Neutrino states in oscillation experiments

– are they pure or mix?

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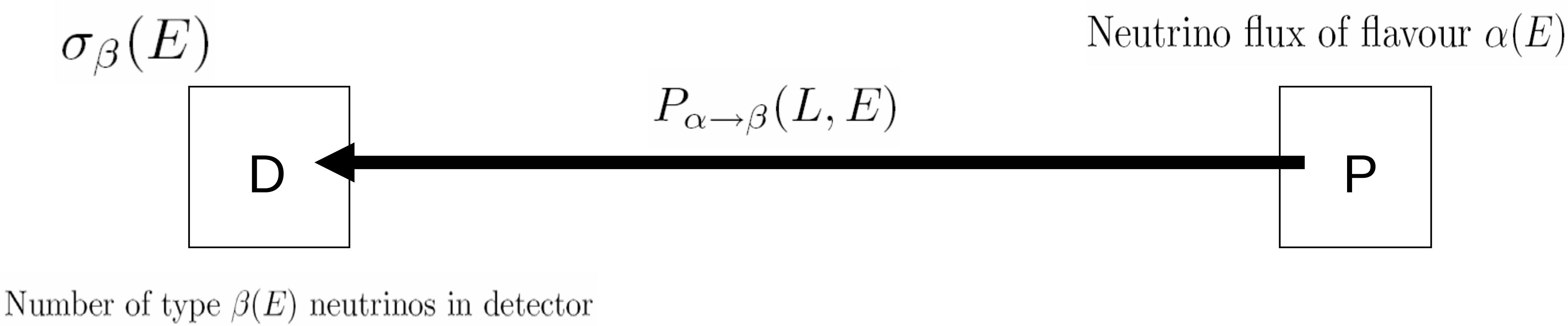
1. INTRODUCTION

Common approach to oscillation phenomena



$$\text{Number of type } \beta(E) \text{ neutrinos in detector} = \text{Neutrino flux of flavour } \alpha(E) \times P_{\alpha\to\beta}(L,E) \times \sigma_\beta(E)$$

FLUX → $\sigma_\alpha(E)$

DETECTOR → $\sigma_\beta(E)$

Calculated for massless neutrinos

1) For production and detection cross section - massless neutrino

$$L_{CC} = \frac{e}{2\sqrt{2}\ sin\theta_W} \sum_{\alpha} \overline{\nu}_\alpha \gamma^\mu (1-\gamma_5) l_\alpha W^+_\mu + h.c.$$

2) Factoryzation

$$N_{\beta\alpha} = Flux(\alpha) \times P_{\beta\alpha} \times \sigma_\beta$$

3) Transition probability

$$P_{\alpha\to\beta}(L) = |A_{\alpha\to\beta}(L)|^2$$

$$A_{\alpha\to\beta}(L) = \langle \nu_\beta(0) | \nu_\alpha(L) \rangle = \langle \nu_\beta(0) | e^{-iH(t=L)} | \nu_\alpha(L) \rangle$$

↑ In vacuum or in matter

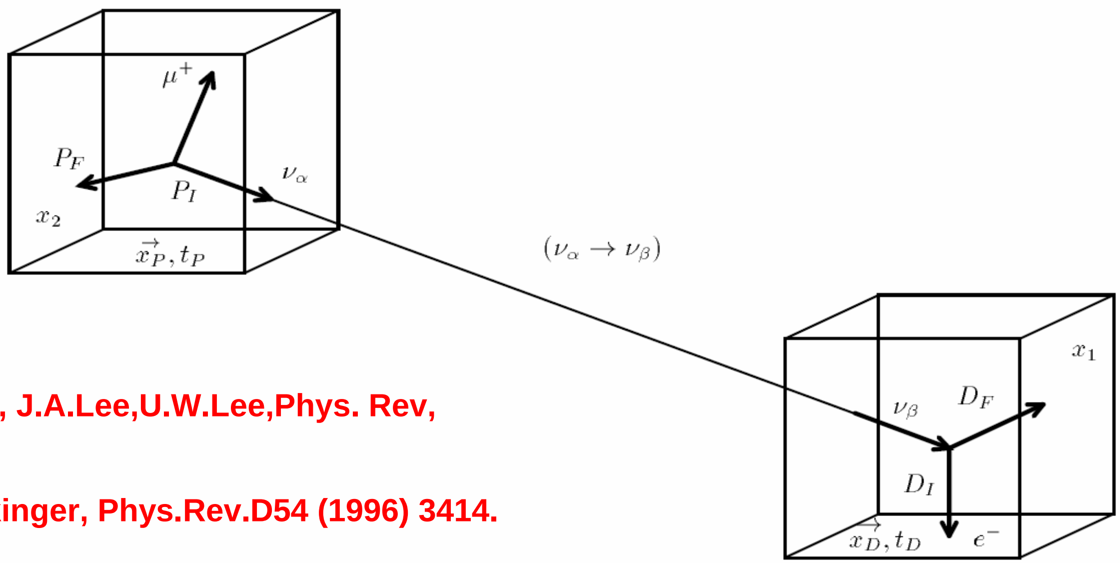
Where flavour states are given by

Z.Maki,M.Nakagawa,S. Sakata, Prog.Theor.Phys. 28(1962)870

$$|\nu_\alpha(0)\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle$$

How we can convince that it is correct?   Full Quantum field theoretical treatment

$$P_I \to P_F + l^+_\alpha + \nu_\alpha \overset{(\nu_\alpha \to \nu_\beta)}{\longrightarrow} \nu_\beta + D_I \to D_F + l^-_\beta$$



C.Giunti, C.W.Kim, J.A.Lee,U.W.Lee,Phys. Rev, D48(1993) 4310.

W.Grimus,P.Stockinger, Phys.Rev.D54 (1996) 3414.

Neutrino propagate over macroscopic distance (sometimes astronomical) → it is unnatural to consider them as virtual

Quantum-field-theoretical model of neutrino oscillation in which the propagating neutrino is described by a wave packet state determined by the production process

C.Giunti JHEP 0211(2002)017

For the process:

$$P_I \longrightarrow P_F + \ell_\alpha^+ + \nu_\alpha$$

It was proposed:

$$|\widetilde{P}_F, \widetilde{\ell}_\alpha^+, \widetilde{\nu}_\alpha\rangle \propto -i \int \mathrm{d}^4x \, \mathcal{H}_I^P(x) \, |P_I\rangle$$

And finally the neutrino states are given by:

$$|\nu_\alpha\rangle = N_\alpha \sum_a U_{\alpha a}^* \int \mathrm{d}^3p \, e^{-S_a^P(\vec{p})} \sum_h \mathcal{A}_a^P(\vec{p}, h) \, |\nu_a(\vec{p}, h)\rangle$$

This approach is not fully correct:

1) Particles which take part in the production and detection processes have spins, we don’t known what to do with them,

2) All time the neutrino state is pure quantum mechanical states, even for non relativistic neutrinos,

3) We don’t know how to incorporate physics beyond tha SM.

We propose to use density matrix approach, then

1. **We know what to do with any properties of accompanied particles,**
2. **We can check, when neutrino state is pure, and when it is mixed,**
3. **Any New Physic (NP) in neutrino interaction can be easy considered,**
4. **In very natural way we are able to take into account neutrino space localization (wave packet approach) ,**
5. **We exactly know, when the formula for neutrino transition factorize,**
6. **For relativistic neutrino and their SM Left-Handed interaction, we reproduce the standard formulae**

2. DENSITY MATRIX FOR PRODUCED NEUTRINOS

We consider production neutrino process:

$$l_\alpha + 1 \to \nu_i + 2$$

$$\alpha = e, \mu, \tau$$

$$i = 1, 2, 3$$

For each particle (without neutrino) we introduce wave packet (given by experimental condition):

$$|\chi_l, \lambda_l\rangle = \int d^3 p_l \Psi_l(\vec{p_l}, \vec{p_{l0}}, \sigma_{lp}) |\vec{p_l}, \lambda_l\rangle$$

$$|\chi_1, \lambda_1\rangle = \int d^3 p_1 \Psi_1(\vec{p_1}, \vec{p_{10}}, \sigma_{1p}) |\vec{p_1}, \lambda_1\rangle$$

$$|\chi_2, \lambda_2\rangle = \int d^3 p_2 \Psi_2(\vec{p_2}, \vec{p_{20}}, \sigma_{2p}) |\vec{p_2}, \lambda_2\rangle$$

In momentum representation: $$\langle \vec{p}, \lambda | \chi, \lambda' \rangle = \delta_{\lambda\lambda'} \Psi(\vec{p}, \vec{p}_{m0}, \sigma_{pm})$$

$$\Psi_m(\vec{p}_m, \vec{p}_{m0}, \sigma_{mp}) = \frac{1}{(2\pi\sigma_{mp}^2)^{\frac{3}{4}}} e^{-\frac{(\vec{p}_m - \vec{p}_{m0})^2}{4\sigma_{mp}^2}}$$

Final results do depend on the shape of wave pockets - we use Gauss distribution.

In coordinate space: $$\langle \vec{x}, \lambda | \chi, \lambda' \rangle = \delta_{\lambda\lambda'} \Psi(\vec{x}, t = 0; \vec{p}_{m0}, \sigma_{xm})$$

$$\langle \vec{x}, \lambda | \chi, \lambda' \rangle = \delta_{\lambda\lambda'} \frac{1}{(2\pi\sigma_{xm}^2)^{\frac{3}{4}}} e^{i\vec{p}_{m0}\vec{x}} e^{-\frac{\vec{x}^2}{4\sigma_{xm}^2}}$$

$$\sigma_{pm}\sigma_{xm} = \frac{1}{2}$$

We calculate:

$$\langle f|S|i\rangle = (\langle \chi_l, \lambda_l| \otimes \langle \chi_1, \lambda_1|) e^{-\int d^4x H(x)} (|\chi_\nu, \lambda_\nu\rangle \otimes |\chi_2, \lambda_2\rangle)$$

Let us assume the effective Hamiltonian:

$$H = g(\bar{l} O_{l\nu} \nu)(\bar{u} O_{du} d) + g^*(\bar{\nu} \bar{O}_{l\nu} l)(\bar{d} \bar{O}_{du} u)$$

$$\bar{O} = \gamma_0 O^+ \gamma_0$$

Then:

$$\langle f|S|i\rangle = -i \int d^3p_2 \int d^3p_1 \int d^3p_l \Psi_2^* \Psi_1 \Psi_l \int d^4x e^{ix(p_l + p_1 - p_\nu - p_2)}$$

$$\times g^*(\bar{u}(\vec{p}_\nu, \lambda_\nu) \bar{O}_{l\nu} u(\vec{p}_l, \lambda_l))(\bar{u}(\vec{p}_2, \lambda_2) \bar{O}_{du} u(\vec{p}_1, \lambda_1))$$

First we integrate over particle momenta:

Or in the other way:

$$\langle f|S|i\rangle = -i\int d^4x e^{-ixp_\nu}$$

$$\times\int d^3p_2\Psi_2(\vec{p_2},\vec{p_{20}},\sigma_{2p})e^{-ixp_2}$$

$$\times\int d^3p_1\Psi_1(\vec{p_1},\vec{p_{10}},\sigma_{1p})e^{ixp_1}$$

$$\times\int d^3p_l\Psi_l(\vec{p_l},\vec{p_{l0}},\sigma_{lp})e^{ixp_l}$$

$$\times M(\vec{p}_l,\vec{p}_1,\vec{p}_\nu,\vec{p}_2,\lambda_l,\lambda_1,\lambda_\nu,\lambda_2)$$

$$E(\overrightarrow{p}) \simeq E_0 + \overrightarrow{v}_0(\overrightarrow{p} - \overrightarrow{p}_0)$$

First we integrate over particle momenta, using:

$$\int d^3p\, \psi_\chi(\overrightarrow{p};\overrightarrow{p}_0,\sigma_p)e^{-ipx} \simeq \frac{1}{(2\pi\sigma_x^2)^{\frac{3}{4}}} e^{-i(E_0 t - \overrightarrow{p}_0 \overrightarrow{x})}\ e^{-\frac{(\vec{x}-\vec{v}_0 t))^2}{4\sigma_x^2}}$$

We obtain:

$$\langle f|S|i\rangle \simeq -i\frac{1}{[(2\pi\sigma_{2x}^2)(2\pi\sigma_{1x}^2)(2\pi\sigma_{lx}^2)]^{\frac{3}{4}}}\int d^4x e^{-ixp_\nu}$$

$$\times e^{-i(E(\vec{p}_{20})t-\vec{x}\vec{p}_{20})} e^{-\frac{(\vec{x}-\vec{v}_{20}t)^2}{4\sigma_{2x}^2}}$$

$$\times e^{i(E(\vec{p}_{10})t-\vec{x}\vec{p}_{10})} e^{-\frac{(\vec{x}-\vec{v}_{10}t)^2}{4\sigma_{1x}^2}}$$

$$\times e^{i(E(\vec{p}_{l0})t-\vec{x}\vec{p}_{l0})} e^{-\frac{(\vec{x}-\vec{v}_{l0}t)^2}{4\sigma_{lx}^2}}$$

$$\times M(\vec{p}_{l0},\vec{p}_{10},\vec{p}_\nu,\vec{p}_{20},\lambda_l,\lambda_1,\lambda_\nu,\lambda_2)$$

If we introduce:

$$E_R = E(\vec{p}_{l0}) - E(\vec{p}_{20}) + E(\vec{p}_{10}) \qquad \vec{p}_R = \vec{p}_{l0} - \vec{p}_{20} + \vec{p}_{10}$$

$$\frac{1}{\sigma_{Rx}^2} = \frac{1}{\sigma_{lx}^2} + \frac{1}{\sigma_{2x}^2} + \frac{1}{\sigma_{1x}^2}$$

$$\vec{v}_R = \sigma_{Rx}^2 \left(\frac{\vec{v}_{l0}}{\sigma_{lx}^2} + \frac{\vec{v}_{20}}{\sigma_{2x}^2} + \frac{\vec{v}_{10}}{\sigma_{1x}^2} \right)$$

$$\Sigma_R = \sigma_{Rx}^2 \left(\frac{\vec{v}_{l0}^2}{\sigma_{lx}^2} + \frac{\vec{v}_{20}^2}{\sigma_{2x}^2} + \frac{\vec{v}_{10}^2}{\sigma_{1x}^2} \right)$$

We can integrate over d^4x:

$$\langle f|S|i\rangle = \frac{2^{\frac{5}{4}}\left[\sigma_{lp}\sigma_{1p}\sigma_{2p}\right]^{3/2}}{\pi^{\frac{1}{4}}\sigma_{Rp}^{4}\sqrt{\Sigma_R - \vec{v}_R^2}}$$

$$\times e^{-\frac{(\vec{p}_\nu - \vec{p}_R)^2}{4\sigma_{Rp}^2} - \frac{(-\vec{v}_R(\vec{p}_\nu - \vec{p}_R) + E_\nu - E_R)^2}{\sigma_{Rp}^2\left(\Sigma_R - \vec{v}_R^2\right)}}$$

$$\times M\left(\vec{p}_{l0}, \vec{p}_{10}, \vec{p}_\nu, \vec{p}_{20}, \lambda_l, \lambda_1, \lambda_\nu, \lambda_2\right) =$$

$$= N_{fi}\ f_i(\vec{p})\ M_i^\alpha(\vec{p}, \lambda; \lambda_2, \lambda_1, \lambda_l) \equiv A_i^\alpha(\vec{p}, \lambda; \lambda_2, \lambda_1, \lambda_l)$$

Where

$$f_i(\vec{p}) = e^{-\frac{(\vec{p}-\vec{p}_R)^2}{4\sigma^2_{pR}} - \frac{(-\vec{v}_R(\vec{p}-\vec{p}_R)-E_i-E_R)^2}{4\sigma^2_{pR}(\Sigma_R-\vec{v}^2_R)}}$$

$$N_{fi} = \frac{2^{\frac{5}{4}}(\sigma_{lP}\sigma_{2P}\sigma_{1P})^{\frac{3}{2}}}{\pi^{\frac{1}{4}}\sigma^4_{pR}\sqrt{\Sigma_R - \vec{v}^2_R}}$$

$$M_i^\alpha(\vec{p}, \lambda; \lambda_2, \lambda_1, \lambda_l) \equiv M(\vec{p}_{l0}, \vec{p}_{10}, \vec{p}_\nu, \vec{p}_{20}, \lambda_l, \lambda_1, \lambda_\nu, \lambda_2)$$

$$\vec{p} = \vec{p}_\nu \qquad \lambda = \lambda_\nu$$

Let us assume now that initial particles are not polarized, we can define density matrix for final neutrino:

$$\varrho^{\alpha}(\vec{p},\lambda,i;\vec{p'},\lambda',i') = \frac{1}{N_{\alpha}} \sum_{\lambda_2,\lambda_1,\lambda_l} A_i^{\alpha}(\vec{p},\lambda;\lambda_2,\lambda_1,\lambda_l) A_{i'}^{\alpha *}(\vec{p'},\lambda';\lambda_2,\lambda_1,\lambda_l)$$

$$\varrho^{\alpha} = \sum_{\lambda,\lambda',i,i'} \int d^3p\, d^3p'\, \varrho^{\alpha}(\vec{p},\lambda,i;\vec{p'},\lambda',i') |\vec{p},\lambda,i\rangle\langle\vec{p'},\lambda',i'|$$

Normalization condition:

$$Tr(\varrho) = \sum_{\lambda=\pm1} \sum_{i=1}^{3} \int d^3p\ \varrho^{\alpha}(\overrightarrow{p},\lambda,i;\overrightarrow{p},\lambda,i) = 1$$

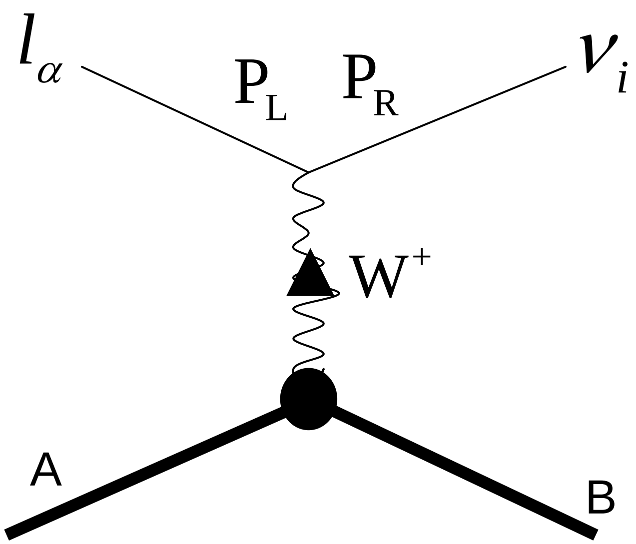
The amplitudes we calculate for general interaction:

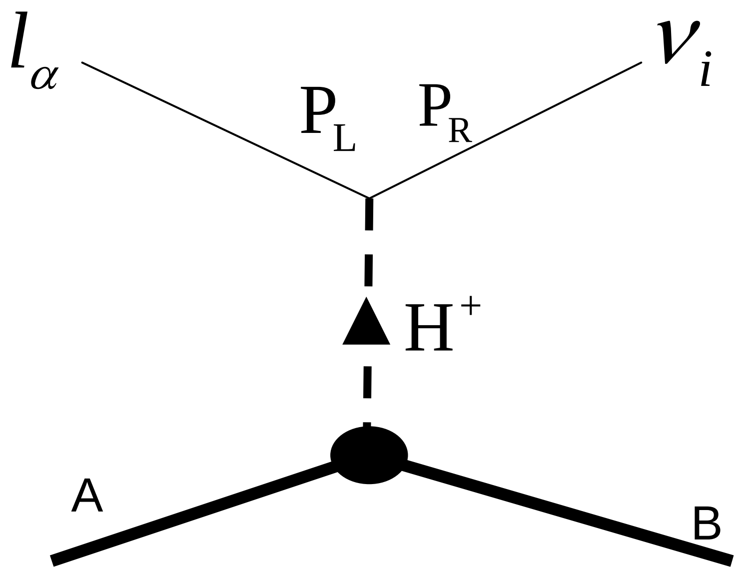
$$\mathcal{L}_{CC} = - \frac{e}{2\sqrt{2}sin\theta_W}\Big\{\sum_{\alpha,i}\bar{\nu_i}\,[\,\gamma^\mu(1-\gamma_5)\epsilon_L^c U_{\alpha i}^{L*} + \gamma^\mu(1+\gamma_5)\epsilon_R^c U_{\alpha i}^{R*}]\; l_\alpha\; W_\mu^+$$

$$+ \sum_{\alpha,i}\bar{\nu_i}\,[\,(1-\gamma_5)\eta_L V_{\alpha i}^{L*} + (1+\gamma_5)\eta_R V_{\alpha i}^{R*}]\; l_\alpha\; H^+$$

$$+ \sum_{u,d}\bar{u}\,[\,\gamma^\mu(1-\gamma_5)\epsilon_L^q U_{ud}^{*} + \gamma^\mu(1+\gamma_5)\epsilon_R^q U_{ud}^{*}]\; d\; W_\mu^+$$

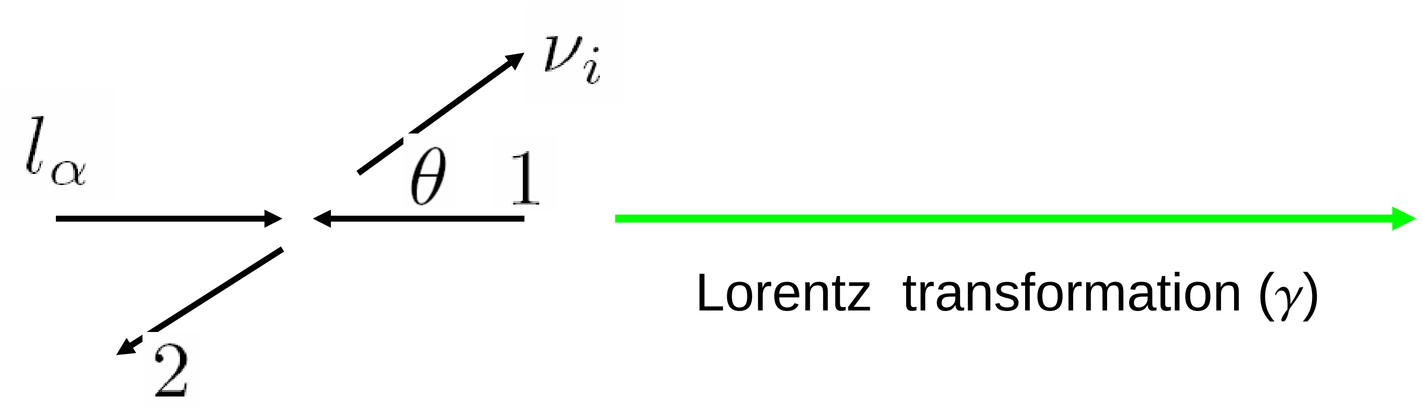
$$+ \sum_{u,d}\bar{u}\,[\,(1-\gamma_5)\tau_L W_{ud}^{L*} + (1+\gamma_5)\tau_R W_{ud}^{R*}]\; d\; H^+\Big\} + h.c$$





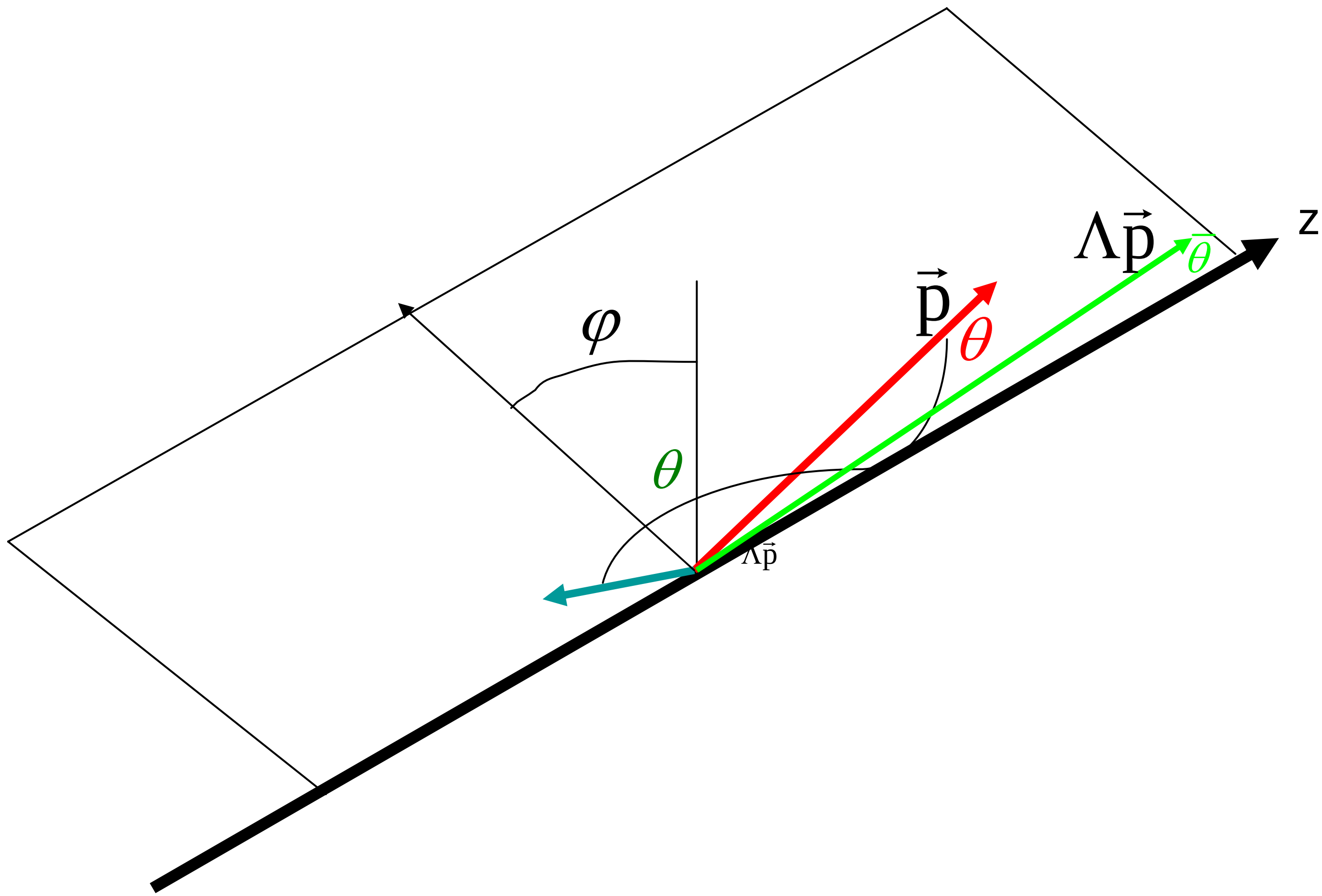
We calculate the amplitude in the CM frame

Everything have to be transformed to the laboratory frame



Helicity states feel Lorentz transformation:

$$
\begin{aligned}
|\overrightarrow{p}, s, \lambda\rangle &= U[L(\overrightarrow{p})]U[R(\varphi, \theta, -\varphi)]|s, \lambda\rangle \\
&= U[R(\varphi, \theta, -\varphi)]U[L_z(p)]|s, \lambda\rangle,
\end{aligned}
$$



$$U[\Lambda]|\overrightarrow{p},s,\lambda\rangle = \sum \mathbf{R}^s_{\sigma\,\lambda}(R(\varphi,\overline{\theta},-\varphi)R_W R(\varphi,\theta,-\varphi)|\overrightarrow{p}'',s,\sigma\rangle$$

$$R_W = L^{-1}(\Lambda\overrightarrow{p})\Lambda L(\overrightarrow{p})$$

Helicity and Wigner rotation

$$D^{\frac{1}{2}}(\overrightarrow{n},\omega) = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$$

$$b(\gamma,\theta,\varphi,E_\nu) = |b|e^{I\varphi}$$

$$|b| = \frac{|S(\theta)[p\beta\gamma + (m+E_\nu\gamma)cos\theta] - C(\theta)(E_\nu + m\gamma)sin\theta|}{\sqrt{2[m+E_\nu\gamma + p\beta\gamma cos\theta][m+E_\nu\gamma + C(\theta)p\beta\gamma + p\beta\gamma cos\theta + C(\theta)(m+E_\nu\gamma)cos\theta + S(\theta)(E_\nu + m\gamma)sin\theta]}}$$

$$a(\gamma,\theta,E_\nu) = \sqrt{\frac{m+E_\nu\gamma + C(\theta)[p\gamma\delta + (m+E_\nu\gamma)cos\theta] + p\gamma\beta cos\theta + S(\theta)(E_\nu + m\gamma)sin\theta}{2[m+E_\nu\gamma + p\gamma\beta cos\theta]}}$$

For Wigner rotation: $S(\theta) \mapsto sin\theta, \quad C(\theta) \mapsto cos\theta$

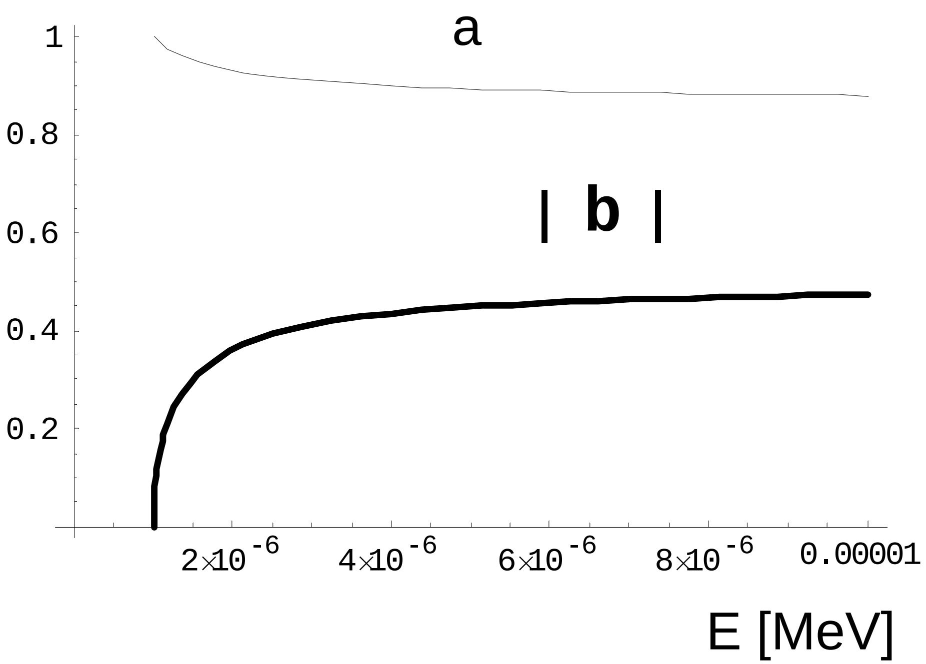
For rotation of helicity states:

$$C(\theta) = \frac{\gamma(E_\nu\beta + p\ cos\theta)}{\sqrt{\gamma^2(E_\nu\beta + p\ cos\theta)^2 + p^2 sin\theta^2}}$$

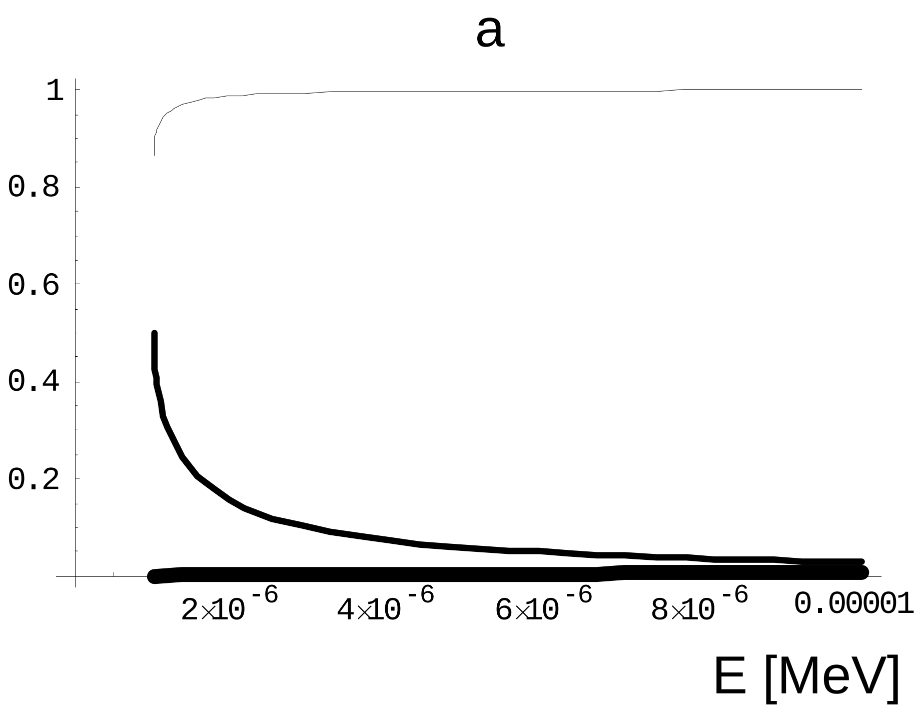
$$S(\theta) = \sqrt{1 - C(\theta)^2}$$

$$\gamma = 500, \quad m_\nu = 1\,\text{eV}, \quad \theta = \frac{\pi}{3}$$

Wigner rotation

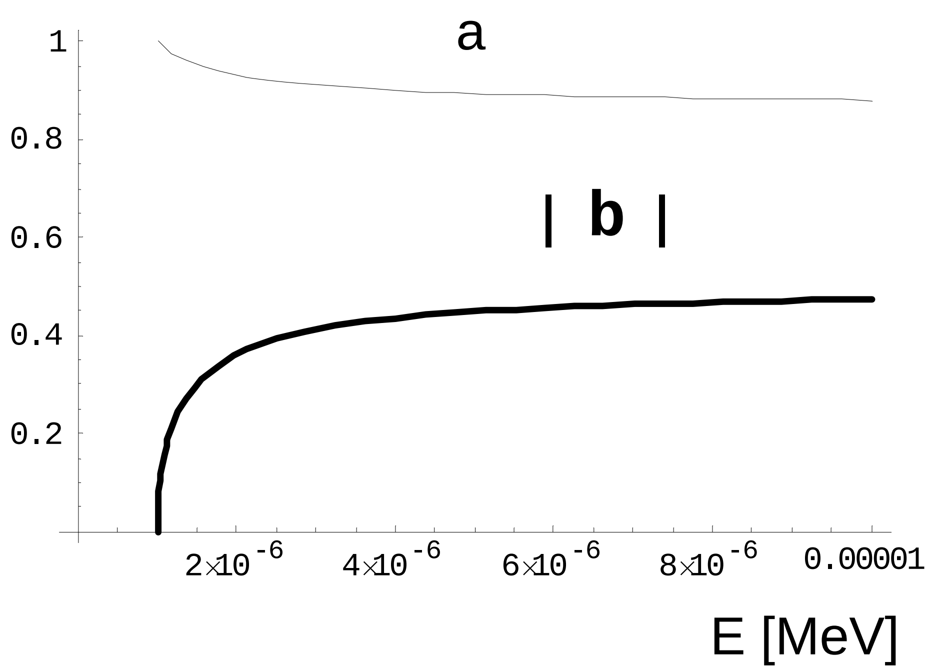


Rotation for helicity states

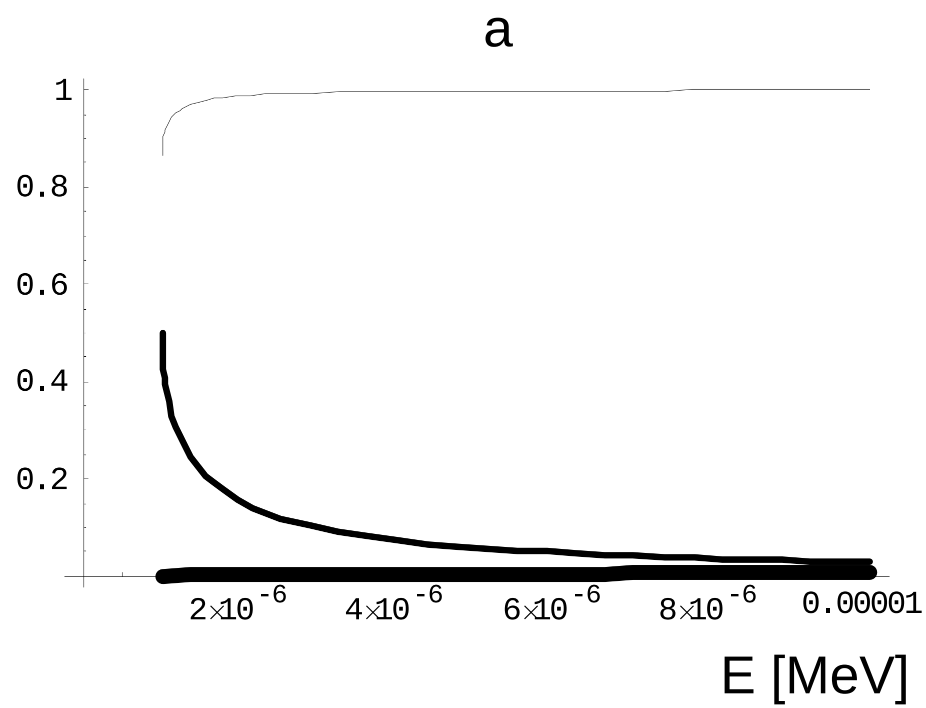


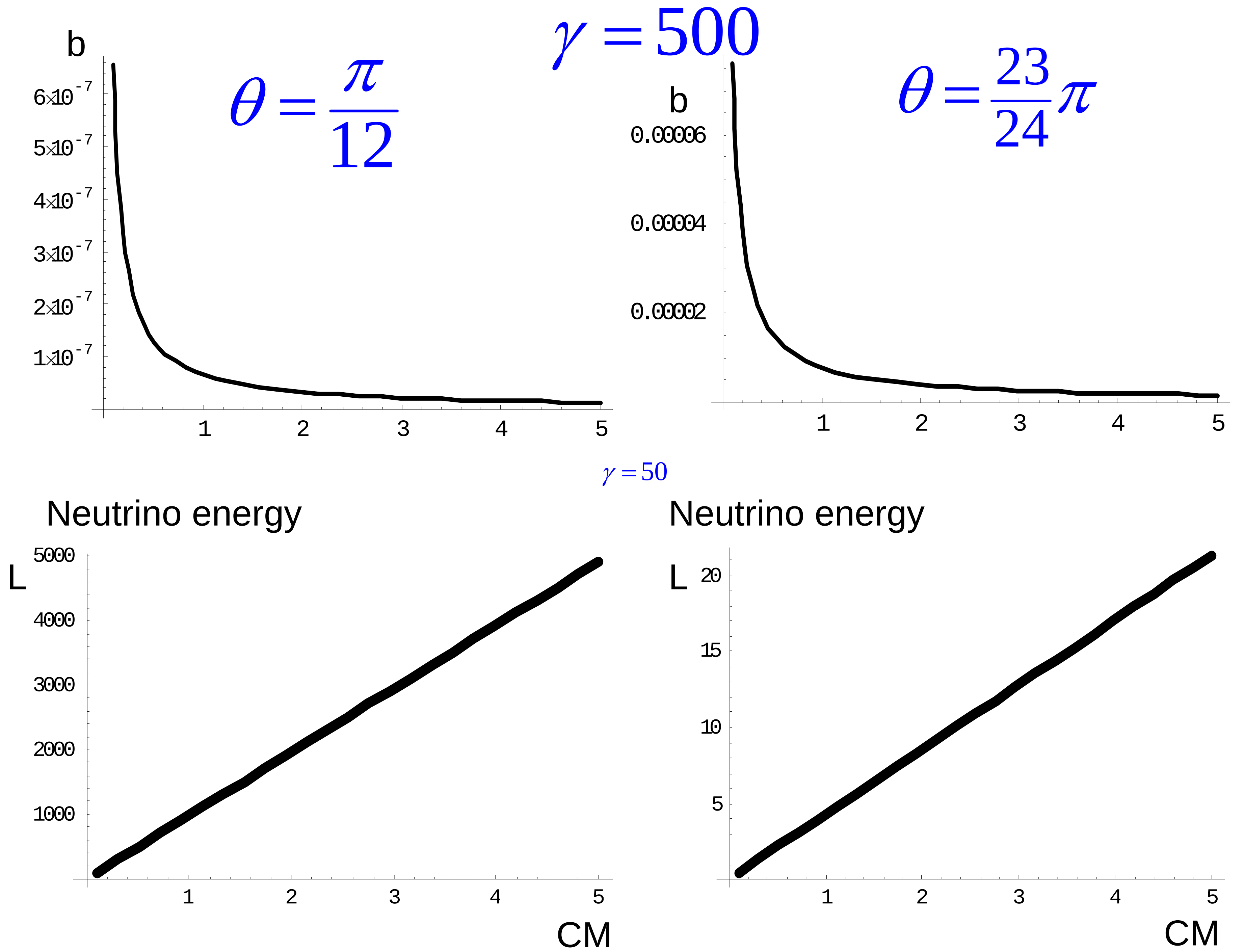
Wigner rotation and rotation for helicity states near threshold

Wigner rotation



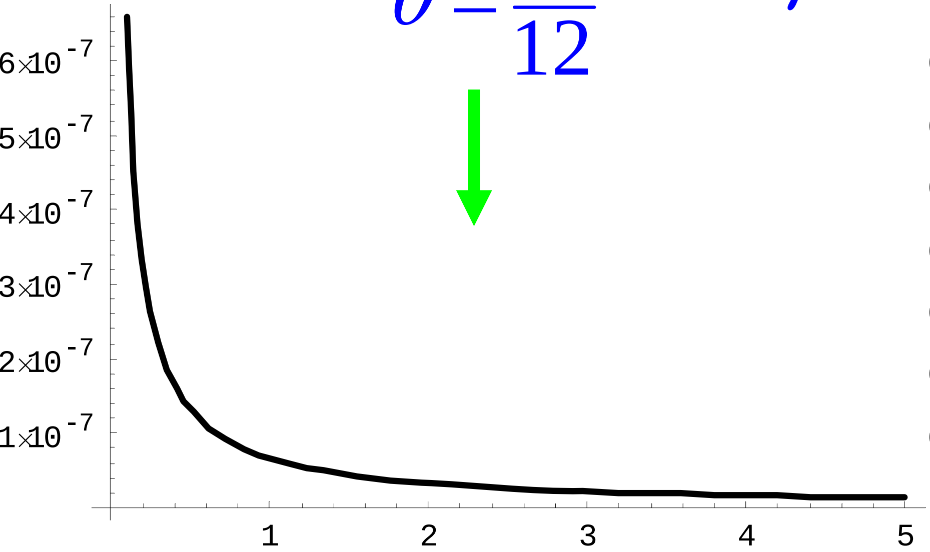
Rotation for helicity states

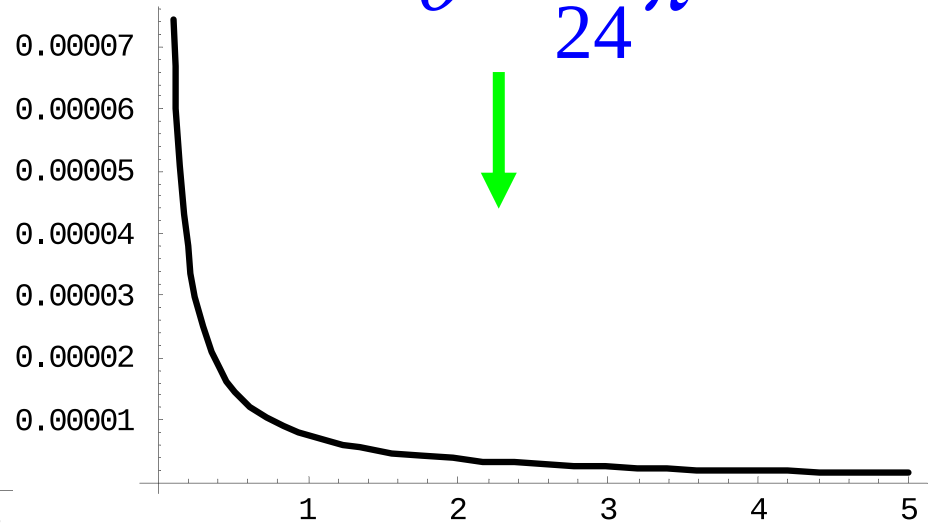




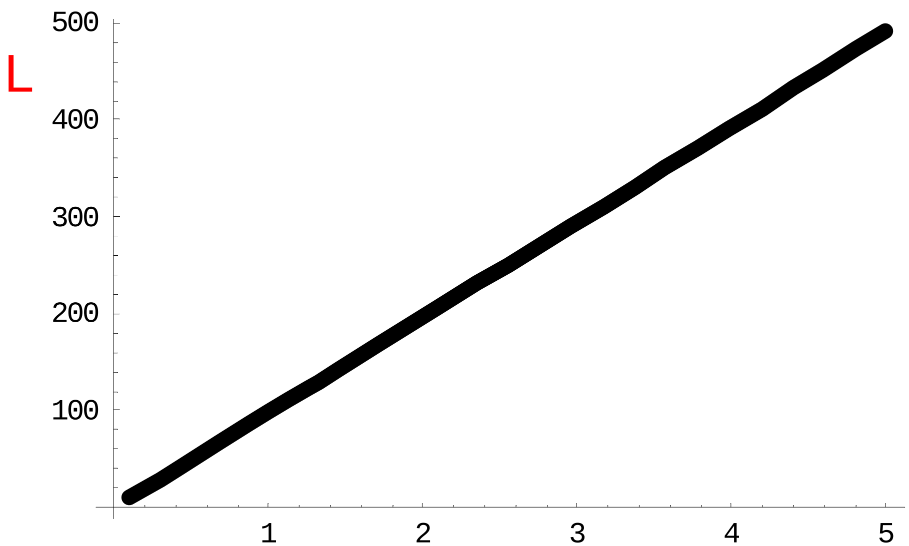
γ = 500
θ = π/12
θ = 23π/24
b
6×10^-7
5×10^-7
4×10^-7
3×10^-7
2×10^-7
1×10^-7
0.00006
0.00004
0.00002
1
2
3
4
5
γ = 50
Neutrino energy
Neutrino energy
L
5000
4000
3000
2000
1000
20
15
10
5
CM
CM

$\theta = \frac{\pi}{12}$ $\gamma = 50$ $\theta = \frac{23}{24}\pi$

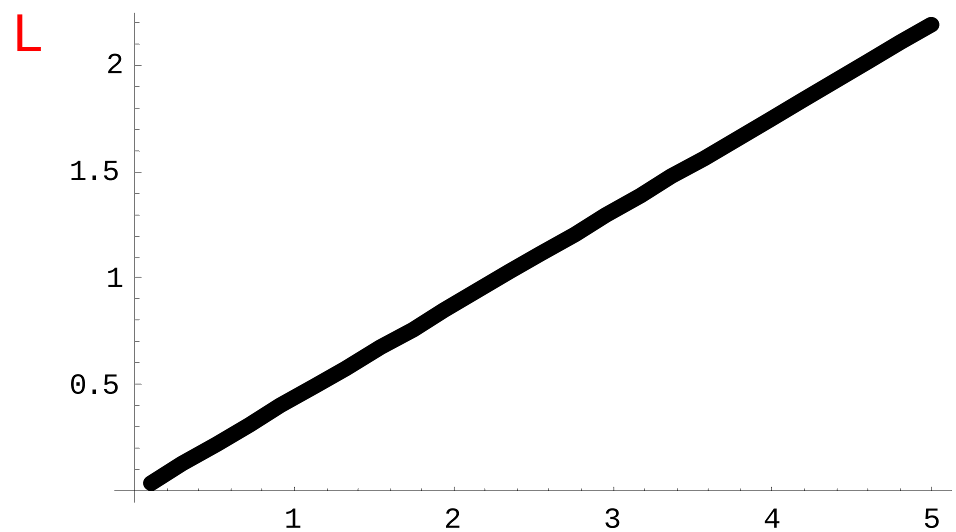




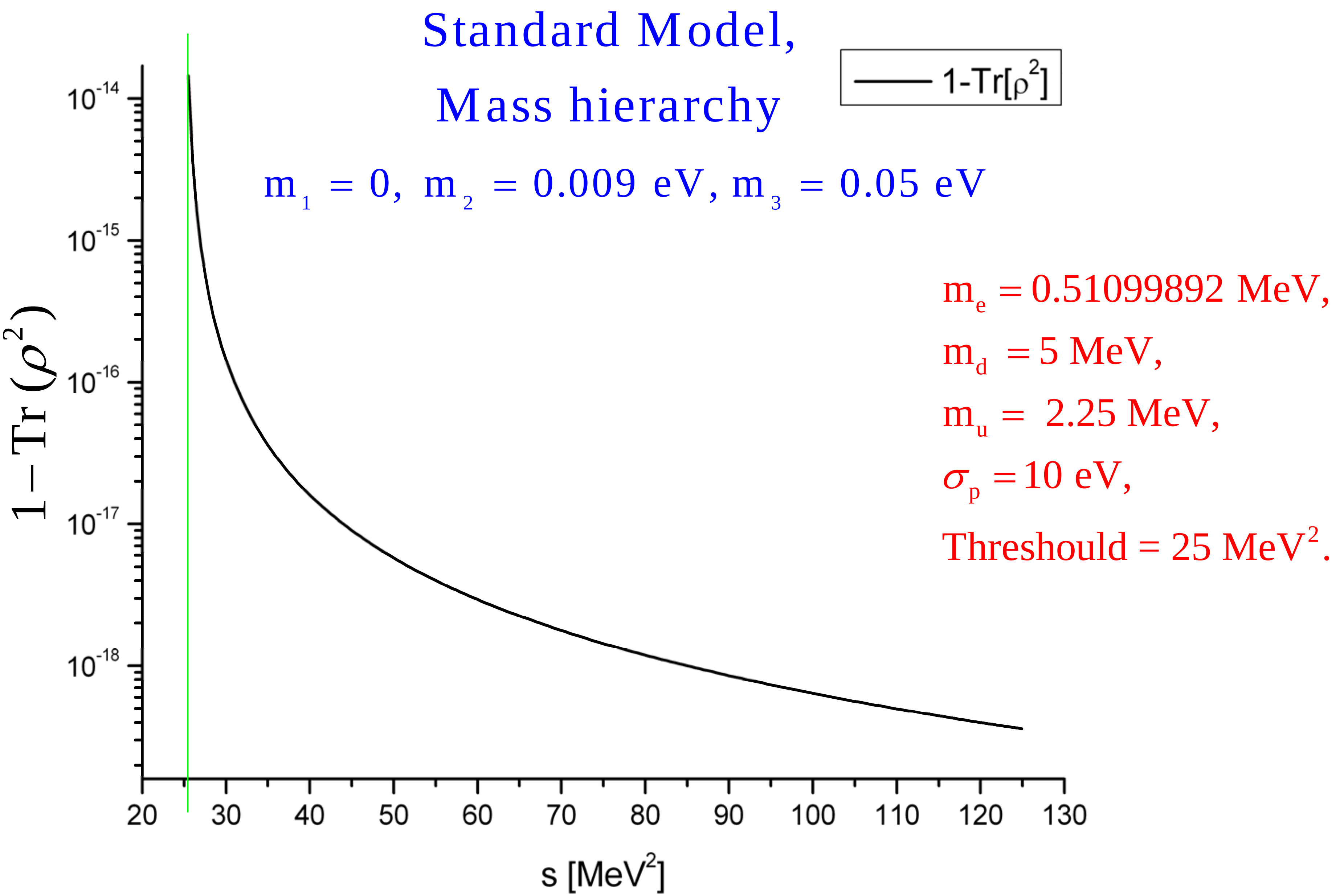
Neutrino energy



Neutrino energy



CM CM

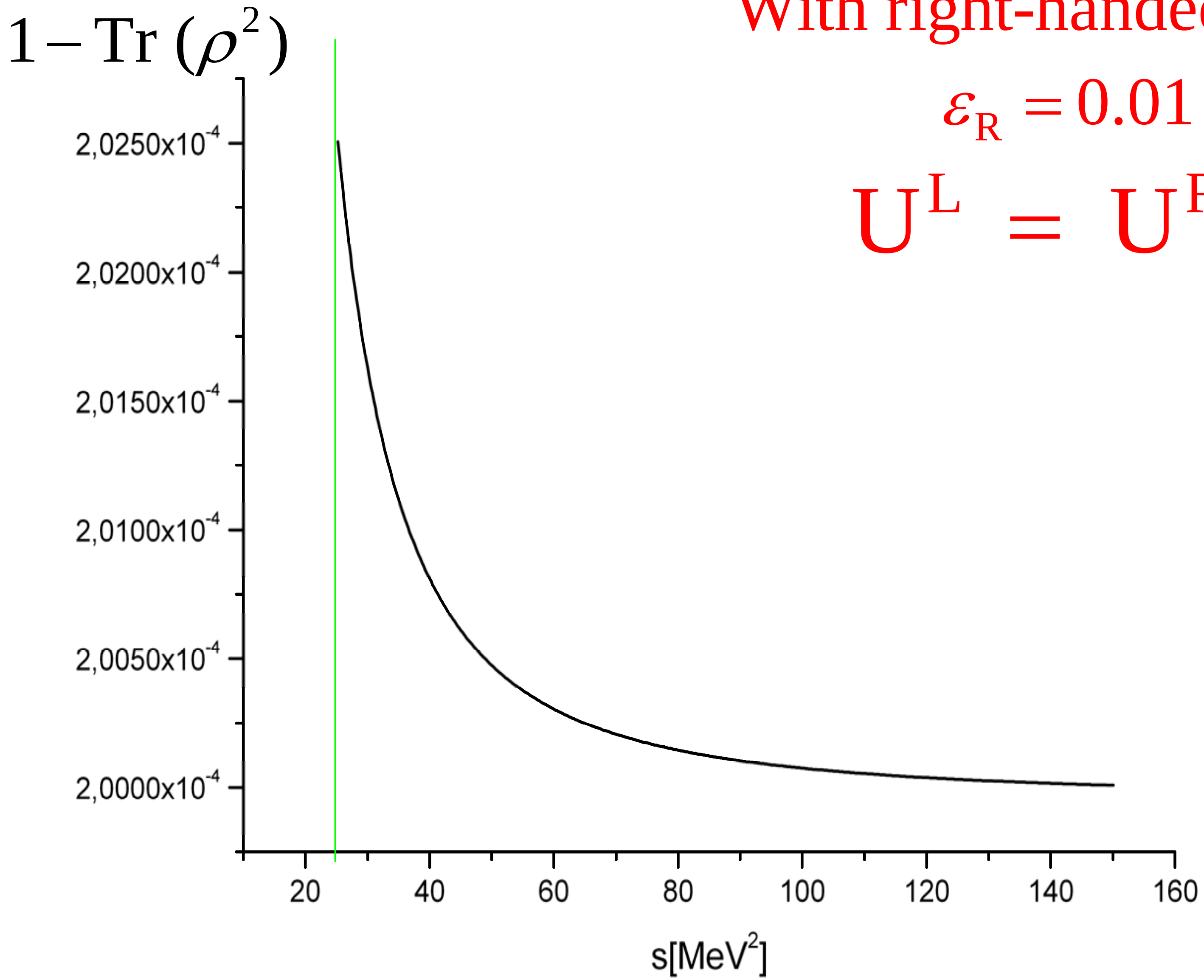


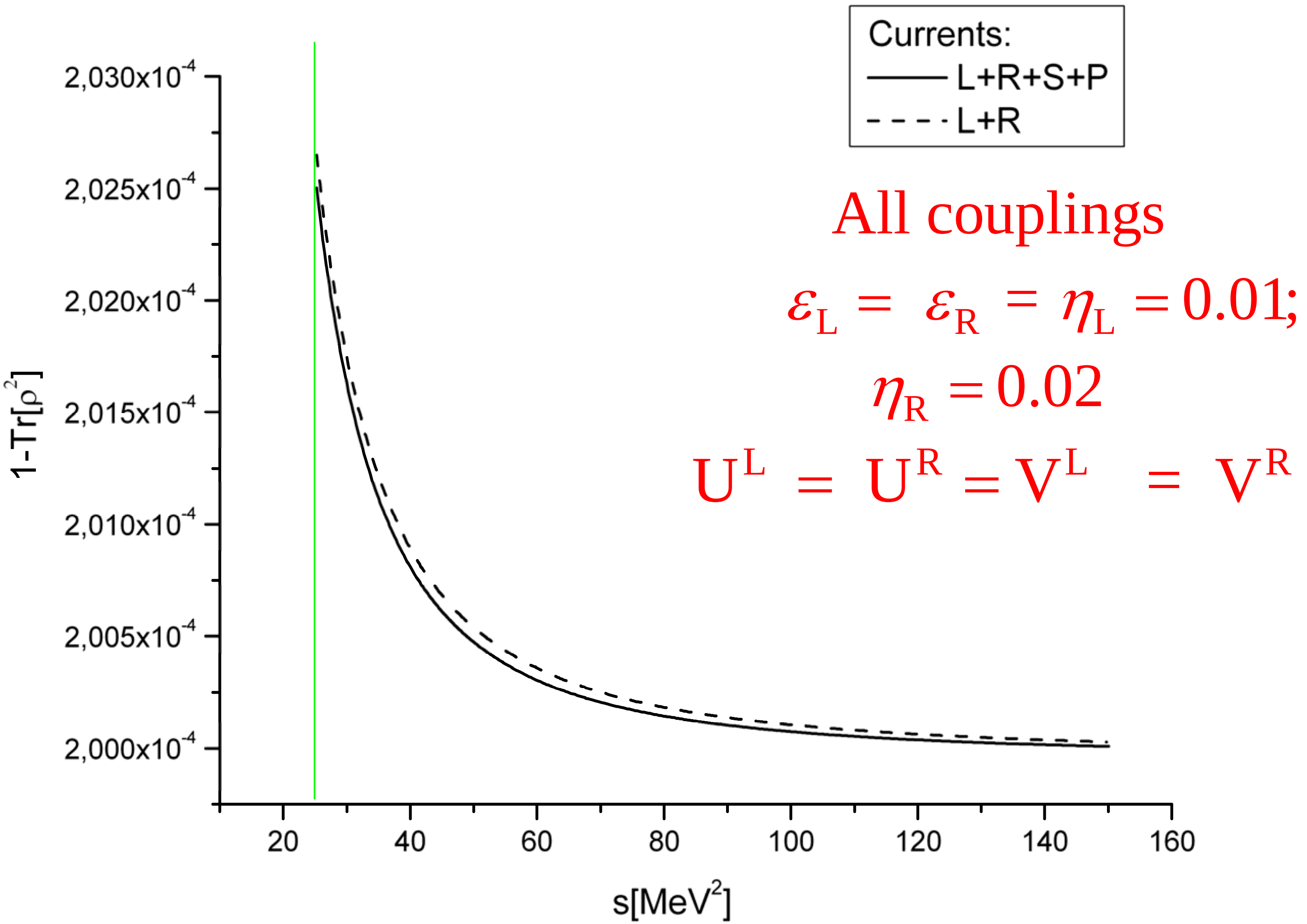
Standard Model,
Mass hierarchy
$m_1 = 0,\ m_2 = 0.009$ eV, $m_3 = 0.05$ eV
1-Tr[ρ²]
$m_e = 0.51099892$ MeV,
$m_d = 5$ MeV,
$m_u = 2.25$ MeV,
$\sigma_p = 10$ eV,
Threshould = 25 MeV².
$1 - \mathrm{Tr}(\rho^2)$
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
20
30
40
50
60
70
80
90
100
110
120
130
s [MeV²]

With right-handed currents,

$\varepsilon_R = 0.01$

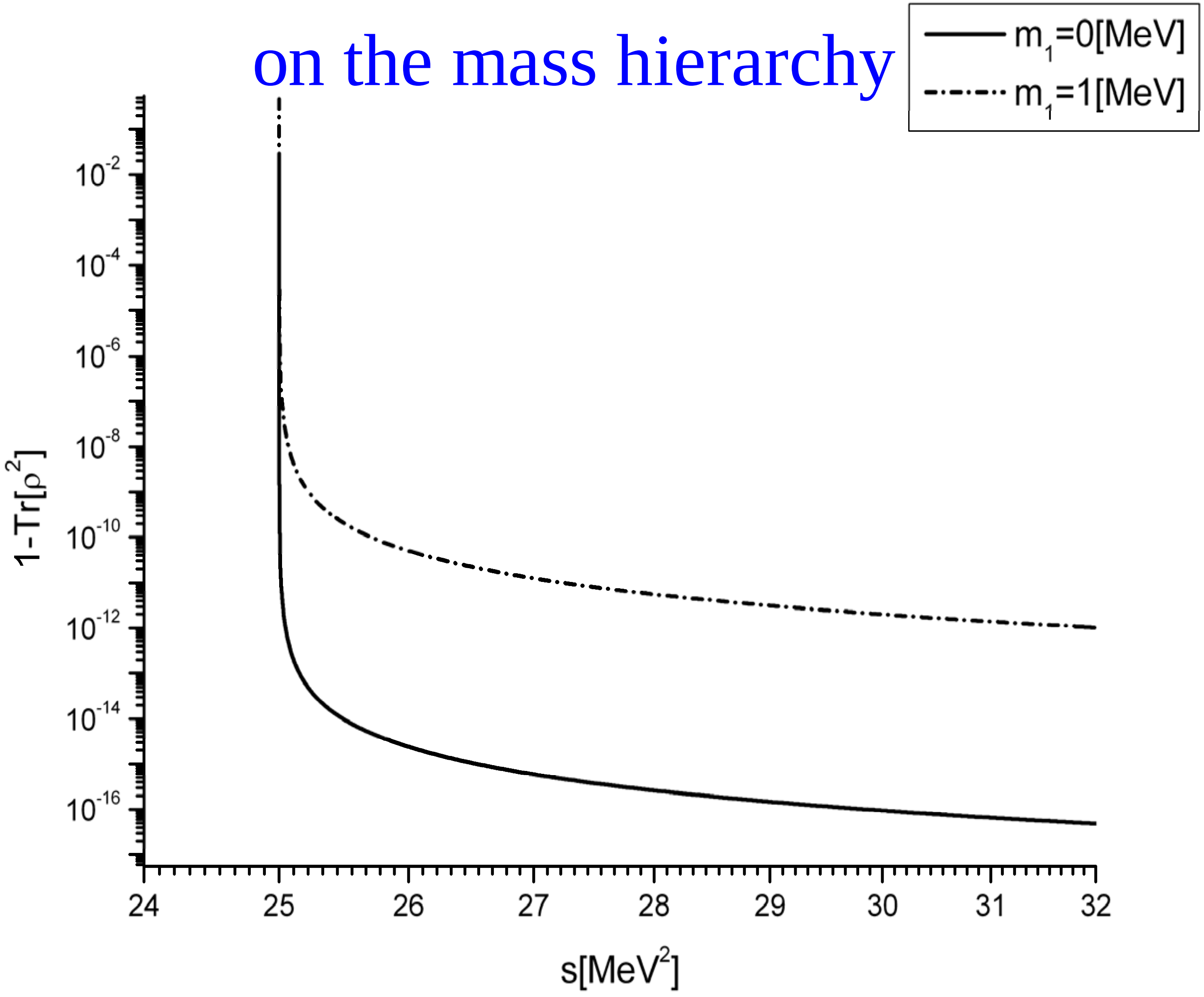
$U^L = U^R$



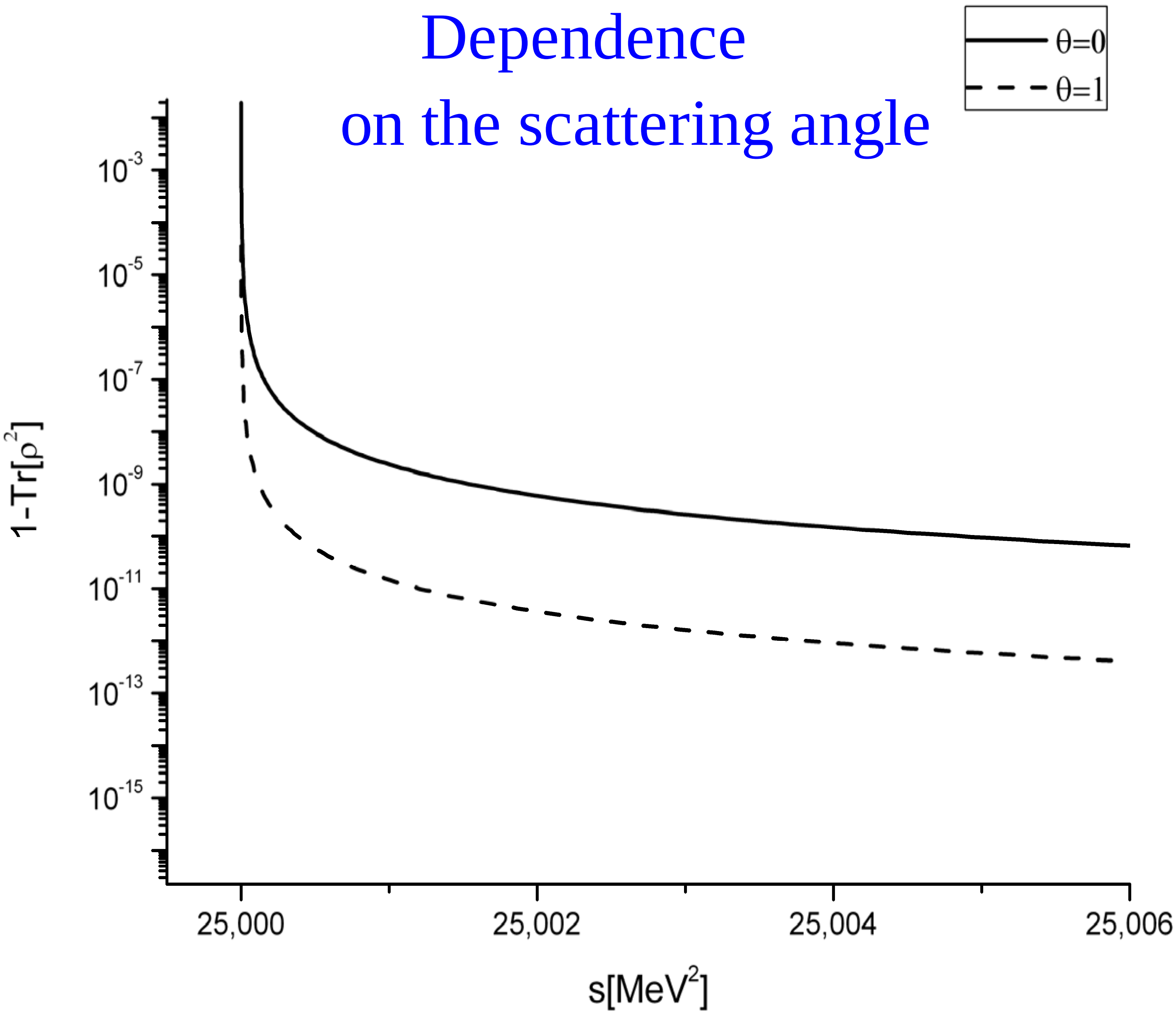


Currents:
L+R+S+P
L+R
All couplings
$\varepsilon_L = \varepsilon_R = \eta_L = 0.01;$
$\eta_R = 0.02$
$U^L = U^R = V^L = V^R$
2,030x10^{-4}
2,025x10^{-4}
2,020x10^{-4}
2,015x10^{-4}
2,010x10^{-4}
2,005x10^{-4}
2,000x10^{-4}
1-Tr[ρ^2]
20
40
60
80
100
120
140
160
s[MeV^2]

Dependence on the mass hierarchy



Dependence on the scattering angle



3. NEUTRINO PROPAGATION AND DETECTION

The statistical operator in the detector place, after time T:

$$\varrho^{\alpha}(\vec{L},T) = e^{-i\hat{H}T+i\hat{\vec{P}}\vec{L}}\varrho^{\alpha}(\vec{0},0)e^{i\hat{H}T-i\hat{\vec{P}}\vec{L}}$$

and density matrix:

$$\varrho^{\alpha}(\overrightarrow{x},t) = \sum_{\lambda,\lambda';i,i'}\int d^3p\, d^3p' e^{-i(E_i(\overrightarrow{p})+\overrightarrow{p}\overrightarrow{x})}\varrho^{\alpha}(\overrightarrow{p},\lambda,i;\overrightarrow{p}',\lambda',i')e^{i(E_{i'}(\overrightarrow{p}')t-\overrightarrow{p}'\overrightarrow{x})}$$

Let us assume, now that neutrinos are detected in the process:

$$\nu_i + A \rightarrow l_\beta + B$$

The transition cross section for flavour β neutrino detection:

$$\sigma_{\beta,\alpha}(\overrightarrow{x},t) \sim$$

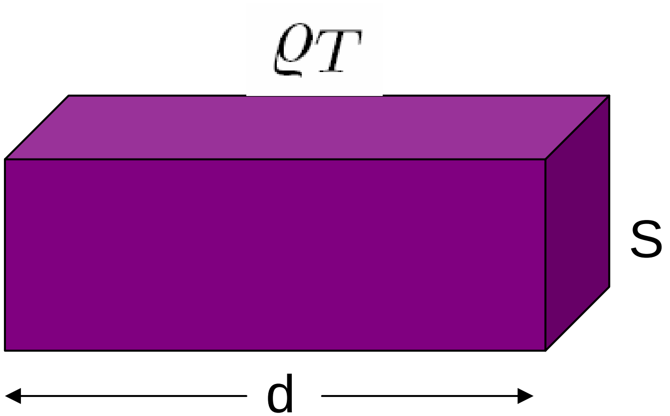
$$\sum_{\lambda_A,\lambda_B,\lambda_\beta,\lambda,\lambda'} \int d^3p\, d^3p' A_i^{\beta}(\overrightarrow{p},\lambda,\lambda_A;\lambda_\beta,\lambda_B)\ \varrho^{\alpha}(\overrightarrow{x},t;\overrightarrow{p},\lambda,i;\overrightarrow{p}',\lambda',i')\ A_{i'}^{\beta *}(\overrightarrow{p}',\lambda',\lambda_A;\lambda_\beta,\lambda_B)$$

To allow full and all wave pockets to pass:

$$\sigma_{\beta,\alpha}(L) = \int_0^{\infty} (\frac{dt}{1sek})\ \sigma_{\beta,\alpha}(\overrightarrow{x} = (0,0,L),t)$$

Number of flavour β neutrinos in the detector:

$v_B \simeq 1$ $\quad \varrho_B$ $\quad N$



$$V = Sd$$

Total number of neutrinos = N

$$N_\beta = N \ \sigma_{\beta,\alpha} \ d \ \varrho_T$$

$$N_\beta = \sigma_{\beta,\alpha} \ \varrho_B \ \varrho_T \ (v_B \approx 1) \ VT$$

Final cross sections:

$$\sigma_{-1,-1} = \frac{1}{32\pi s}\frac{p}{\epsilon}\frac{1}{2s_A+1}$$

$$\frac{1}{3}[AL\varepsilon\varepsilon\ \varepsilon_L \overline{\varepsilon_L}\ U^L_{\beta i} U^{L*}_{\beta k} + AL\varepsilon\eta\ (\eta_L \overline{\varepsilon_L}\ V^L_{\beta i} U^{L*}_{\beta k} + \varepsilon_L \overline{\eta_L}\ U^L_{\beta i} V^{L*}_{\beta k}) + AL\eta\eta\ \eta_L \overline{\eta_L}\ V^L_{\beta i} V^{L*}_{\beta k}]\ \varrho_{-1\,i;-1\,k};$$

$$\sigma_{+1,+1} = \frac{1}{32\pi s}\frac{p}{\epsilon}\frac{1}{2s_A+1}$$

$$\frac{1}{3}[AR\varepsilon\varepsilon\ \varepsilon_R \overline{\varepsilon_R}\ U^R_{\beta i} U^{R*}_{\beta k} + AR\varepsilon\eta\ (\eta_R \overline{\varepsilon_R}\ V^R_{\beta i} U^{R*}_{\beta k} + \varepsilon_R \overline{\eta_R}\ U^R_{\beta i} V^{R*}_{\beta k}) + AR\eta\eta\ \eta_R \overline{\eta_R}\ V^R_{\beta i} V^{R*}_{\beta k}]\ \varrho_{+1\,i;+1\,k};$$

$$\sigma_{-1,+1} + \sigma_{+1,-1} =$$

$$\frac{1}{32\pi s}\frac{p}{\epsilon}\frac{1}{2s_A+1}$$

$$\pi[D1\ \mathrm{Re}\,(\overline{\eta_L}\ \varepsilon_R\ U^R_{\beta i} V^{L*}_{\beta k}\ \varrho_{+1i;-1k}) - D1\ \mathrm{Re}\,(\overline{\varepsilon_L}\ \eta_R\ V^R_{\beta i} U^{L*}_{\beta k}\ \varrho_{+1i;-1k}) + D2\ \mathrm{Re}\,(\overline{\varepsilon_L}\ \varepsilon_R\ U^R_{\beta i} U^{L*}_{\beta k}\ \varrho_{+1i;-1k})]$$

Normalized elements of the production density matrix:

$$\varrho_{(-1,i);(-1,k)} = \frac{1}{\text{Nor}} \left(A_{\varepsilon_L^2}\, \varepsilon_L^2\, U_{\alpha i}^{L*}\, U_{\alpha k}^{L} + A_{\eta_R^2}\, \eta_R^2\, V_{\alpha i}^{R*}\, V_{\alpha k}^{R} + \frac{1}{2} \left(A_{\epsilon_L \eta_R \tau_L}\, \tau_L + A_{\epsilon_L \eta_R \tau_R}\, \tau_R\right) \epsilon_L\, \eta_R \left(U_{\alpha i}^{L*}\, V_{\alpha k}^{R} + V_{\alpha i}^{R*}\, U_{\alpha k}^{L}\right) \right);$$

$$\varrho_{(+1,i);(+1,k)} = \frac{1}{\text{Nor}} \left(A_{\varepsilon_R^2}\, \varepsilon_R^2\, U_{\alpha i}^{R*}\, U_{\alpha k}^{R} + A_{\eta_L^2}\, \eta_L^2\, V_{\alpha i}^{L*}\, V_{\alpha k}^{L} + \frac{1}{2} \left(A_{\epsilon_R \eta_L \tau_L}\, \tau_L + A_{\epsilon_R \eta_L \tau_R}\, \tau_R\right) \epsilon_R\, \eta_L \left(U_{\alpha i}^{R*}\, V_{\alpha k}^{L} + V_{\alpha i}^{L*}\, U_{\alpha k}^{R}\right) \right);$$

$$\sigma_{-1,-1} \quad ==$$

$$[AL\varepsilon\varepsilon\ \varepsilon_L \overline{\varepsilon_L}\ U^L_{\beta i}\ U^{L*}_{\beta k} + AL\varepsilon\eta\ (\eta_L \overline{\varepsilon_L}\ V^L_{\beta i}\ U^{L*}_{\beta k} + \varepsilon_L \overline{\eta_L}\ U^L_{\beta i}\ V^{L*}_{\beta k}) + AL\eta\eta\ \eta_L \overline{\eta_L}\ V^L_{\beta i}\ V^{L*}_{\beta k}]$$

$$X$$

$$\left(A_{\varepsilon_L^2}\ \varepsilon_L^2\ U^{L*}_{\alpha i}\ U^L_{\alpha k} + A_{\eta_R^2}\ \eta_R^2\ V^{R*}_{\alpha i}\ V^R_{\alpha k} + \frac{1}{2}\ (A_{\epsilon_L\ \eta_R\ \tau_L}\ \tau_L + A_{\epsilon_L\ \eta_R\ \tau_R}\ \tau_R)\ \epsilon_L\ \eta_R\ (U^{L*}_{\alpha i}\ V^R_{\alpha k} + V^{R*}_{\alpha i}\ U^L_{\alpha k})\right)$$

$$e^{-I\left[\frac{m_i^2-m_k^2}{2E}\right]L}$$

$$P_{\alpha\to\beta}\ (L) \ = \sum_{i,k=1}^{3} U^L_{\beta i}\ U^{L*}_{\beta k}\ U^{L*}_{\alpha i}\ U^L_{\alpha k}\ e^{-I\left[\frac{m_i^2-m_k^2}{2E}\right]L}$$

4. CONCLUSIONS

1. States are not pure near threshold, pure states appear for relativistic neutrinos and charge current left-handed production and detection mechanism,
2. For searching a physics beyond the SM, neutrino production and detection states are not necessary pure,
3. States are mixed, if right-handed (RH), scalar-LH-RH or pseudoscalar-RH – LH interactions are present,
4. Wigner rotation for helicity neutrino states are completely negligible in practice,
5. Only for relativistic neutrinos produced and detected by the LH mechanism the oscillation rates factorize.