B_s^0-$\bar{B}_s^0$ mixing parameters with $N_f = 2+1$ sea quarks in lattice QCD

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In collaboration with:

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HPQCD Collaboration

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1. Introduction: $B_0 - \bar{B}_0$ mixing parameters

$$|B^0_{s/d}(H)\rangle = p|B^0_{s/d}\rangle + q|\bar{B}^0_{s/d}\rangle$$
$$|B^0_{s/d}(L)\rangle = p|B^0_{s/d}\rangle - q|\bar{B}^0_{s/d}\rangle$$

$$\Delta M_{s/d} = M_{s/d}(H) - M_{s/d}(L)$$
$$\Delta\Gamma_{s/d} = \Gamma_{s/d}(H) - \Gamma_{s/d}(L)$$

- experimentally: very well measured

$$\boxed{\Delta M_d|_{exp.} = 0.508 \pm 0.004}$$ **World average**

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Unofficial world average (**R.v.Kooten, FP& CP, April 2006**)

$$\Delta\Gamma_s = 0.097^{+0.041}_{-0.042}\, ps^{-1} \Longrightarrow \boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)_s \simeq 0.15 \pm 0.06}$$

- theoretically: In the Standard Model

$$\Delta M_s|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \hat{B}_{B_s}$$

where $x_t = m_t^2/M_W^2$, η_2^B is a perturbative QCD correction factor and $S_0(x_t)$ is the `Inami-Lim` function.

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Non-perturbative input

$$\frac{8}{3} f_{B_s}^2 B_{B_s}(\mu) M_{B_s}^2 = \langle \bar{B}_s^0 | O_L | B_s^0 \rangle(\mu) \quad \text{with} \quad O_L \equiv [\overline{b^i}\, s^i]_{V-A} [\overline{b^j}\, s^j]_{V-A}$$

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For $\Delta\Gamma_s$ one needs either O_S and O_L, or O_3 and O_L

$$O_S \equiv [\overline{b^i}\, s^i]_{S-P} [\overline{b^j}\, s^j]_{S-P}$$
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2. Lattice formulations for light and heavy quarks

MILC $N_f^{\mathbf{sea}} = 2 + 1$ configurations

Light quarks (sea and valence): improved staggered quarks (`Asqtad`)

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* accessible dynamical simulations

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$$\Upsilon \quad 2S - 1S\,\texttt{splitting} \rightarrow a^{-1}$$

$$\Upsilon \rightarrow m_b$$

$$\texttt{Kaon} \rightarrow m_s$$

3. Relevant four fermion operators

(for ΔM_s and $\Delta\Gamma_s$)

$$\left.\begin{aligned} O_L &\equiv [\overline{b^i}\, s^i]_{V-A}[\overline{b^j}\, s^j]_{V-A} \\ O_S &\equiv [\overline{b^i}\, s^i]_{S-P}[\overline{b^j}\, s^j]_{S-P} \\ O_3 &\equiv [\overline{b^i}\, s^j]_{S-P}[\overline{b^j}\, s^i]_{S-P} \end{aligned}\right\} \text{lowest order in } 1/M$$

$$O_L^{M1} \equiv \frac{1}{2aM_0}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^i]_{V-A}[\overline{b^j}\, s^j]_{V-A} + [\overline{b^i}\, s^i]_{V-A}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^j]_{V-A}\right\}$$

$$O_S^{M1} \equiv \frac{1}{2aM_0}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^i]_{S-P}[\overline{b^j}\, s^j]_{S-P} + [\overline{b^i}\, s^i]_{S-P}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^j]_{S-P}\right\}$$

$$O_3^{M1} \equiv \frac{1}{2aM_0}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^j]_{S-P}[\overline{b^j}\, s^i]_{S-P} + [\overline{b^i}\, s^j]_{S-P}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^i]_{S-P}\right\}$$

with i, j colour indices and aM_0 the bare b mass in lattice units.

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* Dimension 7 operators O_X^{M1} required at $\mathcal{O}(\Lambda_{QCD}/M)$

* O_3 and O_L lead to smaller theoretical uncertainties in the calculation of $\Delta\Gamma_s$ than O_S and O_L (**Lenz & Nierste**):

$$\langle O_3\rangle = -\langle O_S\rangle - 1/2\langle O_L\rangle + \mathcal{O}(1/M)$$

4. One-loop matching

The input for the SM prediction for ΔM_s is

$$\langle O_L \rangle^{\overline{MS}}(\mu) \equiv \frac{8}{3} f_{B_s}^2 B_{B_s}^{\overline{MS}}(\mu) M_{B_s}^2$$

that is related to the lattice operators through $\mathcal{O}(\alpha_s)$, $\mathcal{O}\left(\frac{\Lambda_{QCD}}{M}\right)$ and $\mathcal{O}\left(\frac{\alpha_s}{aM}\right)$ by

$$\frac{a^3}{2M_{B_s}} \langle O_L \rangle^{\overline{MS}}(\mu) = [1 + \alpha_s \cdot \rho_{LL}]\langle O_L \rangle(1/a) + \alpha_s \cdot \rho_{LS} \langle O_S \rangle(1/a) + \left[\langle O_L^{M1} \rangle(1/a) - \alpha_s \left(\zeta_{10}^{LL} \langle O_L \rangle(1/a) + \zeta_{10}^{LS} \langle O_S \rangle(1/a)\right)\right]$$

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* The one-loop renormalization coefficients $\rho_{XY} = \rho_{XY}^{\overline{MS}}(\mu) - \rho_{XY}^{latt.}(1/a)$
* ζ_{10}^{XY} are necessary to subtract $\mathcal{O}\left(\frac{\alpha_s}{aM}\right)$ power law cont. from $\langle O_L^{M1} \rangle$

⟹ Similarly one can define bag parameters for the operators O_S and O_3 entering in the calculation of $\Delta\Gamma_s$

$$\langle O_S\rangle^{\overline{MS}}_{(\mu)} \equiv -\frac{5}{3} f^2_{B_s} \frac{B^{\overline{MS}}_S(\mu)}{R^2} M^2_{B_s} \,; \qquad \langle O_3\rangle^{\overline{MS}}_{(\mu)} \equiv \frac{1}{3} f^2_{B_s} \frac{\tilde{B}^{\overline{MS}}_S(\mu)}{R^2} M^2_{B_s}$$

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* Analogous matching relations

* Renormalization of these operators at one-loop does not involve new lattice operators

5. Numerical simulations

We calculate both 3-point (for any $\hat{Q} = Q_X, Q_X^{1j}$) and 2-point correlators

$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_s}(\vec{x}_1, t_1) \left[\hat{Q}\right](0) \Phi^{\dagger}_{\bar{B}_s}(\vec{x}_2, -t_2) | 0 \rangle$$

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* Two ensembles of MILC configurations (560 and 414 conf.) with $(m_u^{sea} = m_d^{sea})/m_s = 0.25, 0.50$ and $a^{-1} = 1.6\mathrm{GeV}$.

Fitting

We carried out bayesian **simultaneous** fits of the 3-point and 2-point correlators to the forms

$$C^{(4f)}(t_1, t_2) = \sum_{j,k=0}^{N_{exp}-1} A_{jk} (-1)^{j \cdot t_1} (-1)^{k \cdot t_2} e^{-E_B^{(j)}(t_1-1)} e^{-E_B^{(k)}(t_2-1)}$$

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$$\boxed{\langle \hat{Q} \rangle \equiv \langle \bar{B}_s | \hat{Q} | B_s \rangle = \frac{A_{00}}{\xi_0}}$$

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* Fit directly to $C^{(4f)}$ and C^B rather than take ratios

* Use entire range $1 \leq t_1, t_2 \leq 16$

* We let $N_{exp} \leq 7-9$

6. Main results to date

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ [GeV]	0.281(21)	0.289(22)
$f_{B_s}\sqrt{B_{B_s}^{\overline{MS}}(m_b)}$ [GeV]	0.227(17)	0.233(17)
$f_{B_s}\frac{\sqrt{B_S^{\overline{MS}}(m_b)}}{R}$ [GeV]	0.295(22)	0.301(23)
$f_{B_s}\frac{\sqrt{\tilde{B}_S^{\overline{MS}}(m_b)}}{R}$ [GeV]	0.305(23)	0.310(23)

$$\langle O_L\rangle^{\overline{MS}}(\mu) \equiv \frac{8}{3} f_{B_s}^2 B_{B_s}^{\overline{MS}}(\mu) M_{B_s}^2 \qquad \langle O_S\rangle^{\overline{MS}}_{(\mu)} \equiv -\frac{5}{3} f_{B_s}^2 \frac{B_S^{\overline{MS}}(\mu)}{R^2} M_{B_s}^2$$

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$f_{B_s}\sqrt{B_{B_s}^{\overline{MS}}(m_b)}$ [GeV]	0.227(17)	0.233(17)

Main Errors in $f_{B_s}^2 B_{B_s}(m_b)$

Statistical + Fitting	**9 %**
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Light sea quark mass dependence smaller than current errors (1%-3%)
$\rightarrow$ use the $m_f/m_s = 0.25$ results in the following comparison with experimental data.

Comparison with experiment: ΔM_s

CDF measurement:

$$\boxed{\Delta M_s|_{exp.} = 17.77 \pm 0.10(stat) \pm 0.07(syst)\, ps^{-1}}$$

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Conversely, one can use $\Delta M_s|_{exp.}$ and our value of $f_{B_s}^2 \hat{B}_{B_s}$ to get

$$|V_{ts}^* V_{tb}| = (3.8 \pm 0.3 \pm 0.1) \times 10^{-2}$$

Comparison with experiment: $\Delta\Gamma_s$

Unofficial experimental world average **(R.v.Kooten, FPCP, Vancouver, April 2006)**

$$\Delta\Gamma_s^{exp.} = 0.097^{+0.041}_{-0.042}\, ps^{-1} \Longrightarrow \boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)^{exp.}_{B_s} \simeq 0.15 \pm 0.06}$$

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Use NLO formula of **Lenz& Nierste**

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)^{theor.}_{B_s} = \left(\frac{f_{B_s}}{245\mathrm{MeV}}\right)^2 \left[0.170\, B_{B_s} + 0.059\, \tilde{B}_S - 0.044\right]$$

$$\Longrightarrow \qquad \left(\frac{1}{245\mathrm{MeV}}\right)^2 \left[0.170 \left(f^2_{B_s} B_{B_s}\right) + 0.059 R^2 \left(\frac{f^2_{B_s} \tilde{B}_S}{R^2}\right) - 0.044 f^2_{B_s}\right]$$

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Inserting **HPQCD's** $f_{B_s} = 0.260(29)\mathrm{GeV}$, $R^2 \equiv \frac{(\overline{m}_b+\overline{m}_s)^2}{M^2_{B_s}} = 0.652$ and our results for $f_B B^2_B$

$$\boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)^{theor.}_{B_s} = 0.16 \pm 0.03 \pm 0.02}$$

Comparison with other (lattice) work

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$	JLQCD ($N_f = 2$)
$B_{B_s}^{\overline{MS}}(m_b)$	0.76(11)	0.80(12)	-
$B_{B_s}^{\overline{MS}}(m_b)$ (no 1/M correc.)	0.88(13)	0.92(14)	0.85(6)
$\hat{B}_{B_s}$	1.17(17)	1.23(18)	1.30(9)

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$	Hashimoto et al. (quenched)
$\frac{B_S^{\overline{MS}}(m_b)}{R^2}$	1.29(19)	1.34(20)	1.24(16)
$\frac{\tilde{B}_S^{\overline{MS}}(m_b)}{R^2}$	1.38(21)	1.42(21)	-
			Becirevic et al. (quenched)
$B_S^{\overline{MS}}(m_b)$	0.84(13)	0.87(13)	0.84(2)(4)
$\tilde{B}_S^{\overline{MS}}(m_b)$	0.90(14)	0.93(14)	0.91(3)(8)

7. Summary and future work

Results are presented for the **first $N_f = 2+1$ determination** of the B_s^0 meson mixing parameters $f_{B_s}^2 B_{B_s}$, $f_{B_s}^2 \frac{B_S}{R^2}$ and $f_{B_s}^2 \frac{\tilde{B}_S}{R^2}$

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Repeat calculations with light (down) valence quark masses (corresponding to B_d) and determine $[f^2_{B_s} B_{B_s}]/[f^2_{B_d} B_{B_d}]$.

* (Partial) cancellation of chiral corrections
* (Almost complete) cancellation of a^{-3} and higher order matching uncertainties

Main sources of error reduced $\longrightarrow$ Chiral extrapolation to the physical point using Staggered χPT (incorporates discretization and perturbative corrections).

* More relevant for B_d^0 mixing parameters since we need an extrapolation in both valence and sea quark masses.

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Same analysis using **Fermilab** action to describe b quarks (instead of NRQCD)

* Main advantage: Part of the renormalization can be done non-perturbatively → much smaller matching uncert.

(**R.T. Evans, A.X. El-Khadra and M. Di Pierro**, work in progress)

X

Staggered Asqtad action

(for light u, d and s valence and sea quarks)

→ Advantages of staggered fermions

* **good chiral properties**

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These **problems** can be **reduced** by using **improved staggered fermion actions**

J.F.Lagae and D.K.Sinclair

G.P.Lepage

NRQCD action

(for b valence quarks)

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$$\mathcal{L}_Q = \overline{\psi} \left(D_t - \frac{\vec{D}^2}{2 m_Q a} - c_4 \frac{\vec{\sigma} \cdot \vec{B}}{2 m_Q a} + \ldots \right) \psi$$

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* c_i fixed pert. or non-pert. matching to QCD

Much faster calculation of quark propagators

$$G(\vec{x}, t+1) = \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)^n U^\dagger(\vec{x}, t)\left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2}\right) G(\vec{x}, t)$$

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On lattice, hamiltonian is (improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$):

$$aH_0 = -\frac{\Delta^{(2)}}{2(aM_0)} \quad \textbf{non-relat. kinetic energy oper.}$$

$$a\delta H = -c_1 \frac{(\Delta^{(2)})^2}{8(aM_0)^3} + c_2 \frac{i}{8(aM_0)^2}\left(\nabla\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\nabla\right)$$

$$-c_3 \frac{1}{8(aM_0)^2}\boldsymbol{\sigma}\cdot(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla})$$

$$-c_4 \frac{1}{2(aM_0)}\boldsymbol{\sigma}\cdot\tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24(aM_0)} - c_6 \frac{(\Delta^{(2)})^2}{16n(aM_0)^2} + \cdots$$

relativistic and discretization corrections