Solving problems of 4D minimal SO(10) model in a warped extra dimension

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Based on the work in collaboration with
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Brief review of Minimal SO(10) model

Babu-Mohapatra(93'), Fukuyama-Okada (01'), Dutta-Mimura-Mohapatra(04),…

- This model is constructed to have two Higgs multiplets **10** + **126** in the Yukawa coupling with matter **16**

$$W_Y = Y_{10}^{ij} \mathbf{16}_i H_{10} \mathbf{16}_j + Y_{126}^{ij} \mathbf{16}_i H_{126} \mathbf{16}_j$$

- This leads to some simple relations for the quark and lepton mass matrices:

$$M_e = c_d \left(M_d + \kappa M_u \right)$$

- 13 inputs : 6 quark masses, 3 angles + 1 phase in CKM matrix, 3 charged-lepton masses.

  $\Rightarrow$ fix $|c_d|$ and κ

Fermion Mass matrices

- All the fermions' mass matrices are written in terms of two basic mass matrices

$$
\begin{aligned}
M_u &= c_{10} M_{10} + c_{126} M_{126} \\
M_d &= M_{10} + M_{126} \\
M_D &= c_{10} M_{10} - 3 c_{126} M_{126} \\
M_e &= M_{10} - 3 M_{126} \\
M_R &= c_R M_{126}
\end{aligned}
$$

- Note that M126 is determined so as to reproduce the charged fermion mass matrix data. Accordingly, the right-handed neutrino mass scale is determined to produce the correct scale for the light neutrino masses, which is found to be $M_R = 10^{14}$ GeV.

Predictions for the Neutrino mass matrices in Minimal SO(10) model

- Mass relation for the neutrino sector:

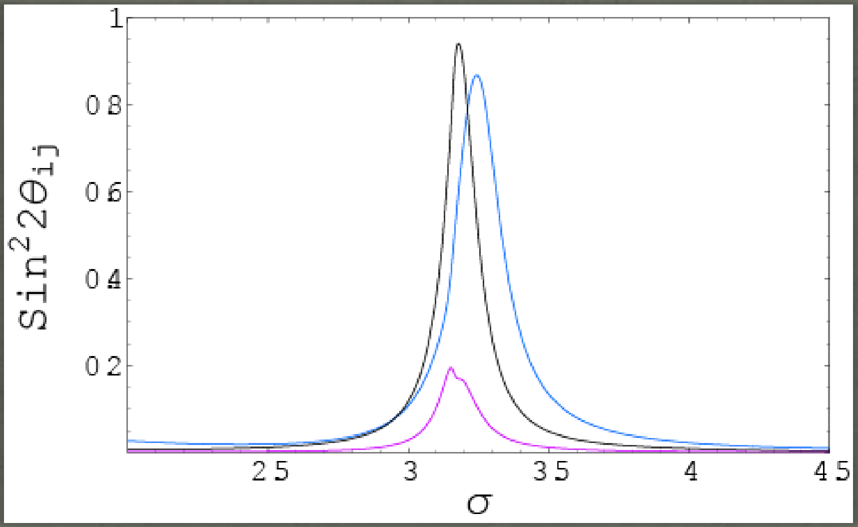
$$M_D = M_u + \frac{1 + 3/c_d}{\kappa} (M_d - M_e)$$

$$M_R = c_R (M_d - M_e)$$

- Type-I See-saw mass matrix: $M_\nu = -M_D M_R^{-1} M_D$
- All lepton mass matrices are determined by only the quark mass matrices!
- ***Predictive theory for the neutrino oscillation data!***
- Other phenomena related to the Yukawa couplings can be evaluated precisely! (LFV, muon g-2, EDM, proton decay)

Predictions in neutrino sector

- We have only one parameter $\sigma = \arg(c_d)$, left free. So, we can make definite predictions.



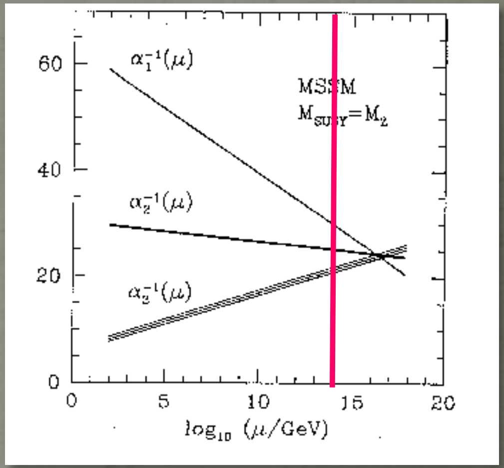
Fukuyama-Okada (01')

For $\sigma = 3.198$ [rad] :

$\sin^2 2\theta_{12} \sim 0.72, \quad \sin^2 2\theta_{23} \sim 0.90, \quad \sin^2 2\theta_{13} \sim 0.16$

Mild mass hierarchy problem: $M_R < M_{GUT}$

- In order to realize the gauge coupling unification, the simplest way is to put all the VEV's at the GUT scale.
- On the other hand, neutrino oscillation data shows the existence of an intermediate mass scale, which may destabilize the successful gauge coupling unification in the MSSM.



$$M_\nu = M_D^T M_R^{-1} M_D \Rightarrow M_R \sim 10^{11-14}\ \mathrm{GeV}$$

- It is necessary to have an additional suppression as follows $M_R = cM_{\mathrm{GUT}},\ c = \mathcal{O}(10^{-2} - 10^{-5})$

Question: *Can we realize $\nu R = M_{GUT}$ while keeping the successful data fitting of the fermion masses and mixings?* The key point of this work is to extend the model into ***a warped five dimensional space.***

Higgs sector of the minimal SO(10) model

- Suppose a minimal Higgs sector which is able to break SO(10) to MSSM

$$\mathbf{10} \oplus \mathbf{126} \oplus \overline{\mathbf{126}} \oplus \mathbf{210}$$

- The minimal Higgs potential constructed from the above Higgs fields is given by

$$W = m_1 \Phi^2 + m_2 \Delta \overline{\Delta} + m_3 H^2 + \lambda_1 \Phi^3 + \lambda_2 \Phi \Delta \overline{\Delta} + \lambda_3 \Phi \Delta H + \lambda_4 \Phi \overline{\Delta} H$$

$$H = \mathbf{10}, \ \Delta = \mathbf{126}, \ \overline{\Delta} = \overline{\mathbf{126}}, \ \Phi = \mathbf{210}$$

Fukuyama-Ilakovac-T.K.-Meljanac-Okada (05')

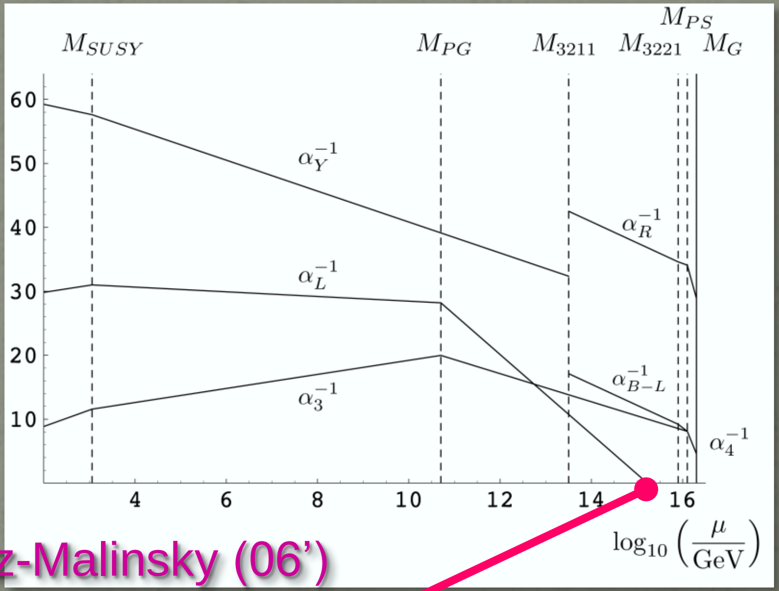
- The minimal SO(10) model utilizes a large representation of the Higgs fields, it means that the gauge coupling blows up soon after the GUT scale! *i.e.* **The cutoff scale of the model is the GUT scale!**

$$\Lambda(\text{4D cutoff scale}) = M_{\text{GUT}}$$

- We need a theory which has a rather low cutoff scale, and not the Planck scale. This can easily be realized in an extra-dimensional set up.

Gauge coupling unification

- Example



Bertolin-Scwetz-Malinsky (06')

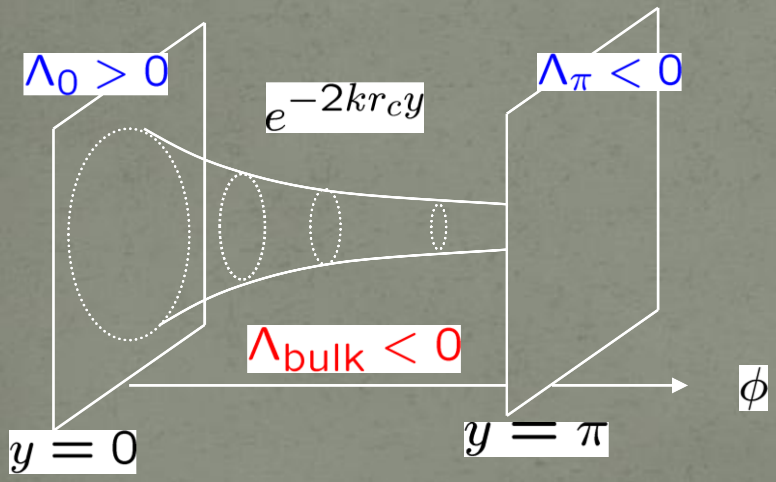
- *Blows up before unify!*
- Intermediate scale (<< M_{GUT}) threshold makes trouble.

Basics of Randall-Sundrum Scenario

- Tuning the bulk and brane cosmological constant, RS found the solution of Einstein equation

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2 \quad (-\pi \le y \le \pi)$$

4D Minkowski part



Model free parameters

- M_5 : 5D Planck scale
- k : AdS curvature
- $e^{-kr_c\pi}$: warp factor or r_c

Basic setup of the 5D warped SUSY model

- Lagrangian for the bulk field:

$$\mathcal{L} = \int dy \left\{ \int d^4\theta \, r_c \, e^{-2kr_c|y|} \left(H^\dagger e^{-V} H + H^c e^V H^{c\dagger} \right) + \int d^2\theta e^{-3kr_c|y|} H^c \left[\partial_y - \left(\frac{3}{2} - c \right) kr_c \epsilon(y) - \frac{\chi}{\sqrt{2}} \right] H + h.c. \right\}$$

- Here an N=2 vector multiplet is decomposed into N=1 vector and adjoint scalar multiplet.

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\theta^2\bar{\theta}^2 D \,,$$

$$\chi = \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F$$

Solution of the bulk field EOM

- Suppose that an adjoint scalar, Σ, which can be arisen from the N=2 vector multiplet, develops the VEV as

$$\langle\Sigma\rangle \equiv 2\alpha k r_c \epsilon(y)$$

- This provides a shift of the bulk mass term

$$\left[\left(\frac{3}{2}-c\right)kr_c\right] H^c H \to \left[\left(\frac{3}{2}-c\right)kr_c + \langle\Sigma\rangle\right] H^c H$$

- A solution for the bulk field's equation of motion is given

$$H = \frac{1}{\sqrt{N}} e^{(3/2-c+\alpha)kr_c|y|}\, h(x^\mu)\ ; \quad \frac{1}{N} = \frac{(1-2c+2\alpha)k}{e^{(1-2c+2\alpha)kr_c\pi}-1}$$

$$H(y=\pi) \simeq \sqrt{(1-2c+2\alpha)k}\ \omega^{-1}\ h(x^\mu)$$

if $e^{(1/2-c+\alpha)kr_c\pi} \gg 1$

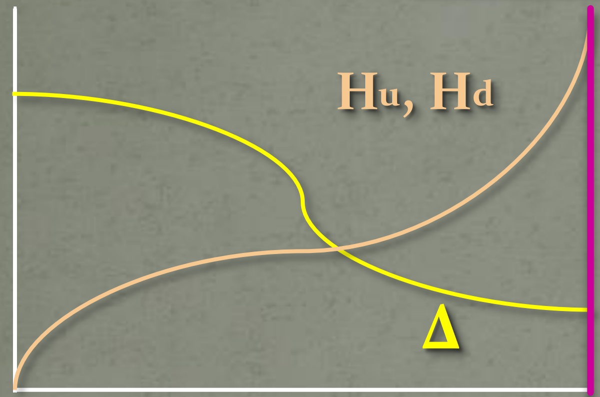
$$H(y=\pi) \simeq \sqrt{-(1-2c+2\alpha)k}\ \omega^{-1} e^{(1/2-c+\alpha)kr_c\pi}\ h(x^\mu)$$

if $e^{(1/2-c+\alpha)kr_c\pi} \ll 1$

Our setup-*wave function profile for the bulk Higgs*

- **Matter is on the IR brane while the Higgs is in the bulk.**

$$\mathbf{16} \ni Q, u, d, L, e, \nu_R$$



y=0
UV brane
(Planck brane)

y=π
IR brane
("GUT brane")

$$e^{-\pi k r_c}\Lambda_{\rm UV} = \Lambda_{\rm IR}$$

- **GUT scale as a 4D cutoff scale:** GUT scale is obtained from Planck scale by a ***mini-warping***, and is found to be the cutoff scale of the 4D effective theory.
- **Higgs doublets** (Hu, Hd) are localized toward IR brane, which enable to produce O(1) Yukawa coupling.
- **Singlet Higgs (Δ)** which gives a right-handed neutrino mass is localized toward UV brane, hence is well separated to the matter on the IR brane.

4D effective action on the IR brane

- Lagrangian for a chiral multiplet on the IR brane (Φ) is given by

$$\mathcal{L}_{\rm IR} = \int d^4\theta \, \omega^\dagger \omega \, \Phi^\dagger \Phi + \left[\int d^2\theta \, \omega^3 \, W(\Phi) + h.c. \right] \quad (\omega \equiv e^{-\pi k r_c})$$

- The interaction between brane field (Φ) and bulk field (H) is given by

$$\mathcal{L}_{int} = \int d^2\theta \omega^3 \frac{Y}{\sqrt{M_5}} \Phi^2 H(y=\pi) + h.c.$$

- After rescaling of brane field, $\Phi \to \Phi/\omega$, the resultant Yukawa coupling could be unsuppressed or suppressed depending on the choice of the bulk mass parameters

$$Y_{4D} \sim Y \; ; \; \text{if } e^{(1/2-c+\alpha)k r_c \pi} \gg 1$$
$$Y_{4D} \sim Y e^{(1/2-c+\alpha)k r_c \pi} \ll Y \; ; \; \text{if } e^{(1/2-c+\alpha)k r_c \pi} \ll 1$$

Effects of the VEV of adjoint scalar

Suppose adjoint scalar gets a non-zero VEV, then it is always possible to have a mass term for the bulk field as follows:

$$\mathcal{L}_{int} \supset \frac{1}{2}\int d^2\theta\omega^3 Q_X \langle\Sigma_X\rangle H^c H + h.c.$$

This gives an additional contribution to the bulk mass term, which is determined by the charge of each bulk field:

$$M_{\text{bulk Higgs}} \sim \left(\frac{3}{2} - c\right) kr_c + \frac{1}{2} Q_X \langle\Sigma_X\rangle$$

Basically, the wave function get suppressed from the bulk mass in the following manner:

$$H \sim e^{-M_{\text{bulk Higgs}}|y|} h(x)$$

Effects of the VEV of adjoint scalar

- Hence, the more charge fields have U(1) charge, the more the wave function of the bulk field receives suppression.

$$\begin{aligned} \mathbf{10} &= \mathbf{5}_{+2} \oplus \overline{\mathbf{5}}_{-2}\,, \\ \overline{\mathbf{126}} &= \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \overline{\mathbf{10}}_{+6} \oplus \mathbf{15}_{-6} \oplus \overline{\mathbf{45}}_{-2} \oplus \mathbf{50}_{+2} \end{aligned}$$

- Due to the large U(1) charge for the singlet, it will be suppressed on the IR brane, that is really what we needed to suppress the Yukawa coupling for the right-handed neutrino!

$$W = M_R \nu_R \nu_R \leftarrow Y_{\nu_R}^{ij} \left\langle \mathbf{1}_{+10} \right\rangle \mathbf{1}^i_{-5} \mathbf{1}^j_{-5}$$

Effectively suppressed Yukawa couplings for the right-handed neutrinos

- The Majorana mass for the right-handed neutrino is described by

$$W = Y_{126} \langle \Delta \rangle \nu_R \nu_R$$
$$M_R = Y_{126} v_R \; ; \;\; \langle \Delta \rangle = v_R = 10^{14} \text{ GeV}$$

- We wanted to construct a model with an extra suppression factor

$$M_R = \epsilon \times Y_{126} v_R \; ; \quad v_R = M_{\text{GUT}}$$

- Such a suppression factor can be achieved by localizing the Higgs in the bulk, that can also be related to the hierarchy between the GUT scale and the Planck scale:

$$M_R = Y^{ij}_{126} v_R \rightarrow M_R = Y^{ij}_{126}(\epsilon v_R); \quad v_R = M_{\text{GUT}}$$

- One interesting example:

$$\epsilon = \omega = M_{\text{GUT}}/M_P \simeq 10^{-2} \quad (\text{for } c = -7/2, \;\; \langle \Sigma \rangle = -k r_c)$$

Summary

- To fit the neutrino oscillation data, the mass scale of right-handed neutrinos lies at the intermediate scale.
- Such a new scale below the GUT scale may spoil the gauge coupling unification.
- We have considered the minimal SO(10) model in the warped extra dimension. The warped geometry leads to a low scale effective cutoff in effective four dimensional theory, and we fix it at the GUT scale:

$$M_{\mathsf{GUT}} = e^{-kr_c\pi} M_{\mathsf{PL}}$$

- The effective right-handed neutrino Yukawa couplings can be lowered while keeping any VEV's at the GUT scale.

$$M_R = e^{-kr_c\pi} M_{\mathsf{GUT}} = e^{-2kr_c\pi} M_{\mathsf{PL}}$$