# Holographic Mixing Quantified *

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# Warped extra dimension or strongly coupled theory?

- AdS/CFT duality: Extra dimension is a calculational tool
- Randall-Sundrum models equivalent to SM partial compositeness
- How to quantify elementary/composite mixing?
  - Understand structure and phenomenology of 4D dual theory
- Answer: The Holographic Basis:

$$\Phi(x,y) = \varphi^s(x) g^s(y) + \sum_{n=1}^{\infty} \varphi^n_{CFT}(x) g^n(y)$$

# A slice of AdS$_5$

(Randall, Sundrum '97)

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

- Warped geometry $\Longrightarrow$ Energy scales depend on location $y$
- Planck/Weak scale hierachy due to geometry
- Put the SM in the bulk $\Longrightarrow$
  - explain fermion masses
  - gauge coupling unification
  - meet EWPT, signals at LHC

# Holographic dual interpretation

Gravity in warped 5D dual to strongly coupled 4D gauge theory

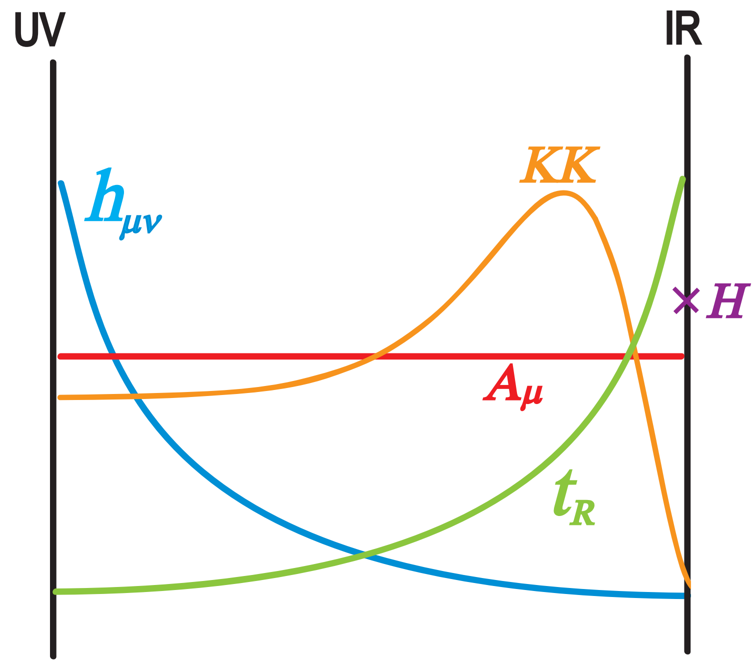
(Maldacena '97)

## AdS/CFT dictionary:

| 5D | | 4D |
|---|---|---|
| bulk field $\Phi(x,y)$ | $\Longleftrightarrow$ | CFT operator $\mathcal{O}(x)$ |
| BC $\Phi(x,y_0) = \varphi^s(x)$ | $\Longleftrightarrow$ | source: $\varphi^s(x)\mathcal{O}(x)$ |
| UV brane | $\Longleftrightarrow$ | UV cutoff for CFT |
| IR brane | $\Longleftrightarrow$ | conformal symmetry breaking |

- Finite UV cutoff $\Longrightarrow$ dynamical source field (elementary)
- Broken conformal symmetry $\Longrightarrow$ bound states (composites)
- Mass eigenstates are elementary/composite mixtures

# Partial compositeness of SM fields




- UV localized $\Longleftrightarrow$ mostly elementary
- IR localized $\Longleftrightarrow$ mostly composite

Can we quantify source/CFT (elementary/composite) mixing?

# Kaluza-Klein mass eigenbasis

Scalar field with tuned bulk and boundary masses

$$S = \int d^5x \sqrt{-g} \left[ -\frac{1}{2}(\partial_M \phi)^2 - \frac{1}{2} a k^2 \phi^2 - b k \phi^2 \left( \delta(y) - \delta(y - \pi R) \right) \right]$$

KK decomposition

$$\phi(x,y) = \sum_{n=0}^{\infty} \phi^n(x) f^n(y),$$

$$\mathrm{BC:} \;\; (++)$$

$$(\partial_5 - bk) f^n(y)\big|_{0,\pi R} = 0$$

Localized massless mode:

$$f^0(y) \sim e^{bky}, \qquad -\infty < b < \infty$$

The fields $\phi^n(x)$ are the mass eigenstates

# Holographic basis

Basic idea:

Expand the bulk field directly in terms of a source field $\varphi^s(x)$ and composite CFT states $\varphi^n_{CFT}(x)$:

$$\Phi(x,y) = \varphi^s(x)g^s(y) + \sum \varphi^n_{CFT}(x)g^n(y)$$

- Leads to kinetic and mass mixing in 4D effective theory
- Mass Eigenstates will be a mixture of $\varphi^s(x)$ and $\varphi^n_{CFT}(x)$
- What about the profiles $g^n(y)$?

# Source profile

Near the UV boundary, AdS/CFT prescribes

$$\Phi(x,y) \to e^{(4-\Delta)ky}\varphi^s(x) + e^{\Delta ky}A(x)$$

(Klebanov, Witten '97);

$\Delta$ is the scaling dimension of $\mathcal{O}$

$$\Delta = 2 + |2-b|$$

$$\Longrightarrow \quad g^s(y) \sim e^{(4-\Delta)ky} = \begin{cases} e^{bky} & \text{for} \quad b < 2 \\ e^{(4-b)ky} & \text{for} \quad b > 2 \end{cases}$$

# CFT composite profiles

Two-point function in dual theory:

$$\langle\mathcal{O}\mathcal{O}\rangle(p) = \mp ip\, \frac{J_{b-1}\left(\frac{ip}{k}\right) Y_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right) - Y_{b-1}\left(\frac{ip}{k}\right) J_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right)}{J_{b-2}\left(\frac{ip}{k}\right) Y_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right) - Y_{b-2}\left(\frac{ip}{k}\right) J_{b-1}\left(\frac{ipe^{\pi kR}}{k}\right)}$$

Poles of $\langle\mathcal{O}\mathcal{O}\rangle$ correspond to masses of composite states.

Identical to the spectrum one gets with the following BC for $g^n(y)$:

$$\begin{aligned} \mathrm{BC}: &\quad (-+) \\ g^n(y)\Big|_0 &= 0 \\ (\partial_5 - bk)g^n(y)\Big|_{\pi R} &= 0 \end{aligned} \tag{1}$$

# Effective 4D Lagrangian in the holographic basis

$$\mathcal{L} = \tfrac{1}{2}\vec{\varphi}^{\mathrm{T}}\mathbf{Z}\Box\vec{\varphi} - \tfrac{1}{2}\vec{\varphi}^{\mathrm{T}}\mathbf{M}^2\vec{\varphi},$$

where $\vec{\varphi}^{\mathrm{T}} = (\varphi^s, \varphi^1_{CFT}, \varphi^2_{CFT}, \cdots)$

$$\mathbf{Z} = \begin{pmatrix} 1 & z_1 & z_2 & z_3 & \cdots \\ z_1 & 1 & 0 & 0 & \cdots \\ z_2 & 0 & 1 & 0 & \cdots \\ z_3 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad \mathbf{M}^2 = \begin{pmatrix} M_s^2 & \mu_1^2 & \mu_2^2 & \mu_3^2 & \cdots \\ \mu_1^2 & M_1^2 & 0 & 0 & \cdots \\ \mu_2^2 & 0 & M_2^2 & 0 & \cdots \\ \mu_3^2 & 0 & 0 & M_3^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Notice kinetic mixing!

$z_n$ and $\mu_n^2$ computed from wavefunction overlap integrals

Diagonalization leads to KK basis

## Examples - Gauge field $A_\mu$

$$\tilde{f}^0(y) = \frac{1}{\sqrt{\pi R}} \qquad (2)$$

$$\begin{pmatrix} A_\mu^0 \\ A_\mu^1 \\ A_\mu^2 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 & -0.19 & 0.13 & \cdots \\ 0 & -0.98 & -0.03 & \cdots \\ 0 & 0.01 & -0.99 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} A_\mu^s \\ A_\mu^{1(CFT)} \\ A_\mu^{1(CFT)} \\ \vdots \end{pmatrix}$$

- massless eigenstate $A_\mu^0(x)$ is primarily elementary
- KK modes are purely composite

# Examples - Right-handed top $t_R$

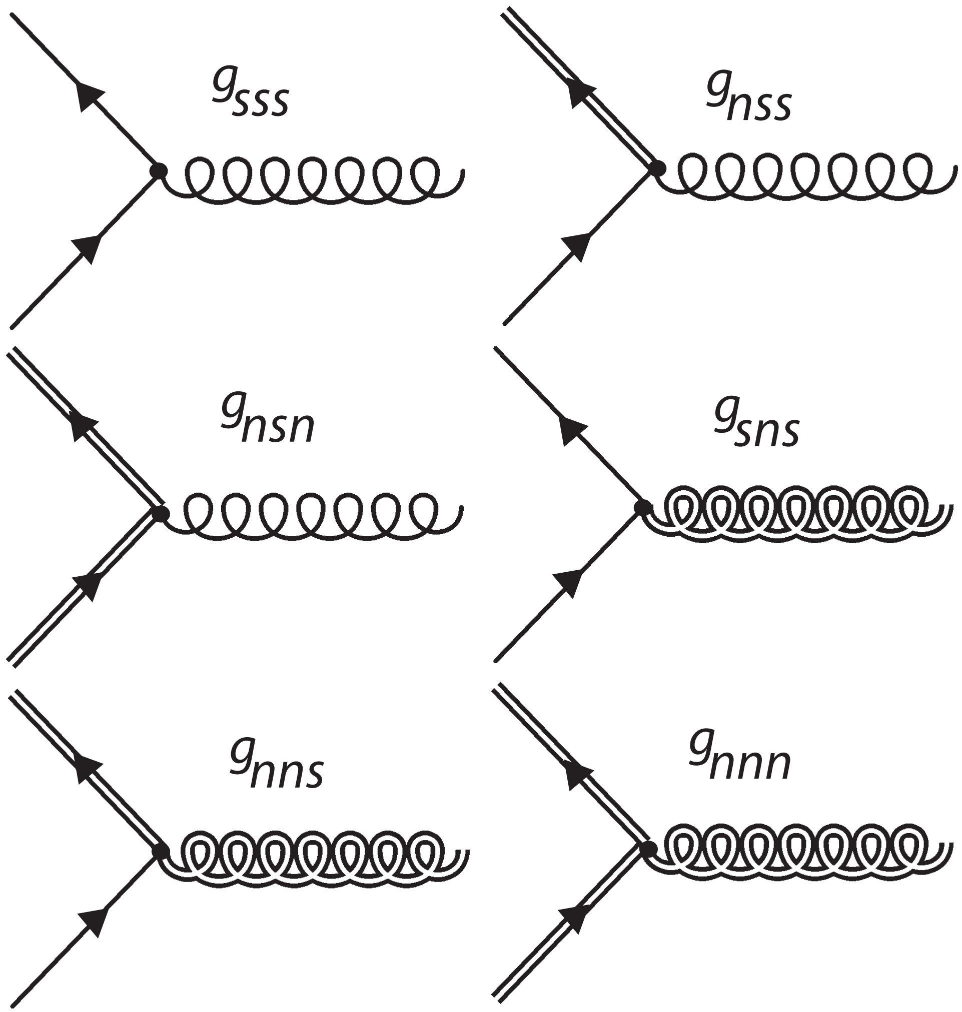
$$\tilde{f}^0(y) = e^{(\frac{1}{2}-c)ky} \qquad m_\psi = ck$$

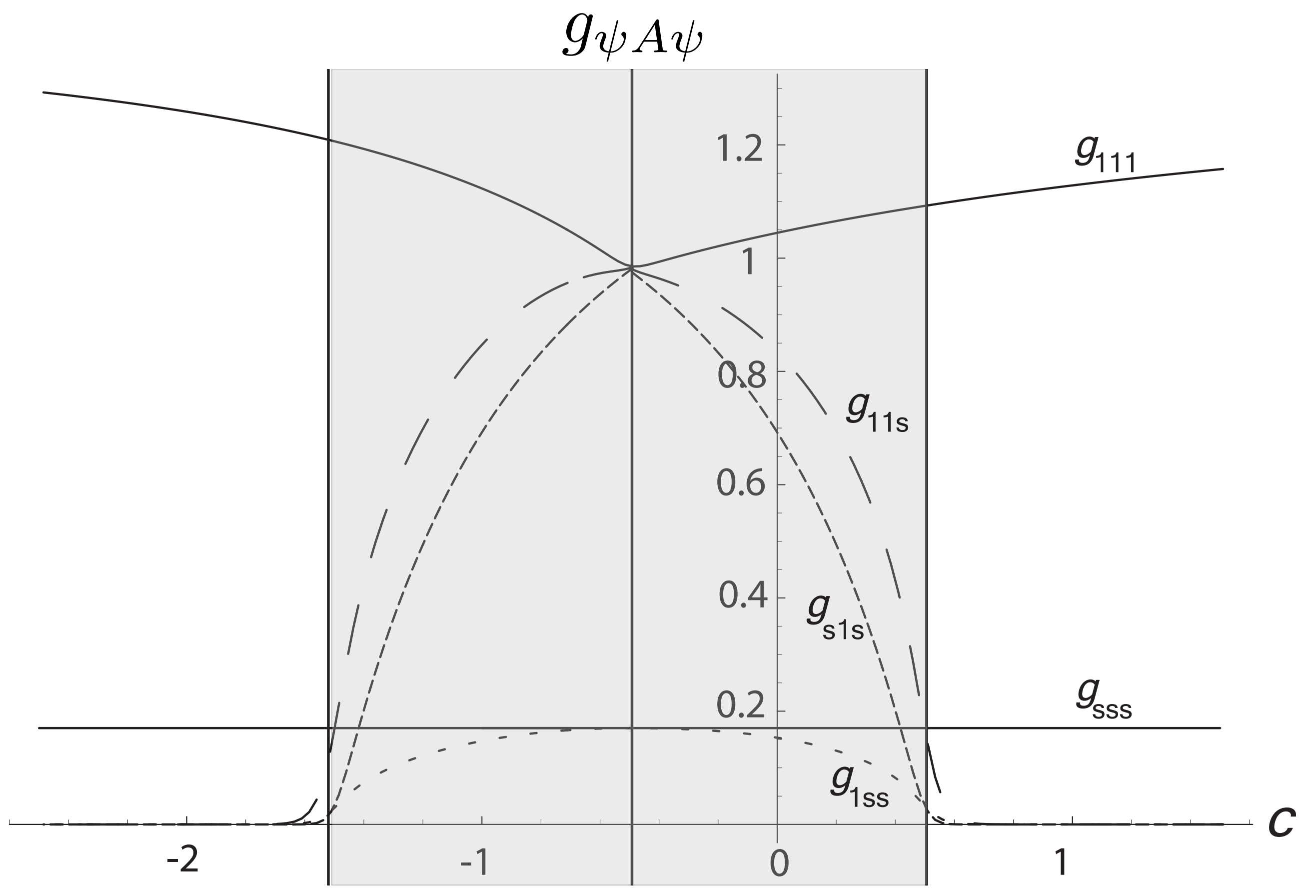
Take e.g. $c = -0.7$

$$\begin{pmatrix} t_R^{(0)} \\ t_R^{(1)} \\ t_R^{(2)} \\ t_R^{(3)} \\ \vdots \end{pmatrix} = \begin{pmatrix} 0.9796 & \sim -1 & \sim 0 & \sim 0 & \cdots \\ -0.1816 & \sim 0 & \sim -1 & \sim 0 & \cdots \\ 0.0514 & \sim 0 & \sim 0 & \sim -1 & \cdots \\ 0.0471 & \sim 0 & \sim 0 & \sim 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} t_R^s \\ t_R^{CFT(1)} \\ t_R^{CFT(2)} \\ t_R^{CFT(3)} \\ \vdots \end{pmatrix}$$

- massless eigenstate $t_R^0(x)$ roughly equal mixture of source/CFT
- KK modes contain elementary component

# Gauge Interactions $g_{\psi A \psi}$





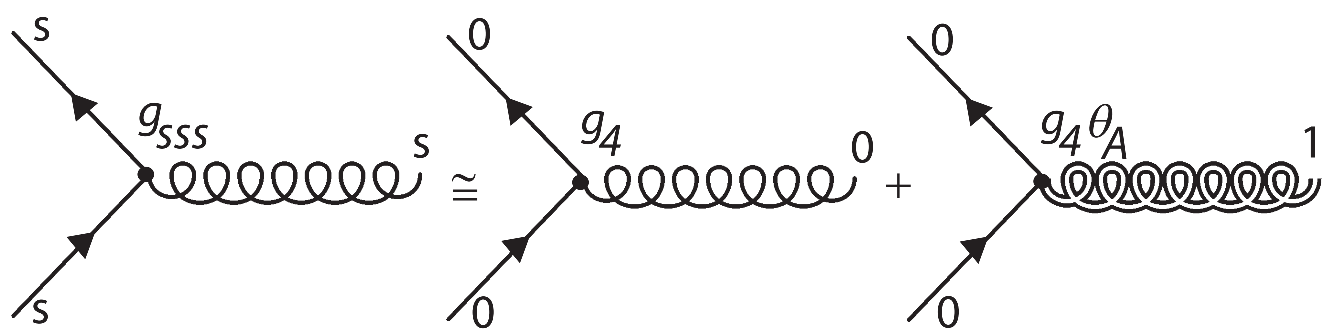

- Light fermions have exponentially suppressed couplings to composites
- Top quark couples strongly to composite sector

# Example: RS GIM mechanism

Occurs because KK gauge bosons couple universally to light fermions

- Light fermions are almost entirely source field: $\psi^0(x) \sim \psi^s(x)$
- Gauge bosons mostly elementary: $A^s_\mu = A^0_\mu + \theta_A A^1_\mu + \ldots$

Only 3-source vertex is relevant:




KK gauge boson couplings are universal for light fermions

$\Longrightarrow$ FCNCs suppressed

# Two sector model of partial compositeness

(Contino, Kramer, Son, Sundrum '06)

$$\mathcal{L} = \mathcal{L}_{elementary} + \mathcal{L}_{composite} + \mathcal{L}_{mix}$$

Goals:

- Capture generic features of models with partial compositeness/warped dimension
- Simplify collider phenomenology/automate computations

Compare with holographic basis

- $\mathcal{L}_{mix}$ contains only mass mixing - no kinetic mixing
- Interpretation of mass eigenstate differs
- Effect on 1) allowed parameter space? 2) Collider signals?

# Conclusions

- Holographic basis: bulk field expanded directly in source and CFT states
- A new tool to quantitatively describe elementary/composite mixing in warped duals
- Warped physics can be precisely translated to language of strongly coupled composites interacting with an elementary sector