Natural Realization of Seesaw in Mini-Warped Minimal $\mathrm{SO}(10)$

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Neutrino Mass and Mixing

- Experiments tell us neutrinos are massive and there is mixing in the leptonic sector.
- In SM, neutrinos are massless.But we can write down $\frac{1}{2}\frac{(LH)^2}{\Lambda}$,If the light neutrino mass scale is ~ 0.1 eV, $\Lambda \sim 10^{14}$ GeV.
- Seesaw mechanism is used to explain the smallness of masses of light neutrinos by introducing heavy right-handed neutrinos or $\mathrm{SU}(2)$ triplet.

$\mathrm{SO}(10)$ gauge group as candidate of grand unification models

- The **16** dimensional spinor representation includes right handed neutrino that is an essential part of the seesaw mechanism.
- The seesaw scale which is close to the GUT scale receives natural explanation as the GUT symmetry breaking scale. $\mathrm{U}(1)_{\mathrm{B-L}}$ is gauged as subgroup of $\mathrm{SO}(10)$.

Minimal $\mathrm{SO}(10)$

- Minimal $\mathrm{SO}(10)$ model includes matter field $\mathbf{16}$ for each generation, Higgs fields $\mathbf{126}, \overline{\mathbf{126}}, \mathbf{10}$ and $\mathbf{210}$.
- Renormalizable operators $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$ and $\mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}}$ generate fermions masses.
- $\mathbf{210}$ is needed to break $\mathrm{SO}(10)$ to SM group.

Decompositions of SO(10) multiplets under $SU(5) \times U(1)_X$

$$\begin{aligned}
\mathbf{16} &= \mathbf{1}_{-5} \oplus \overline{\mathbf{5}}_{+3} \oplus \mathbf{10}_{-1} \\
\mathbf{210} &= \mathbf{1}_0 \oplus \mathbf{5}_{-8} \oplus \overline{\mathbf{5}}_8 \oplus \mathbf{10}_4 \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \oplus \mathbf{75}_0 \oplus \mathbf{40}_{-4} \oplus \overline{\mathbf{40}}_4 \\
\mathbf{126} &= \mathbf{1}_{-10} \oplus \overline{\mathbf{5}}_{-2} \oplus \mathbf{10}_{-6} \oplus \overline{\mathbf{15}}_{+6} \oplus \mathbf{45}_{+2} \oplus \overline{\mathbf{50}}_{-2} \\
\mathbf{10} &= \mathbf{5}_2 \oplus \overline{\mathbf{5}}_{-2}
\end{aligned}$$

Why minimal $\mathrm{SO}(10)$?

- The $\mathrm{U}(1)_{\mathrm{B-L}}$ breaking VEV carries $B-L=2$, so R-parity is conserved.
- The model is very predictive. There are only 13 parameters.
- Large atmospherical mixing angle and small reactor angle are naturally explained by $b-\tau$ unification if type II seesaw dominates the contribution to the light neutrino mass.

Fermions mass sum rule and neutrino mass in minimal SO(10) model

The general neutrino mass formulae $M_\nu = fv_L - M_D^T (fv_R)^{-1} M_D$

$$M_u = h\kappa_u + fv_u, M_d = h\kappa_d + fv_d$$
$$M_\ell = h\kappa_d - 3fv_d, M_D = h\kappa_u - 3fv_u$$

The type II contribution: $M_\nu \sim (M_d - M_l)$.

$M_d \sim m_b \begin{bmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}$,and $M_l \sim M_d$ for $m_b \simeq m_\tau$,

$$M_\nu \simeq m_0 \begin{bmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{bmatrix}$$

Problems

- Gauge couplings blow up beyond GUT scale due to large representations.
- To explain discrepancy between seesaw scale and GUT scale, one needs to fine-tune the parameters.
- In the minimal $\mathrm{SO}(10)$ model, type II seesaw can not dominate contribution to the light neutrino mass.

The type II contribution can be written as

$$M_\nu \sim f\frac{v_{wk}^2}{M_T}$$

where M_T is SU(2) triplet mass.

- If $f \sim 1$,we need $M_T \sim 10^{14}\mathrm{GeV}$.
- $\overline{\mathbf{126}} = \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \mathbf{10}_{+6} \oplus \mathbf{15}_{-6} \oplus \mathbf{45}_{-2} \oplus \mathbf{50}_{+2}$. This triplet belongs to **15**, whose mass has the same order as masses of **45** and **50**.
- This model is so constraint that there is no way to achieve type-II dominance even by tuning the parameters.

Motivation

- Mini-Warping provides the solution to the mini hierarchy between M_P and M_{GUT}. $M_{GUT} = e^{-k\pi r_c} M_P$ is the cutoff of 4D theory. (Fukuyama, Kikuchi and Okada PRD 2007)
- Overlapping of of bulk fields configuration provides a way to understand mini-fine tuning needed in 4D minimal SO(10). Particularly it may be helpful for the realization of type II dominance or mixed case.

Warped extra dimension

As in the RS model, we use the warped metric,

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2 \ ,$$

with $-\pi \le y \le \pi$ and $\eta_{\mu\nu} = (+,-,-,-)$. In the above expression, k is the AdS curvature, and r_c and y are the radius and the angle of S^1, respectively.

SUSY in 5D

$$\begin{aligned}\mathcal{L} &= \int dy \left\{ \int d^4\theta \; r_c \; e^{-2kr_c|y|} \left(H_i^\dagger e^{-Q_i V} H_i + H_i^c e^{Q_i V} H_i^{c\dagger} \right) \right. \\ &+ \left. \int d^2\theta e^{-3kr_c|y|} H_i^c \left[\partial_y - (1 + C_i)\, kr_c \epsilon(y) - Q_i \frac{\chi}{\sqrt{2}} \right] H_i + h.c. \right\}\end{aligned}$$

Zero mode

The zero mode wave function of H_i satisfies the following equation of motion:

$$[\partial_y - (1 + C_i + Q_i\alpha)\, kr_c\epsilon(y)]\, H_i = 0$$

which yields $H_i = \frac{1}{\sqrt{N_i}} e^{(1+C_i+Q_i\alpha)kr_c|y|}\ h_i(x^\mu)$, where $h_i(x^\mu)$ is the chiral multiplet in four dimensions. Here, N_i is a normalization constant, $\frac{1}{N_i} = \frac{2(C_i+Q_i\alpha)k}{e^{2(C_i+Q_i\alpha)kr_c\pi}-1}$.

Setup of mini-warped minimal $\mathrm{SO}(10)$

- Put matter fields and Higgs fields in the bulk. Write superpotential on both UV brane and IR brane. All of the coupling constants are $\mathcal{O}(1)$.
- Bulk adjoint field gets VEV, and breaks $\mathrm{SO}(10)$ to $\mathrm{SU}(5) \times \mathrm{U}(1)_{\mathrm{X}}$. $\mathrm{SU}(5)$ submultiplets within one $\mathrm{SO}(10)$ multiplet get different effective 5D mass parameters due to different $\mathrm{U}(1)_{\mathrm{X}}$ charge.
- Choose 5D mass parameters for $\mathrm{SO}(10)$ multiplets C_i and bulk adjoint field VEV parameter α.

Example

We take $\alpha = -1/4$ and C_i as follows

H_i components	C_i
$\mathbf{16}$	$1/2$
$\mathbf{10}$	$1/2$
$\overline{\mathbf{126}}$	1
$\mathbf{126}$	0
$\mathbf{210}$	-2

Masses of submultiplets of $\overline{\mathbf{126}}$

$$\overline{\mathbf{126}} = \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \mathbf{10}_{+6} \oplus \mathbf{15}_{-6} \oplus \mathbf{45}_{-2} \oplus \mathbf{50}_{+2}$$

$$\mathbf{15} \sim \omega^{3/2} M_{GUT} \sim 10^{13}\mathrm{GeV}$$
$$\mathbf{45} \sim \omega^{1/2} M_{GUT} \sim 10^{15}\mathrm{GeV}$$
$$\mathbf{50} \sim \quad M_{GUT} \quad \sim 10^{16}\ \mathrm{GeV}$$

Type II seesaw contribution is estimated as

$$M_\nu^{II} \simeq \frac{2(f_1)_{33}\omega^{1/2} v_{10} v_{210}\alpha_2}{M_{GUT}\omega^{3/2}},$$

where $\omega = M_P/M_{GUT} \sim 10^{-2}$. If we take $(f_1)_{33} \sim 1$, $\alpha_2 \sim 0.5$ and assume $v_{10} \simeq v_{210} \sim 100$ GeV, we get the reasonable value for the atmospheric neutrino oscillation data, $M_\nu^{II} \simeq 0.05$ eV.

The type I seesaw contribution

$$M_\nu^{I} = M_D^T M_R^{-1} M_D \simeq \frac{m_t^2\omega^{1/2}}{2(f_1)_{33} M_{GUT}\omega^{3/2}},$$

$m_t \sim 100$ GeV at the GUT scale, the type I seesaw gives the contribution to the "heaviest" light neutrino mass as $m_3 \simeq 0.025$ eV.

Conclusions

- Minimal $\mathrm{SO}(10)$ provides way to understand neutrino mass and mixing.
- There are some problems in the minimal SO(10) models.
- Embedding 4D minimal $\mathrm{SO}(10)$ into mini-warped 5D space provides a way to solve these problems.