$SU(4)_L \times U(1)_X$ electroweak gauge group with little Higgs

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Introduction

- As an alternative solution to the naturalness problem (and the fine-tuning issue), little Higgs models (LHM) has been recently introduced.
  (Arkani-Hamed, Cohen, and Georgi 2001)
- LHMs adopts the early idea that Higgs can be considered as a Nambu Goldstone boson from global symmetry breaking at some higher scale $\Lambda \sim 4\pi f$.
  (Dimopoulos, Preskill 1982; Georgi, Kaplan 1984; Banks 1984)
- Higgs acquires a mass radiatively through symmetry breaking at the EW scale v, by collective breaking.
- There are several LHMs such as “minimal moose”, “product group”, and “simple group” models.
- In this talk, we discuss the simple group model with $SU(4)_L \times U(1)_X$ electroweak gauge group (suggested by Kaplan and Schmaltz, 2003) by embedding the anomaly-free fermion spectra.

Model Construction - Scalar Sector

- The Coleman-Weinberg mechanism (radiative corrections) produces the Higgs potential:

$$V = \delta m^2 h^\dagger h + \delta\lambda (h^\dagger h)^2$$

- In the $SU(3)$ LHM (simplest), generated Higgs mass is somewhat too large, so a tree level "μ" term is included by hand and partially cancels the Higgs mass.

- "μ" term explicitly breaks the spontaneously broken global symmetry and gives masses to the would-be Nambu-Goldstone bosons:

$$\begin{aligned} -V &= \mu^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \\ &= 2 f_1 f_2 \mu^2 \cos\left(\frac{f\eta}{\sqrt{2} f_1 f_2}\right)\left[1 - \frac{f^2}{2 f_1^2 f_2^2}(h^\dagger h) + \frac{f^4}{24 f_1^3 f_2^3}(h^\dagger h)^2 + \ldots\right] \end{aligned}$$

- In the $SU(4)$ LHM, however, the model produces a tree-level quartic coupling from the following terms, so doesn't reqire the above μ term:

$$\kappa_{11}|\Phi_{11}^\dagger \Phi_{21}|^2 + \kappa_{12}|\Phi_{11}^\dagger \Phi_{22}|^2 + \kappa_{21}|\Phi_{12}^\dagger \Phi_{21}|^2 + \kappa_{22}|\Phi_{12}^\dagger \Phi_{22}|^2$$

Model Construction - Scalar Sector

- Start from a non-linear sigma model $[SU(4)/SU(3)]^4$ with four complex quadruplets scalar fields Φ_{ij} $(i, j = 1, 2)$

  $\Longrightarrow$ Diagonal $SU(4)$ is gauged.

- Gauge symmetry breaking: $SU(4)_w \times U(1)_X \rightarrow SU(2)_w \times U(1)_Y$

  $\Longrightarrow$ 12 new gauge bosons with masses of order the scale f.

- Global symmetry breaking: $[SU(4)]^4 \rightarrow [SU(3)]^4$

  $\Longrightarrow$ 12 of the 28 degrees of freedom in the Φ_{ij} are eaten by the Higgs mechanism when $SU(4)_w$ is broken.

  $\Longrightarrow$ Remaining 16 consist of two complex doublets h_u and h_d, three complex $SU(2)$ singlets σ_1, σ_2 and σ_3, and two real scalars η_u and η_d.

  (Kaplan and Schmaltz 2003)

Model Construction - Scalar Sector

One possible parametrization of the non-linear sigma model fields Φ_{ij} with general f_{ij} ($SU(4)$ breaking is not aligned):

$$\Phi_{11} = e^{+i\mathcal{H}_u \frac{f_{12}}{f_{11}}} \begin{pmatrix} 0 \\ 0 \\ f_{11} \\ 0 \end{pmatrix} \qquad \Phi_{12} = e^{-i\mathcal{H}_u \frac{f_{11}}{f_{12}}} \begin{pmatrix} 0 \\ 0 \\ f_{12} \\ 0 \end{pmatrix}$$

$$\Phi_{21} = e^{+i\mathcal{H}_d \frac{f_{22}}{f_{21}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{21} \end{pmatrix} \qquad \Phi_{22} = e^{-i\mathcal{H}_d \frac{f_{21}}{f_{22}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{22} \end{pmatrix}$$

where

$$\mathcal{H}_u = \begin{pmatrix} 0 & 0 & h_u & 0 \\ 0 & 0 & & 0 \\ & h_u^\dagger & -\eta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} /2f_1 \qquad \mathcal{H}_d = \begin{pmatrix} 0 & 0 & 0 & h_d \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \\ & h_d^\dagger & 0 & \eta \end{pmatrix} /2f_2$$

$$f_i^2 = \frac{1}{2} \sum_{j=1,2} f_{ij}^2, \qquad \langle h_1 \rangle = v_u, \qquad \langle h_2 \rangle = v_d$$

Model Construction - Scalar Sector

- Most general expression for the electric charge generator:

$$Q = aT_{3L} + \frac{1}{\sqrt{3}} bT_{8L} + \frac{1}{\sqrt{6}} cT_{15L} + XI_4$$

- Correct embedding of the SM isospin $SU(2)_L$ doublets gives $a = 1$ ($Q = T_{3L} + Y$).
- Charge operator acting on the representations 4 and $\bar{4}$ of $SU(4)_L$:

$$Q[4] = Diag\left(\frac{1}{2} + \frac{b}{6} + \frac{c}{12} + X, -\frac{1}{2} + \frac{b}{6} + \frac{c}{12} + X, -\frac{2b}{6} + \frac{c}{12} + X, -\frac{3c}{12} + X\right)$$

$$Q[\bar{4}] = Diag\left(-\frac{1}{2} - \frac{b}{6} - \frac{c}{12} + X, \frac{1}{2} - \frac{b}{6} - \frac{c}{12} + X, \frac{2b}{6} - \frac{c}{12} + X, \frac{3c}{12} + X\right)$$

- If we assume that all gauge bosons have electric charges 0 and ± 1 only, there are not more than four different possibilities (Ponce, Gutiérrez, and Sánchez 2004):

$$b = c = 1; \quad b = c = -1; \quad b = 1 \;\&\; c = -2; \quad \underbrace{b = -1 \;\&\; c = 2}_{\text{We focus on this scenario}}$$

- For $b = -1 \& c = 2$, $X(\Phi_{1j}) = -\frac{1}{2}$ & $X(\Phi_{2j}) = \frac{1}{2}$.

Model Construction - Fermion Sector

- For example, each fermion 4-plet takes the following form:

$$b = c = 1 : \begin{pmatrix} u \\ d \\ D \\ D' \end{pmatrix} \quad or \quad b = c = -1 : \begin{pmatrix} d \\ u \\ U \\ U' \end{pmatrix}$$

$$b = 1 \& \, c = -2 : \begin{pmatrix} u \\ d \\ D \\ U \end{pmatrix} \quad or \quad b = -1 \& \, c = 2 : \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix}$$

- Our choice is to have the duplicated extra heavy fermions which can remove the quadratic divergences due to their SM fermion partners.

Model Construction - Fermion Sector

- There are several possible ways to construct anomaly-free fermion spectra for $b = -1$ & $c = 2$.
- We choose the following set of fermion spectra which would be feasible enough to complete this little Higgs model (Kong 2003):

	$U(1)_Y$-states		
$(3_C, 4_L, \frac{1}{6})$	$\frac{1}{6}[Q]$	$\frac{2}{3}(T)$	$\frac{-1}{3}(B)$
$2\ (3_C, \bar{4}_L, \frac{1}{6})$	$2\ \frac{1}{6}[2\ Q]$	$2\ \frac{-1}{3}(D, S)$	$2\ \frac{2}{3}(U, C)$
$3\ (I_C, 4_L, \frac{-1}{2})$	$3\ \frac{-1}{2}[3\ L]$	$3\ \ 0(3\ N)$	$3\ \ \text{-}1(3\ E^-)$
$6\ (\bar{3}_C, 1_L, \frac{-2}{3})$	$6\ \frac{-2}{3}\ (\bar{u}, \bar{c}, \bar{t}, \bar{U}, \bar{C}, \bar{T})$		
$6\ (\bar{3}_C, 1_L, \frac{1}{3})$	$6\ \frac{1}{3}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S}, \bar{B})$		
$6\ (1_C, 1_L, 1)$	$3\ 1\ (e^+, \mu^+, \tau^+)$		$3\ \ 1(3\ E^+)$

$\Longrightarrow$ Anomaly cancellation is achieved when $N_f = N_c = 3 \Rightarrow$ one of the best features of this model.

Model Construction - Gauge Boson Sector

- Covariant derivative for 4-plets:

$$iD_\mu = i\partial_\mu + gT^\alpha A_\mu^\alpha + g_X X A_\mu^X$$

- There are 15 gauge bosons associated with $SU(4)_L$:

$$T^\alpha A_\mu^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} Z^0_{1\mu} & W^+_\mu & Y^0_\mu & X'^+_\mu \\ W^-_\mu & Z^0_{2\mu} & X^-_{1\mu} & Y'^0_\mu \\ \bar{Y}^0_\mu & X^+_{1\mu} & Z^0_{3\mu} & W'^+_\mu \\ X'^-_\mu & \bar{Y}'^0_\mu & W'^-_\mu & Z^0_{4\mu} \end{pmatrix}$$

where

$$Z_1^{0\mu} = A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12} \qquad Z_2^{0\mu} = -A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12}$$

$$Z_3^{0\mu} = -2A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12} \qquad Z_4^{0\mu} = -3A_{15}^\mu/\sqrt{12}$$

Model Construction - Gauge Boson Sector

- Mass matrix for the neutral Hermitian gauge bosons:

$$\frac{1}{4}g^2 \begin{pmatrix} 4t^2 f^2 & -tv^2 & t(4f^2 - v^2) & 2\sqrt{2}t\triangle f^2 \\ -tv^2 & v^2 & 0 & \frac{\triangle v^2}{\sqrt{2}} \\ t(4f^2 - v^2) & 0 & 4f^2 - v^2 & 2\sqrt{2}\triangle f^2 \\ 2\sqrt{2}t\triangle f^2 & \frac{\triangle v^2}{\sqrt{2}} & 2\sqrt{2}\triangle f^2 & 2f^2 \end{pmatrix} \quad \begin{matrix} A_X \\ A_3 \\ \frac{1}{\sqrt{3}}A_8 - \sqrt{\frac{2}{3}}A_{15} \\ \sqrt{\frac{2}{3}}A_8 + \frac{1}{\sqrt{3}}A_{15} \end{matrix}$$

where

$$f^2 = f_1^2 + f_2^2 \gg \triangle f^2 = f_1^2 - f_2^2, \; v^2 = v_1^2 + v_2^2 \gg \triangle v^2 = v_1^2 - v_2^2, \quad t \equiv g_X/g$$

Model Construction - Gauge Boson Sector

- Physical Gauge Boson Masses after diagonalizing the mass matrix:
  - $M_W^2 = \frac{1}{4} g^2 v^2, \qquad M_{W'}^2 = \frac{1}{4} g^2 \left(4f^2 - v^2\right)$
  - $M_X^2 = \frac{1}{4} g^2 \left(4f_1^2 - v_1^2 + v_2^2\right), \qquad M_{X'}^2 = \frac{1}{4} g^2 \left(4f_2^2 + v_1^2 - v_2^2\right)$
  - $M_Y^2 = g^2 f_1^2, \qquad M_{Y'}^2 = g^2 f_2^2$
  - $M_Z^2 = \frac{g^2 v^2}{4 \cos^2 \theta_W} \left(1 - \frac{v^2}{4f^2} \tan^4 \theta_W\right), \qquad M_{Z'}^2 = g^2 (1 + t^2) f^2 - M_Z^2$
  - $M_{Z''}^2 = \frac{1}{2} g^2 f^2$

  where

$$\cos \theta_W = \sqrt{(1 + t^2)/(1 + 2t^2)},$$

$$v_1^2 \equiv v_u^2 - \frac{v_u^2}{12 f_1^2} \left(\frac{f_{12}^2}{f_{11}^2} + \frac{f_{11}^2}{f_{12}^2} - 1 \right), \quad v_2^2 \equiv v_d^2 - \frac{v_d^2}{12 f_2^2} \left(\frac{f_{22}^2}{f_{21}^2} + \frac{f_{21}^2}{f_{22}^2} - 1 \right)$$

Phenomenological Constraints

- Custodial $SU(2)$ symmetry violating shift in the Z mass:

$$\delta\rho \equiv \alpha T \simeq \frac{t^4}{4(1+t^2)^2}\frac{v^2}{f^2} \qquad (f^2 = f_1^2 + f_2^2)$$

- For $T \lesssim 0.15$ (95% CL), $f_{SU(4)} \gtrsim 1.1$ TeV

  $\Longrightarrow$ comparable to $f_{SU(3)} \gtrsim 1.3$ TeV

- Also, combined analysis of Z pole observables and the atomic parity violation data gives (for $f_1 \approx f_2$):

$$M_{Z'} \gtrsim 0.7\ \mathrm{TeV} \quad \Rightarrow \quad f_{SU(4)} \gtrsim 0.9\ \mathrm{TeV}$$

- Further phenomenological study (especially of fermion sector) is required to test the model itself - *In progress*

Summary

- As an alternative to SUSY, a little Higgs model is proposed to describe a TeV scale effective theory.
- It is natural to complete the LHM with all nice features of the SM such as the gauge anomaly cancellation conditions which dictate the SM fermionic spectrum.
- Sensible phenomenological analysis of such models has to start with feasible complete global quantum number assignment to all multiplets.
- Custodial $SU(2)$ symmetry violating shift in the Z mass gives $\Lambda_{SU(4)} \sim 4\pi f \gtrsim 14$ TeV.
- Further phenomenological study is in progress.