Lepton masses and mixing without Yukawa hierarchies

W.A.P. and O.Z. Phys. Rev. D74, 093007 (2006)

.

William A. Ponce
Instituto de Física.
Universidad de Antioquia.
Medellín Colombia
PHENO 07 Madison

May 8, 2007

Experimental results

Super-Kamiokande, KamLAND and SNO:

$$\begin{aligned}
\Delta m^2_{atm} &= 2.4(1^{+0.21}_{-0.26}) \times 10^{-3}\text{eV}^2, \\
\Delta m^2_{sol} &= 7.92(1 \pm 0.09) \times 10^{-5}\text{eV}^2, \\
\sin^2\theta_{atm} &= 0.44(1^{+0.44}_{-0.22}), \\
\sin^2\theta_{sol} &= 0.314(1^{+0.18}_{-0.15}), \\
\sin^2\theta_{chooz} &\approx 0.009 \ .
\end{aligned} \tag{1}$$

Implies neutrino oscillations. At least two neutrinos with very small masses.

The explanation requires physics beyond the SM

- Existence of ν_L^c.
- Breaking of $B - L$.

Masses for ν_L

- With ν_L^c we can have $\nu_L^c C \nu_L$ Dirac Masses. Requires $h_\nu^\phi \leq 10^{-13}$.
- But $\nu_L^c C \nu_L^c$ Majorana Masses. $\rightarrow$ See saw. Requires $M_{\nu^c} \sim 10^{11}$ GeV.
- Alternative: Majorana masses for the $I = 1$, $\psi_{lL} C \psi_{lL}^c \rightarrow$ triplet Scalar Higgs. Requires a new very small mass scale.
- The Zee Mechanism (one loop): Couple the $L = 2$ Lorentz scalar $\psi_{lL} C \psi_{lL}$ to h^+ a charged singlet scalar.
- The Zee-Babu mechanism (two loop): couple $e_R^- C e_R^-$ to k^{++} a double charged singlet scalar.

3-3-1

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X$$

- Free of gauge anomalies *iff* N_F multiple of 3.
- Peccei-Quinn symmetry can be easily implemented.
- Third quark family different from other two.
- Scalar sector with good candidates for dark matter.
- Suitable for explaining neutrino properties.
- Strong hierarchy in the Yukawas can be avoided.

Electric Charge Operator

$$Q = \frac{\lambda_{3L}}{2} + \frac{b\lambda_{8L}}{\sqrt{3}} + XI_3$$

$b = 1/2, 3/2, 5/2, \ldots n/2$

- $b = 1/2$ models without exotic electric charges.
- $b = 3/2$ the Pisano-Pleitez-Frampton model.

For $b = 1/2$ there are:

- 2 one family models reported in the literature.
- 4 different 3 family models reported in the literature.
- There are 4 more three family models.

If exotic electric charges are allowed, there are an ∞ number of models.

Model with Exotic electrons

M.Özer, Phys. Rev. D**54**, 1143 (1996); J.C.Salazar, W.A.Ponce and D.A.Gutierez, Phys. Rev D**75**, 075016 (2007).

Quarks:

$Q^i_L = (u^i, d^i, U^i)_L \sim (3, 3^*, 1/3),\ i = 1, 2.$ Two families.
$Q^3_L = (u^3, d^3, D)_L \sim (3, 3, 0).$ Third family.
$U^i_L,\ i = 1, 2$ two exotic Up quarks.
D_L an exotic Down quark.
Right-Handed quarks $u^{ac}_L \sim (3^*, 1, -2/3),\ d^{ac}_L \sim (3^*, 1, 1/3)$
$a = 1, 2, 3,, D^c_L \sim (3^*, 1, 1/3), U^{ic}_L \sim (3^*, 1, -2/3). i = 1, 2.$

Leptons:

$L_{lL} = (\nu^0_l, l^-, E^-_l)_L \sim (1, 3, -2/3), l = e, \mu, \tau.$
Singlets $l^+_L \sim (1, 1, 1)\ E^+_{lL} \sim (1, 1, 1).$
E_{lL} three exotic charged electrons.
Right-handed neutrinos are not present.

Scalar sector

$$
\begin{aligned}
\langle\phi_1\rangle^T &= \langle(\phi_1^+, \phi_1^0, \phi_1'^0)\rangle = \langle(0, v_1, 0)\rangle \sim (1, 3, 1/3) \\
\langle\phi_2\rangle^T &= \langle(\phi_2^0, \phi_2^-, \phi_2'^-)\rangle = \langle(v_2, 0, 0)\rangle \sim (1, 3, -2/3) \\
\langle\phi_3\rangle^T &= \langle(\phi_3^+, \phi_3^0, \phi_3'^0)\rangle = \langle(0, 0, V)\rangle \sim (1, 3, 1/3), \\
\langle\phi_4\rangle^T &= \langle(\phi_4^+, \phi_4^0, \phi_4'^0)\rangle = \langle(0, 0, v_4)\rangle \sim (1, 3, 1/3).
\end{aligned}
$$

with the hierarchy: $v_1 \sim v_2 \sim v_3 << V \sim$ a few TeV.

The anomaly free discrete Z_2 symmetry

$$
Z_2(\phi_1, \phi_2, \phi_3, E_{lL}^+) = 1, \quad Z_2(\phi_4, L_{lL}, l_L^+) = 0.
$$

(F. Yin, Phys. Rev. D75, 073010 (2007): discrete symmetry is A_4).

The charged lepton sector 1

$$(2)\ \mathcal{L}^l_Y = \sum_{\alpha=1,3,4}\sum_{l,l'=e,\mu,\tau} L_{lL}\phi^*_\alpha C(h^{E\alpha}_{ll'}E^+_{l'} + h^{e\alpha}_{ll'}l'^+)_L + h.c.,$$

In the flavor basis $\vec{E}_6 = (e,\ \mu,\ \tau,\ E_e,\ E_\mu,\ E_\tau)$, with the discrete symmetry Z_2 enforced and $v_1 = v_2 = v_3 \equiv v \sim 10^2$ GeV

$$M^e = v\begin{pmatrix} 0 & 0 & 0 & h^{E1}_{ee} & h^{E1}_{e\mu} & h^{E1}_{e\tau} \\ 0 & 0 & 0 & h^{E1}_{\mu e} & h^{E1}_{\mu\mu} & h^{E1}_{\mu\tau} \\ 0 & 0 & 0 & h^{E1}_{\tau e} & h^{E1}_{\tau\mu} & h^{E1}_{\tau\tau} \\ h^{e4}_{ee} & h^{e4}_{e\mu} & h^{e4}_{e\tau} & h^{E3}_{ee}\delta^{-1} & h^{E3}_{3e\mu}\delta^{-1} & h^{E3}_{e\tau}\delta^{-1} \\ h^{e4}_{\mu e} & h^{e4}_{\mu\mu} & h^{e4}_{\mu\tau} & h^{E3}_{\mu e}\delta^{-1} & h^{E3}_{\mu\mu}\delta^{-1} & h^{E3}_{\mu\tau}\delta^{-1} \\ h^{e4}_{\tau e} & h^{e4}_{\tau\mu} & h^{e4}_{\tau\tau} & h^{E3}_{\tau e}\delta^{-1} & h^{E3}_{\tau\mu}\delta^{-1} & h^{E3}_{\tau\tau}\delta^{-1} \end{pmatrix}$$

(3)

where $\delta = v/V$ is an expansion parameter.

Charged lepton sector sector 2

$M_e M_e^\dagger$ for Yukawas of order one produce

- Three heavy leptons (masses of order V)
- The symmetry $e \leftrightarrow \mu \leftrightarrow \tau$ implies one see saw mass (for τ).
- $e \leftrightarrow \mu$ two see saw masses (for μ and τ).
- Without flavor symmetry: three see saw eigenvalues.

PROGRAM: Two see saw eigenvalues for μ and τ and generate mass for e with a radiative mechanism.

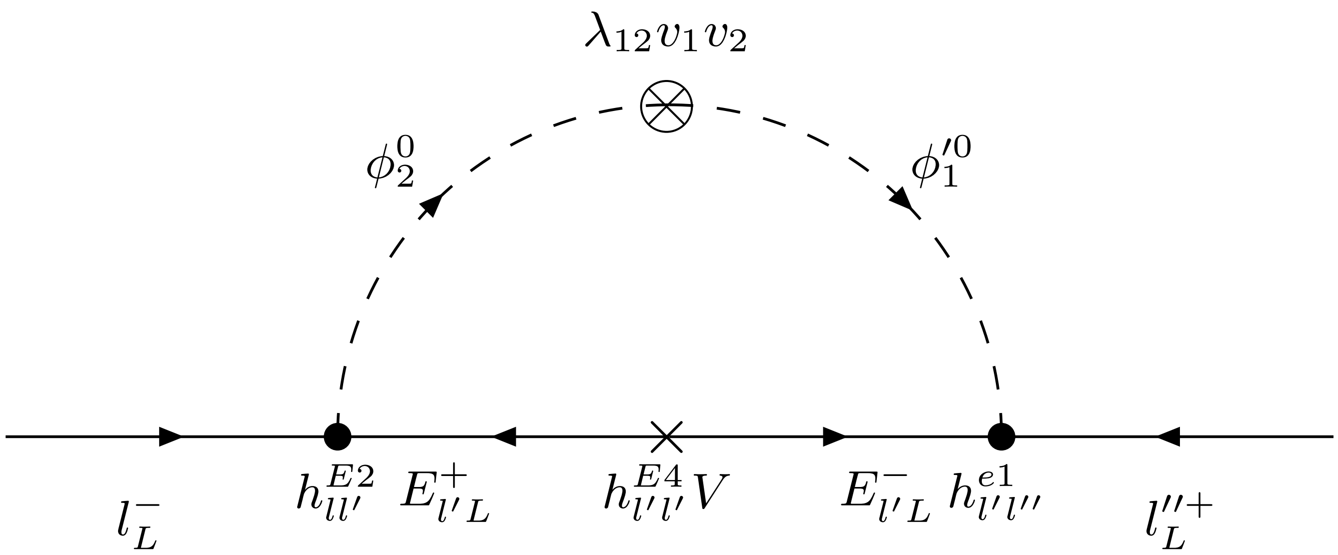

Figure 1: Loop diagram for the electron mass.

Neutrino masses 1

Radiative Majorana masses use a new scalar triplet

$\phi_5 = (\phi_5^{++}, \phi_5^{+}, \phi_5^{\prime +}) \sim (1, 3, 4/3)$, with $Z_2(\phi_5) = 0$ and $\langle \phi_5 \rangle = 0$.

$$\mathcal{L}_Y^{\nu} = \sum_{ll'} h_{ll'}^{\nu} L_{lL} L_{l'L} \phi_5 = \sum_{ll'} h_{ll'}^{\nu} [\phi_5^{++} (l_L^- E_{l'L}^- - l_L^{\prime -} E_{lL}^-)$$

$$+ \quad \phi_5^{+} (E_{lL}^- \nu_{l'L} - E_{l'L}^- \nu_{lL}) + \phi_5^{\prime +} (\nu_{lL} l_L^{\prime -} - \nu_{l'L} l_L^-)], \tag{4}$$

for $l \neq l' = e, \mu, \tau$. 3 parameters $h_{ll'}^{\nu}$ fixed by phenomenology.

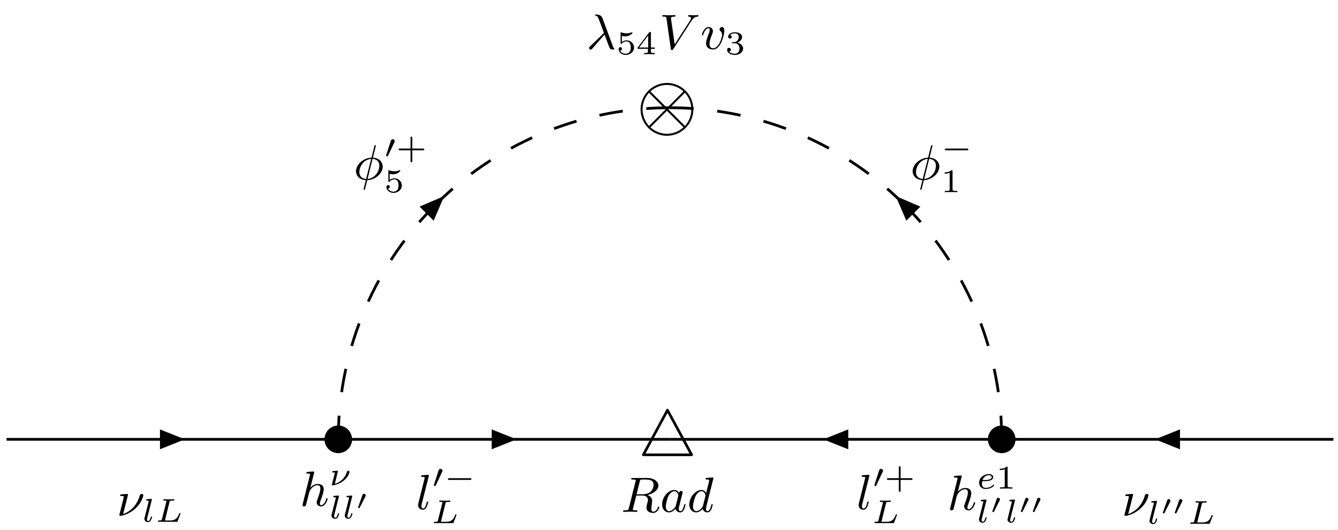

Figure 2: Loop diagrams contributing to the radiative generation of Majorana masses for the neutrinos.

Neutrino masses 2

The lepton mass terms are

$$\mathcal{L}_m = \vec{\nu}^T_{Lf} M'^{\nu} C \vec{\nu}_{Lf} + \vec{E}^T_{6L} M''_e C \vec{E}^c_{6L} + h.c., \tag{5}$$

$\vec{\nu}^T_{Lf}$ and $\vec{E}_6$ are vectors in the flavor basis.
$M'_\nu = (M_\nu + M^\nu_T)/2$ is the symmetric 3×3 neutrino mass matrix constructed from the second order radiative corrections.
M''_e is the 6×6 charged lepton mass matrix

Relations between the flavor states and the mass eigenstates are:

$$\vec{\nu}_{Lf} = U_{PMNS} \vec{\nu}_L,$$

U_{PMNS} is the 3×3 Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix.

Neutrino masses 3

The mass matrices are diagonalized by

$$U_\nu^T M'_\nu U_\nu = M_\nu^d, \qquad U_l M''_e V_l^\dagger = M_e^d,$$

- $M_e^d = Diag.(m_e, m_\mu, m_\tau, M_{E_e}, M_{E_\mu}, M_{E_\tau})$.
- $M_\nu^d = Diag.(m_1, m_2, m_3)$

U_ν is a 3×3 rotation matrix and U_l and V_l are two 6×6 rotation matrices

$$U_{PMNS} = U_{3l}^\dagger U_\nu, \qquad (6)$$

where U_{3l} is the 3×3 upper left submatrix of U_l.

U_{PMNS} is not unitary (differs from unitary by terms proportional to $\delta^2 \sim 10^{-4}$).

Perturbative analysis

$$U^0_{3l} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}, \tag{7}$$

The tribimaximal mixing matrix which is unitary.

$$U^0_{PMNS} = U_{23}(\theta_{23})U_{13}(\theta_{13}, \delta)U_{12}(\theta_{12})I_\phi,$$

U_{ij} are rotation matrices in the ij plane by the angle θ_{ij} and δ and I_ϕ are Dirac and Majorana CP violating phases.

$\theta_{23} = \theta_{atm},\ \theta_{12} = \theta_{sol}$ and $\theta_{13} = \theta_{chooz}$.

Numerical analysis

Perform a random numerical analysis to fix $h^{\nu}_{ll'}$.
with $h^{\nu}_{ll'}$ (for $0.05 \leq h^{\nu}_{ll'} \leq 1.0$)
The experimental results are reproduced, up to 3σ deviations, for

$$h^{\nu}_{e\mu} \approx h^{\nu}_{\mu\tau} \approx h^{\nu}_{e\tau} \approx 0.1$$

With inverted hierarchy.
and neutrino masses predicted as:

- $m_{\nu_1} = 0.0491 \pm 0.0001$ eV.
- $m_{\nu_2} = 0.0483 \pm 0.0001$ eV.
- $m_{\nu_3} = 0.0016278 \pm 3 \times 10^{-7}$ eV.

CONCLUSIONS

The neutrino mixing angles and mass differences were reproduced in the context of this 3-3-1 model with:

ALL THE YUKAWA C.C. OF ORDER 0.1

WITH NEAT PREDICTIONS FOR THE NEUTRINO MASSES.