Instantons, brane-worlds, and violations of Lorentz invariance

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Strong CP problem

In QCD, it is possible to associate a phase with an instanton transition. Experimentally,

$$|\theta| < 10^{-9} \ ,$$

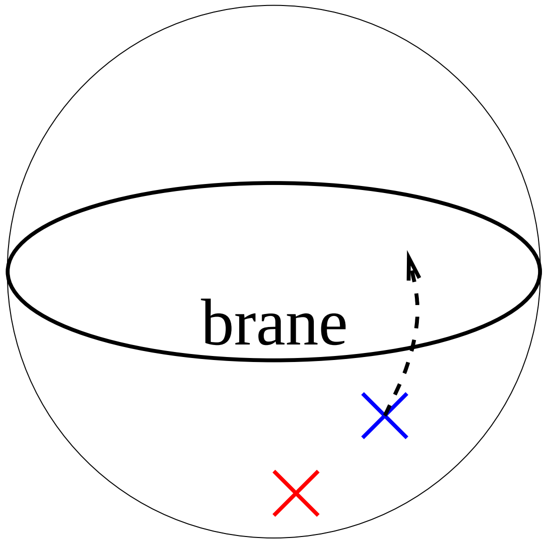
while the "natural" range is $[-\pi, \pi]$.

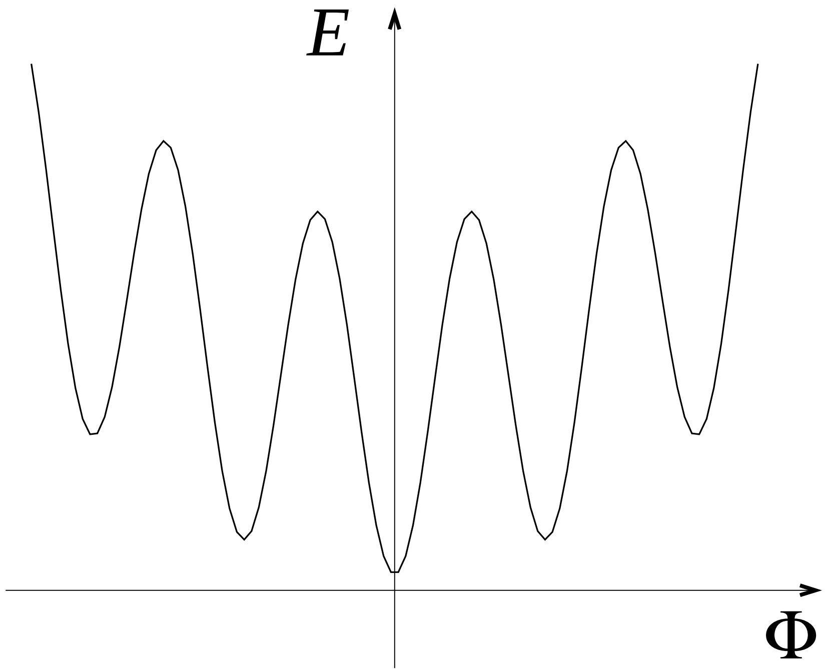
Outline of the talk

1. Why are extra dimensions helpful?
2. "Global axion" and the low-energy theorems.
3. Bulk black hole and escaping photons.
4. Bounds on violations of Lorentz invariance.
5. Conclusions.

The role of extra dimensions

A (1+1)-dimensional Abelian example:





Instantons on the brane = transport of topological charge (vortices) through the brane. Certain topologies (e.g., a sphere) have finite inductances (vortices cost energy). As a result, the effective theta-angle becomes time-dependent and can relax to zero---a "global axion" (S.K. and M. Shaposhnikov, 2004).

This mode is discrete---there is no corresponding particle. This is possible due to a (small) violation of Lorentz invariance.

"Global axion" and the low-energy theorems

Topological susceptibility of the QCD vacuum:

$$K = i \int \langle T \frac{g^2}{8\pi^2} F\tilde{F}(x) \frac{g^2}{8\pi^2} F\tilde{F}(0) \rangle_0 d^4x \; .$$

The low-energy theorem (Crewther, 1977; Shifman, Vainshtein, and Zakharov, 1980) connects K to the successful solution to the U(1) problem:

$$K = 4 \langle \sum_f m_f \bar{q}_f q_f \rangle_0 + 4i \int \langle T O(x) O(0) \rangle_0 d^4x \; ,$$

where $O = i \sum_f m_f \bar{q}_f \gamma_5 q_f$.

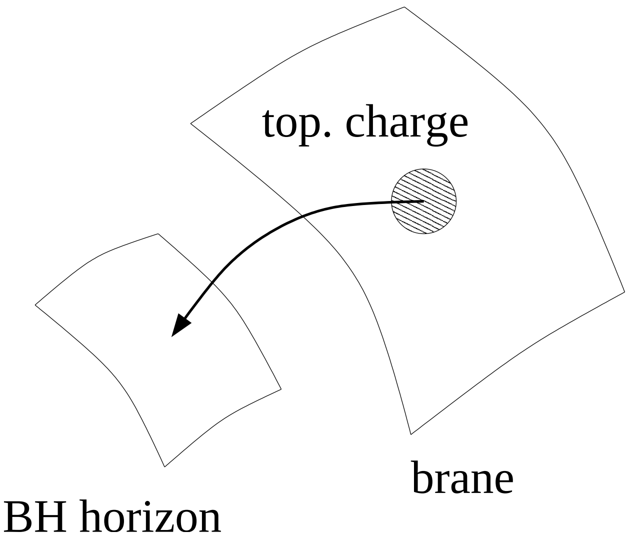
However, the global axion contributes to the correlator, as can be seen in the 2-flavor massive Schwinger model (S.K., 2006). Add to the action a finite inductance term

$$\Delta S = -(1/2I) \int dt \Phi^2 \; .$$

Calculating via bosonization (in the strong-coupling limit) gives $K = 0$.

Bulk black hole

This mechanism may also work simply due to the presence of bulk black hole.



Consider for example the metric (of AdS-Schwarzschild type)

$$ds^2 = -\frac{f(r)}{R^2}dt^2 + \frac{1}{\kappa^2 f(r)}dr^2 + r^2 \sum_{i=1}^{d} d\xi_i^2 \ ,$$

$$f(r) = r^2\left(1 - \frac{r_0^{d+1}}{r^{d+1}}\right) \ .$$

The horizon is at $r = r_0,$ and the brane is at $r = R$.

Escaping photons

For $d > 3$, the photon is quasilocalized on the brane but can leak into the black hole. Typical behavior of the photon mass is (S.K., 2007)

$$m^2(k) \sim \kappa^2 e^{-i\pi\mu} \left(\frac{\epsilon k^2}{\kappa^2} \right)^{\mu} , \tag{1}$$

where $0 < \mu < 1$, $\epsilon = (r_0/R)^{d+1} \ll 1$.

The imaginary part of (1) is at least comparable to the real part. As a result, the existence of photons that reach us from distant sources imposes strong bounds on kinematical violations of Lorentz invariance (such as dependence of the speed of light on momentum or difference between the limiting speeds of different particles).

For example, for an "arm" of 1 Gpc

$$\left| \frac{d\mathrm{Re}\omega}{dk}(k) - c \right| \lesssim 10^{-32} \ \frac{\mathrm{eV}}{k} \ .$$

Conclusions

- There is a generic mechanism by which extra dimensions (with a suitable topology) can solve the strong CP problem, without a need for an axion particle.
- There is instead a single degree of freedom---a "global axion" (with respect to which the world is a giant resonator). It can be precisely identified in low-dimensional models.
- A necessary condition for this type of scenarios is a weak violation of Lorentz invariance.
- In the simplest version of the bulk-black-hole scenario, the effect easiest to observe seems to be the photon extinction (over astronomical distances), rather than kinematical LI violations.