\sin\beta\sin\alpha\sin\theta_L\cos\phi + \cos\beta\cos\theta_L)$$
$$- \sin\xi\sin\beta\cos\alpha\sin\phi$$

- Special cases

$$\text{Downward}(\beta = 0):\ \cos\Theta = -(\cos\xi\sin\theta_L + \sin\xi\cos\theta_L\cos\phi)$$
$$\text{Upward}(\beta = \pi):\ \cos\Theta = (\cos\xi\sin\theta_L + \sin\xi\cos\theta_L\cos\phi)$$
$$\text{Horizontal}(\beta = \frac{\pi}{2}):\ \cos\Theta = \cos\xi\cos\theta_L\sin\alpha - \sin\xi(\sin\theta_L\cos\phi\sin\alpha + \sin\phi\cos\alpha)$$

- Upward and downward events have same $\cos^2\Theta \Longrightarrow$ same δm^2_{eff}

Case 1: $\xi = 0$ (Kostelecky & Mewes)

- $\cos^2\Theta = (\sin\beta\sin\alpha\cos\theta_L - \cos\beta\sin\theta_L)^2$ (independent of time of day)

$$\text{Atmos } \nu \text{ up/down events} : \cos^2\Theta = \sin^2\theta_L$$
$$\text{LBL (K2K or MINOS)} : \cos^2\Theta = \sin^2\alpha\cos^2\theta_L$$

- Can infer value of $m_0 = a^2/c$ from each experiment

Experiment	θ_L	α	$\cos^2\Theta$	$m_0^2 = \delta m^2_{exp}/\cos^2\Theta$
SuperK up/down	36°	$-$	3.5×10^{-1}	7.0×10^{-3} eV^2
K2K	36°	174°	7.2×10^{-3}	4.0×10^{-1} eV^2
MINOS	48°	124°	4.5×10^{-1}	8.0×10^{-3} eV^2

- Inferred values of m_0^2 inconsistent $\Longrightarrow \xi = 0$ ruled out!

Case 2: $\xi \neq 0$

- Dependence on ϕ (time of sidereal day)
- Up/down atmos ν's

(i) $|\tan\xi| < |\tan\theta_L|$

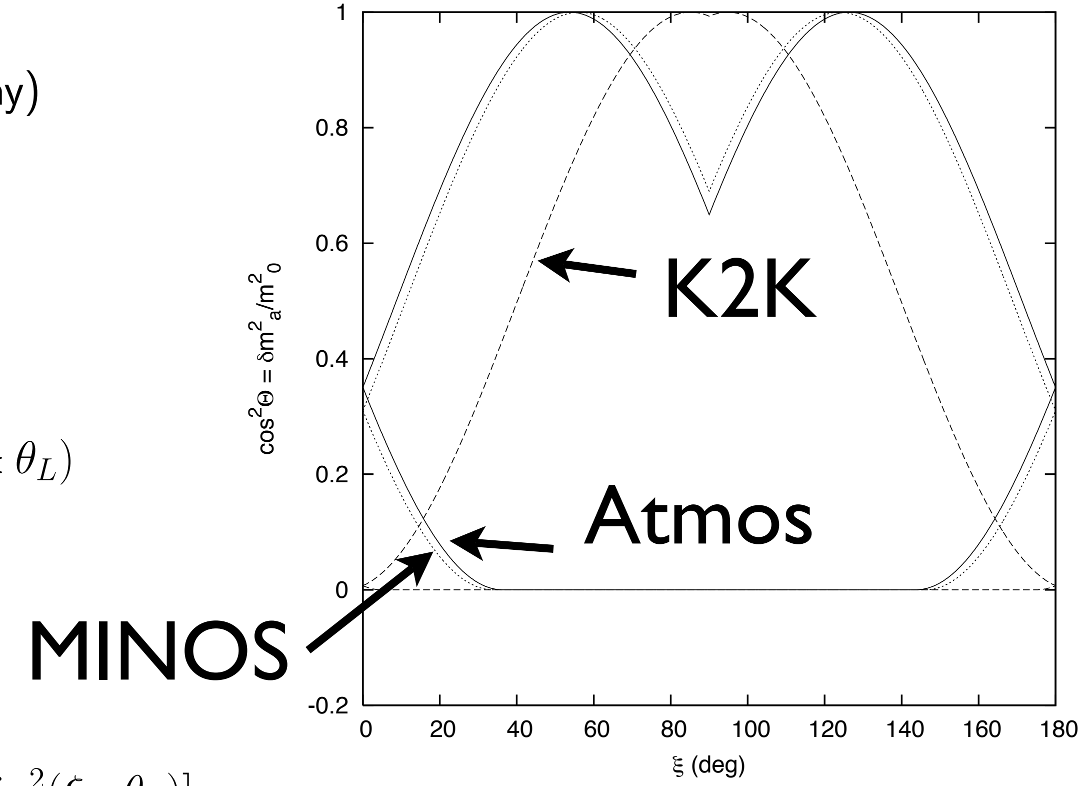
$$\sin^2(\xi \mp \theta_L) \leq \cos^2\Theta \leq \sin^2(\xi \pm \theta_L)$$

(ii) $|\tan\xi| > |\tan\theta_L|$

$$0 \leq \cos^2\Theta \leq \max[\sin^2(\xi+\theta_L), \sin^2(\xi-\theta_L)]$$

- K2K/MINOS: similar situation with $\theta_L \to \alpha$
- Horizontal atmospheric ν events have complicated compass and time-of-day dependence

Daily ranges of $\cos^2\Theta$



- Only $15^\circ \lesssim \xi \lesssim 165^\circ$ possible
- K2K: $\delta m^2_{eff} = 0$ twice per sidereal day
- Atmos ν's/MINOS:
  Large fluctuations in δm^2_{eff} during day
  For $36^\circ \lesssim \xi \lesssim 144^\circ$, $\delta m^2_{eff} = 0$ twice per day
- Hard to test without data vs. sidereal time

Solar ν's

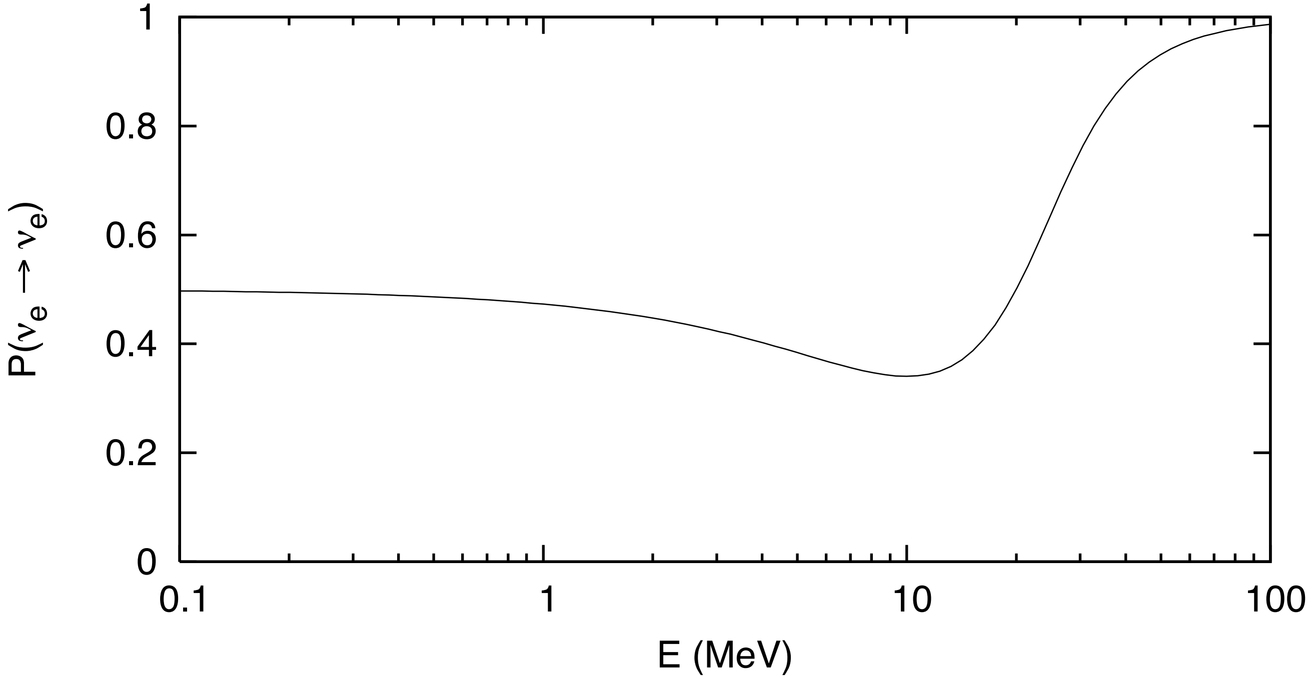
- Include matter effects ($N_e = e$ number density)

$$h_{eff} = \begin{pmatrix} \sqrt{2}G_F N_e - 2cE & \frac{1}{\sqrt{2}}a\cos\Theta & \frac{1}{\sqrt{2}}a\cos\Theta \\ \frac{1}{\sqrt{2}}a\cos\Theta & 0 & 0 \\ \frac{1}{\sqrt{2}}a\cos\Theta & 0 & 0 \end{pmatrix}$$

- resonance occurs for $\sqrt{2}G_F N_e = 2cE$
- $c \sim 10^{-19}$ gives resonance in sun
- For adiabatic propagation, $N_e^0 =$ initial N_e

$$\begin{aligned} P(\nu_e \to \nu_e) &= \cos^2\theta\cos^2\theta_0 + \sin^2\theta\sin^2\theta_0 \\ &= \frac{1}{2} + \frac{1}{2}\,\frac{cE(cE - G_F N_e^0/\sqrt{2})}{\sqrt{(cE)^2 + a^2\cos^2\Theta}\sqrt{(cE - G_F N_e^0/\sqrt{2})^2 + a^2\cos^2\Theta}} \end{aligned}$$

- For ν's starting above resonance, $P < \frac{1}{2}$



- Position of minimum in probability fixed by c:

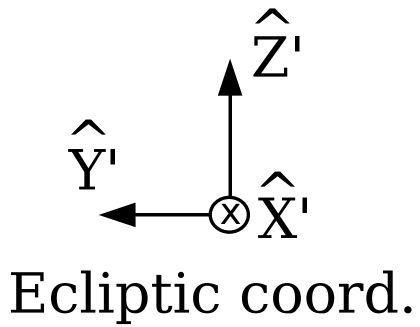
$$E_{min} = \frac{G_F N_e^0}{2\sqrt{2}} \frac{1}{c}$$

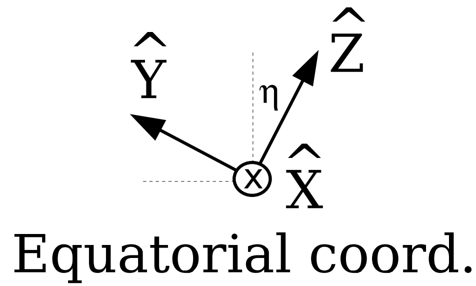
- No E dependence seen in ^{8}B ν's

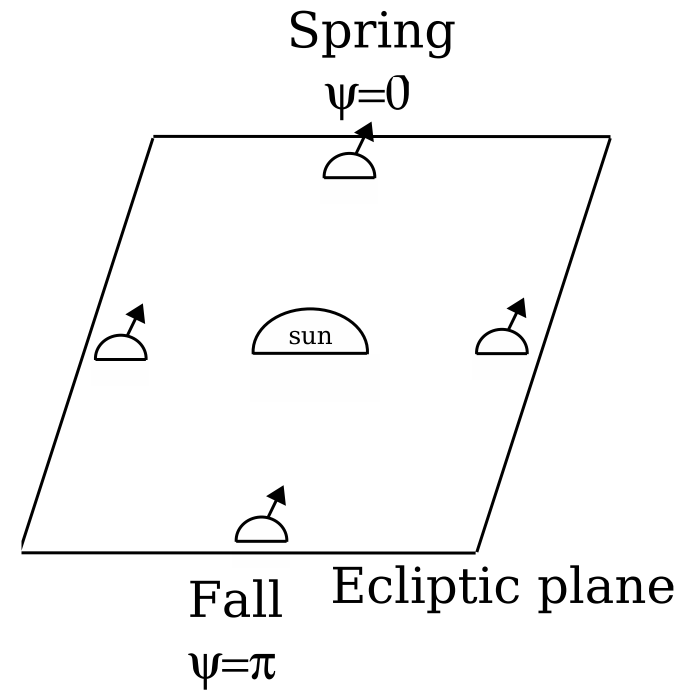
  In order to fit data, E_{min} must lie in middle of SuperK and SNO spectra

  $E_{min} \simeq 10$ MeV $\quad \Longrightarrow \quad c \simeq 1.7 \times 10^{-19}$ (independent of ν direction)

- Probability at minimum fixed by $|a \cos \Theta|$ (depends on neutrino direction)







- Solar ν's propagate in ecliptic plane

$$\hat{p} \;=\; \cos\psi\ \hat{X}' + \sin\psi\ \hat{Y}' = \cos\psi\ \hat{X} + \sin\psi(\cos\eta\ \hat{Y} + \sin\eta\ \hat{Z})$$

$$\cos\Theta = \hat{n}\cdot\hat{p} \;=\; \cos\psi\cos\chi\sin\xi + \sin\psi(\sin\chi\sin\xi\cos\eta - \cos\xi\sin\eta)$$

- Averaged over year, $\langle P_{min}\rangle$ depends on a and preferred direction (ξ,χ)

$$\langle P_{min}\rangle = \frac{1}{2}\left[1 - \frac{G_F N_e^0}{\sqrt{(G_F N_e^0)^2 + 8a^2 D^2}}\right] \qquad D^2 \;\equiv\; \cos^2\chi\sin^2\xi + (\sin\chi\sin\xi\cos\eta - \cos\xi\sin\eta)^2$$

- $\langle P_{min}\rangle \simeq 0.34 \qquad \Longrightarrow \qquad a = (5.0\times 10^{-12}\ \text{eV})/D$

Annual variations in solar ν probability

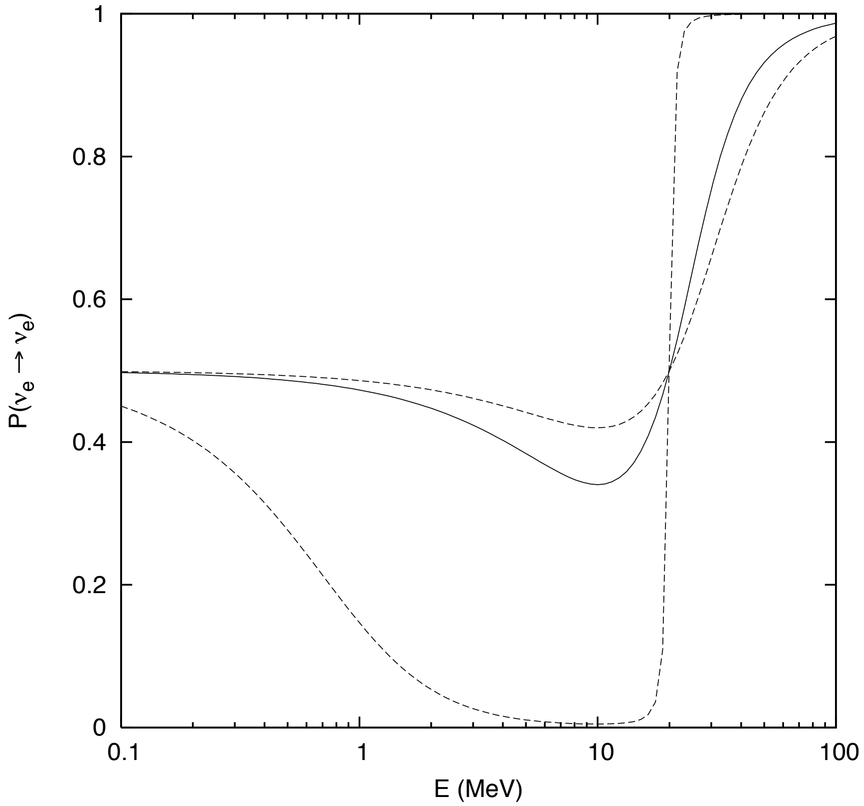
- Remember for solar ν's

$$\cos\Theta = \cos\chi\sin\xi\cos\psi + (\sin\chi\sin\xi\cos\eta - \cos\xi\sin\eta)\sin\psi \equiv A\cos\psi + B\sin\psi$$

If $A \equiv D\sin\delta,\ B \equiv D\cos\delta$ and $D \equiv \sqrt{A^2+B^2}$, then

$$\cos\Theta = D\sin(\psi+\delta)$$

- *Always* have $\cos\Theta = 0$ twice during year ($\psi = -\delta$ and $\pi - \delta$)
- $P_{min} = 0$ for ν's starting above resonance



- Can show

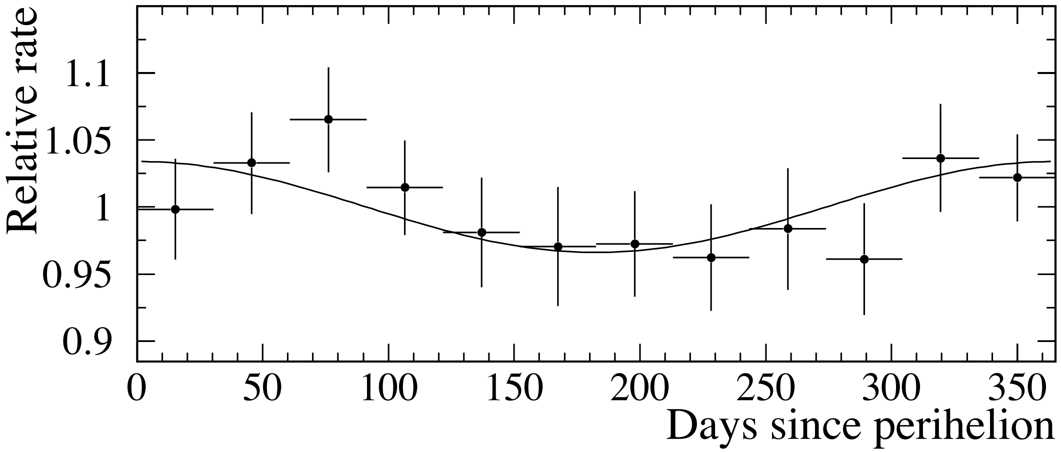
$$0 \le P_{min} \le 2\langle P_{min}\rangle(1-\langle P_{min}\rangle) \simeq 0.45$$
$$0 \le \frac{P_{min}}{\langle P_{min}\rangle} \le 2(1-\langle P_{min}\rangle) \simeq 1.32$$

Test annual variation with SNO data

- SNO measured time dependence of combined data (CC + NC + ES + Bckgrd)

$$R = \frac{N(t)}{\langle N \rangle} = \frac{N^0_{NC} + N^0_{CC}P + N^0_{ES}[P + r(1-P)] + N^0_B}{N^0_{NC} + N^0_{CC}\langle P \rangle + N^0_{ES}[\langle P \rangle + r(1-\langle P \rangle)] + N^0_B} \qquad r = \frac{\sigma_{NC}}{\sigma_{CC}} \simeq \frac{1}{6.48}$$

- D_2O phase (572 d) $N_{CC} : N_{NC} : N_{ES} : N_B = 1968 : 576 : 264 : 116$
- Salt phase (763 d) $N_{CC} : N_{NC} : N_{ES} : N_B = 2176 : 2010 : 279 : 257$



- SNO data clearly shows $1/r^2$ dependence due to Earth's eccentric orbit
- For $\langle P_{min} \rangle = 0.34$, bicycle model predicts $0.42 \leq R \leq 1.19$
  $\Longrightarrow$ direction-dependent bicycle model excluded by SNO data!

Direction-independent bicycle model

- Replace $(\vec{a}_L)_{e\mu} = (\vec{a}_L)_{e\mu}$ with $(a^T)_{e\mu} = (a^T)_{e\tau} = a/\sqrt{2}$
- Equivalent to $\cos\Theta \equiv 1$ in direction-dependent case
- Atmos and LBL now agree

$$P(\nu_\mu \to \nu_\tau) = \sin^2(\delta m^2_{eff} L/4E) \quad \text{with} \quad \delta m^2_{eff} = a^2/c$$

- No annual variation in solar ν's

$$E_{min} = \frac{G_F N_e^0}{2\sqrt{2}} \frac{1}{c} \simeq 10 \text{ MeV} \quad \Longrightarrow \quad c = 1.7 \times 0^{-19}$$

$$P_{min} = \frac{1}{2} \frac{8a^2}{8a^2 + (G_F N_e^0)^2} \simeq 0.34 \quad \Longrightarrow \quad a = 2.5 \times 10^{-12} \text{ eV}$$

$\Longrightarrow$ predicts $\delta m^2_{atm} = 3.6 \times 10^{-5}$ eV^2

Inconsistent with measured values for atmos and LBL ν's

- Alternatively $a^2/c \simeq 2.5 \times 10^{-3}$ eV^2 $\Longrightarrow$ $a = 2.1 \times 10^{-11}$ eV $\Longrightarrow$ $P_{min} = 0.497$

Inconsistent with $P = 0.34$ in SNO

Open questions

- Partial direction dependence?

  $(a_L)^\mu p_\mu/E \rightarrow a^T \pm |\vec{a}| \cos\Theta$

  Lack of annual variation in SNO $\implies$ $|\vec{a}|^2 \ll (a^T)^2$

  Similar to direction-independent case

  Preliminary indications are that this case is also ruled out

- Other direction-independent models

  Can other 3-ν textures for h_{eff} (including cE, a, $and\, m^2/E$ terms)
  produce the proper E dependence for both solar and atmos ν's?

  Difficult to produce a see-saw for atmos ν's and proper E dependence for solar ν's

Conclusions

- SME allows new terms in effective Hamiltonian for ν propagation

- Different energy dependence from ordinary oscillations (cE, a vs. m^2/E)

- Bicycle model has no ν mass terms, but reproduces $1/E$ dependence at high E from see-saw

  Pure direction-dependent case ruled out from lack of annual variation in SNO data

  Pure direction-independent case ruled out from conflict between solar and atmos ν data

- Large parameter space remains

- Direction-dependent models face severe experimental constraints