Anomalies and little higgs models

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Based on:

C.T. Hill and R.J. Hill,

hep-ph/0701044 (Jan 2007, to appear in PRD),

hep-ph/0705.0697 (May 2007)

Very incomplete list of references

Composite models:

Kaplan, Georgi (84), Kaplan, Georgi, Dimopoulos (84), Georgi, Kaplan, Galison (84), Dugan, Georgi Kaplan (85)

Arkani-Hamed, Cohen, Georgi (01) Arkani-Hamed, Cohen, Katz, Kaplan, Schmaltz (03)

Nelson, Gregoire, Wacker (02)

Contino, Nomura, Pomaral (03) Agashe, Contino, Sundrum (2005)

Arkani-Hamed, Cohen, Katz, Nelson (02)

Low, Skiba, Smith (02)

Pierce, Perelstein, Peskin (04)

Han, Logan, Wang (06)

T parity:

Cheng, Low (03,04)

Birkedal-Hansen, Wacker (04) Birkedal-Hansen, Noble, Perelstein, Spray (06) Carena, Hubisz, Perelstein, Verdier (06)

KK parity:

Cheng, Feng, Matchev (02)

Servant, Tait (02) Bertone, Hooper, Silk (05)

Take home messages

- anomaly physics probes fundamental features of a light composite higgs boson
- at a practical level, this is nothing subtle or complicated: just an application of effective field theory

Why consider a composite or "little" higgs ?

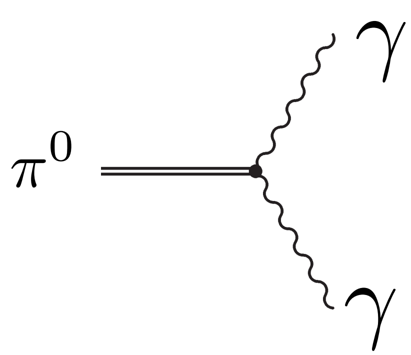
- fermion condensation is the one mechanism we have observed that breaks EW symmetry

- composite particles (pions) can be naturally light compared to other new physics (Λ_{QCD})

Supposing a Higgs boson is found,

- need to distinguish between SM/SUSY/composite/other

IR probes of UV physics



theory: $\Gamma(\pi^0 \to \gamma\gamma) = \frac{N_c \alpha^2}{96\pi^2} \frac{m_\pi^3}{f_\pi^2} = N_c \times 2.4\,\mathrm{eV}$

experiment: $7.7 \pm 0.6\,\mathrm{eV}$

$\Rightarrow$ *number of colors = 3 !*

Recall QCD: important to know # colors (=3) to find out what's going on:

- baryons = 3 quarks
- structure of lepton sector highly constrained by anomaly cancellation: $3(-2(1/6)^3+(2/3)^3+(-1/3)^3)-2(-1/2)^3+(-1)^3 = 0$

What are the analogous probes for a composite Higgs?

Ingredients of a "little higgs" model *

- 1) higgs is light (it's an NGB)
  - 2) EW-symmetric vacuum is destabilized (e.g. coupling to heavy top sector, or gauging broken generators)
    - 3) higgs potential is generated (e.g. after integrating out heavy scalars, or radiative corrections to chiral lagrangian)
      - 4) SM fermions get mass (coupling to Higgs = kaon)

* little higgs = light composite higgs = "higgs is a kaon"

An important minus sign

gauged generators:

$$\Lambda = \underbrace{\Lambda_V}_{\text{unbroken}} + \underbrace{\Lambda_A}_{\text{broken}}$$

one-loop contribution to scalar masses:

$$m^2_{ab} = \underbrace{M^2}_{\text{positive}} \sum_\Lambda \underbrace{\mathrm{Tr}\left\{[\Lambda_V, [\Lambda_V, t^a_A]]t^b_A\right\}} - \underbrace{\mathrm{Tr}\left\{[\Lambda_A, [\Lambda_A, t^a_A]]t^b_A\right\}}$$

positive and nonzero
if NGB charged

[e.g. Peskin 1980]

e.g, π^+ heavier than π^0

- "LH cancellation": *two terms cancel, $m^2 \approx 0$*
- "EWSB by vacuum misalignment": *second term overwhelms first, EWSB*

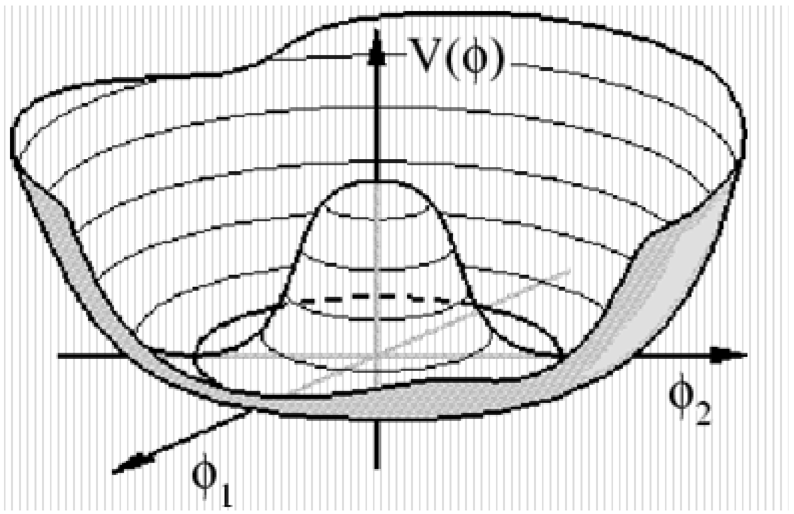
In general, need to gauge broken symmetry generators

The corresponding gauge bosons will "eat" scalars to acquire mass (Higgs mechanism)

- what are the topological interactions of H, W, Z, γ, heavy B' ?

Topological interactions

Consider the NGB's of a spontaneously broken "flavor" symmetry, e.g. SU(3)→SU(2)



Field space M = space of degenerate vacua, e.g. SU(3)/SU(2)

What is the most general action that is:

- *globally SU(3) invariant*
- *four dimensional*
- *local*

Our field space for SU(3)/SU(2) is the five-sphere

$$\Phi = \begin{pmatrix} \phi^1 + i\phi^2 \\ \phi^3 + i\phi^4 \\ \phi^5 + i\phi^6 \end{pmatrix} \qquad \Phi^\dagger \Phi = \sum_{i=1}^{6} (\phi^i)^2 = 1$$

$$\Phi = \exp\left[i \begin{pmatrix} \eta & \cdot & \\ \cdot & \eta & H \\ & H^\dagger & -2\eta \end{pmatrix} \right] \begin{pmatrix} \cdot \\ \cdot \\ 1 \end{pmatrix}$$

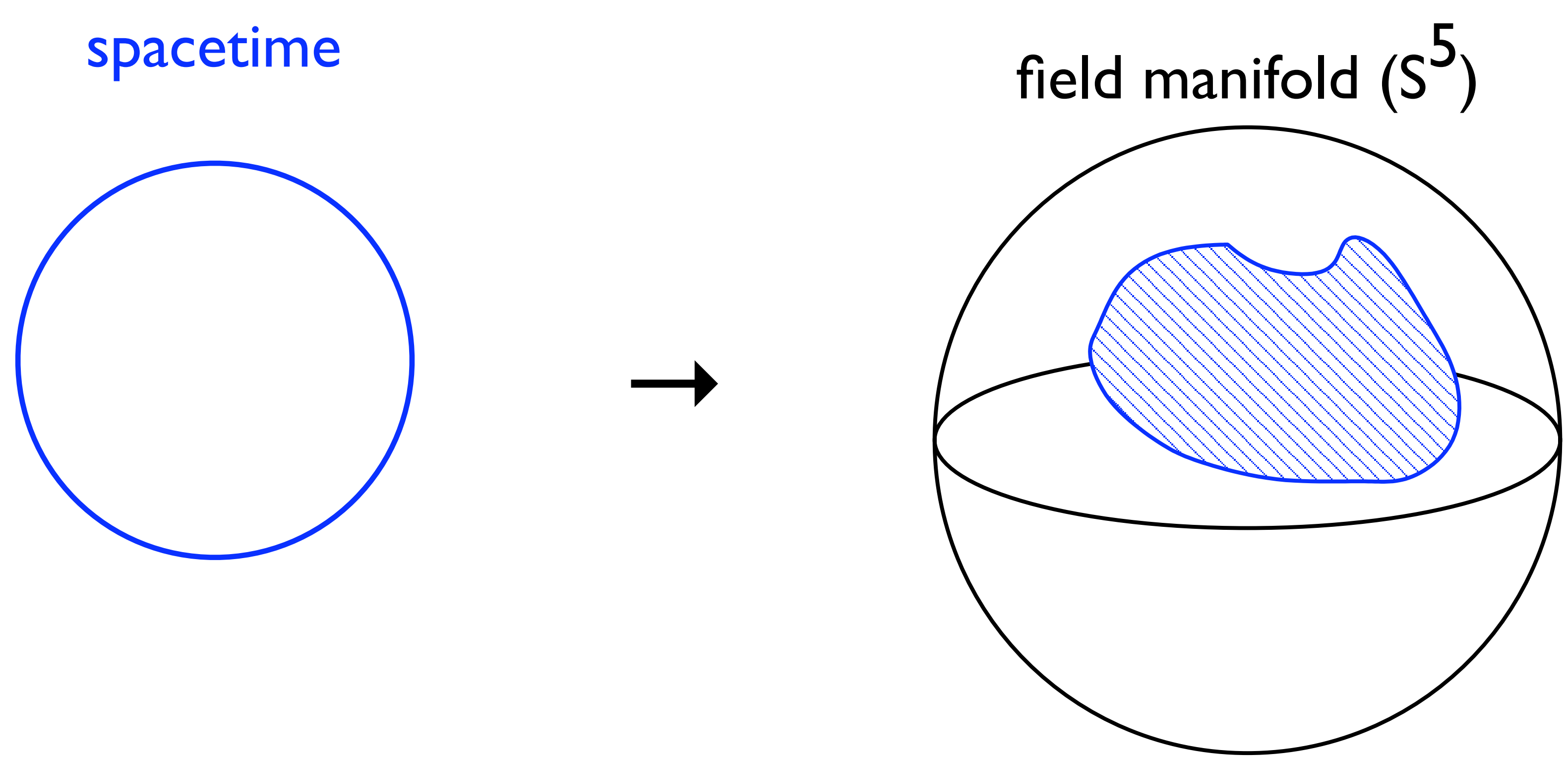
First pass:

$$\Gamma(\Phi) = \int d^4x \, |\partial_\mu \Phi|^2 + c_1 |\partial_\mu \Phi|^4 + c_2 \Phi^\dagger \partial^4 \Phi + \dots$$

Second pass:

$$\Gamma'(\Phi) = \text{number} \times \text{“area bounded by the image of spacetime on } S^5\text{”}$$

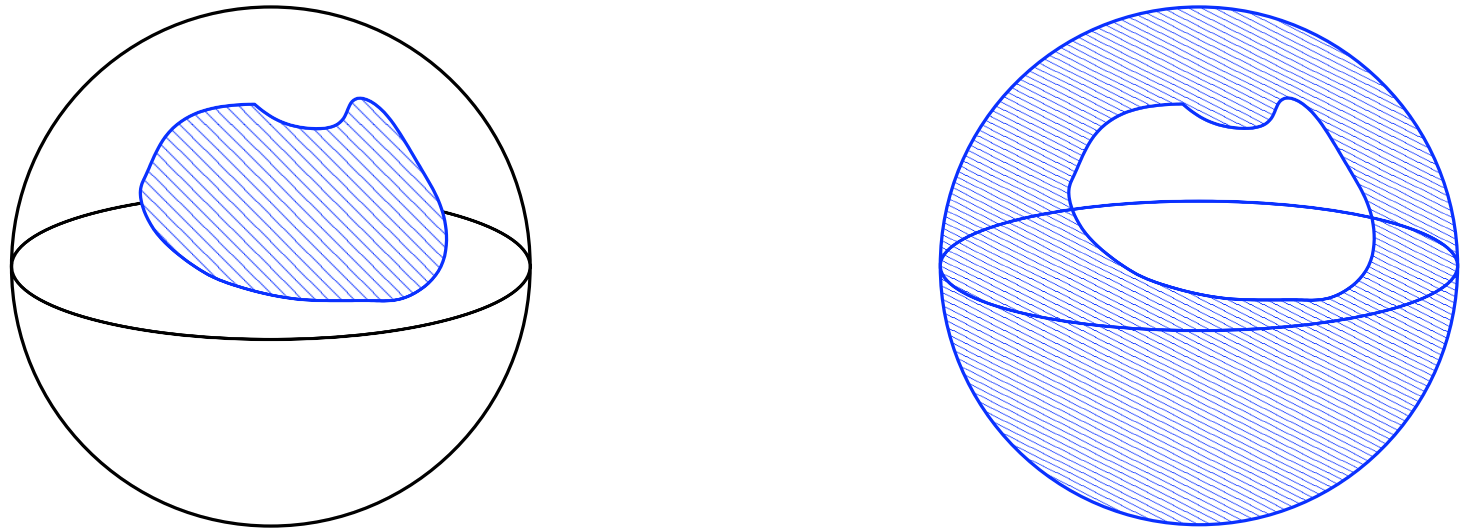
Together, Γ and Γ' give the general effective action for Φ



Nothing subtle, just another way an action that is:

- *globally SU(3) invariant*
- *four dimensional*
- *local*

Quantization:



Can only be consistent if difference between choices of bounding surface is 2π x integer

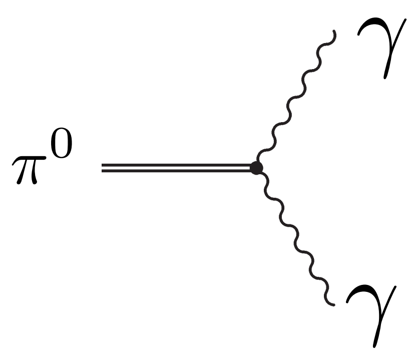
[Witten 1982]

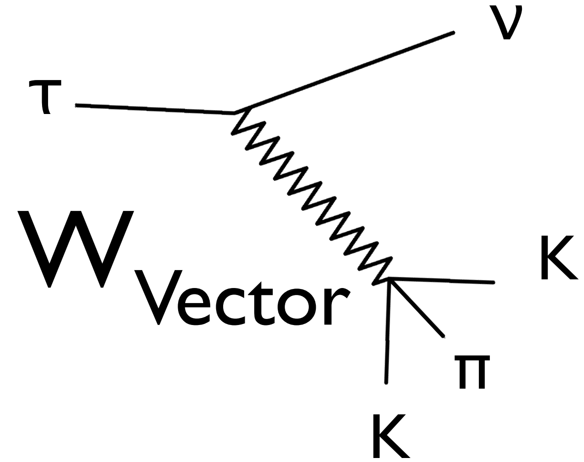
[Volume of S^5] = π^3 ⇒

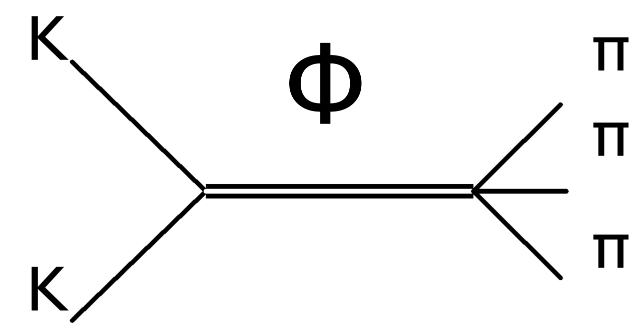
$$\Gamma'(\Phi) = \text{integer} \times 2\pi \times \frac{1}{\pi^3} \int_{M^5} -\frac{i}{8} \Phi^\dagger d\Phi d\Phi^\dagger d\Phi d\Phi^\dagger d\Phi$$

Any candidate UV completion theory is labeled by an integer: 0, 1, 2, 3, ...

Observing the topological interactions







recall QCD: topological interactions of kaons

- single K interactions ruled out: isospin
- two K interactions ruled out: parity
- need at least two K and additional π,η, or axial gauge bosons

Gauge anomalies

When we include the topological term,
we must also deal with gauge anomalies

(Recall that we have gauged broken "axial" symmetry generators)

not gauge invariant,
but independent of H

$$
\begin{aligned}
\Gamma_{WZW} \;=\; & \int d^4x\, \frac{\tilde{g} N_c}{24\sqrt{3}\pi^2}\epsilon^{\mu\nu\rho\sigma}\tilde{B}_\mu \Big[\\
& -\frac{1}{3}g_1^2[B_\nu \partial_\rho B_\sigma] + 2g_2^2 \mathrm{Tr}[W_\nu \partial_\rho W_\sigma] - \frac{3ig_2^3}{2}\mathrm{Tr}[W_\nu W_\rho W_\sigma] \\
& \underbrace{-\frac{ig_1}{4F^2}F^B_{\nu\rho}[H^\dagger(D_\sigma H) - (D_\sigma H^\dagger)H] - \frac{ig_2}{F^2}[H^\dagger F^W_{\nu\rho}(D_\sigma H) - (D_\nu H^\dagger)F^W_{\rho\sigma}H]}\Big]
\end{aligned}
$$

topological interaction
of Higgs

Higgs topological interaction

After cancelling the gauge anomalies, what remains is the Higgs topological interaction

In gory detail:

$$\Gamma_{WZW} = \frac{-\tilde{g}g_2^2 N_c}{96\sqrt{3}\pi^2 F^2}\int d^4x\,(v+h^0)^2\epsilon^{\mu\nu\rho\sigma}\tilde{B}_\mu \times$$
$$\Big[2\sqrt{1+\tan^2\theta}\left(\partial_\nu Z^0_\rho\cos\theta + \partial_\nu A_\rho \sin\theta - ig_2 W^+_\nu W^-_\rho\right)Z^0_\sigma$$
$$+ 2\left[(D^A_\nu W^+_\rho)W^-_\sigma + (D^A_\nu W^-_\rho)W^+_\sigma\right] - 4ig_2\cos\theta Z^0_\nu W^+_\rho W^-_\sigma$$
$$- \tan\theta\sqrt{1+\tan^2\theta}\left(\partial_\nu Z^0_\rho \sin\theta - \partial_\nu A_\rho\cos\theta\right)Z^0_\sigma\Big]$$

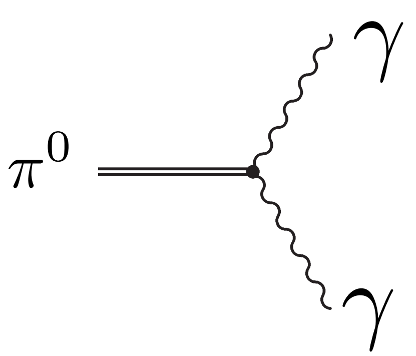
Chiral lagrangians and parities

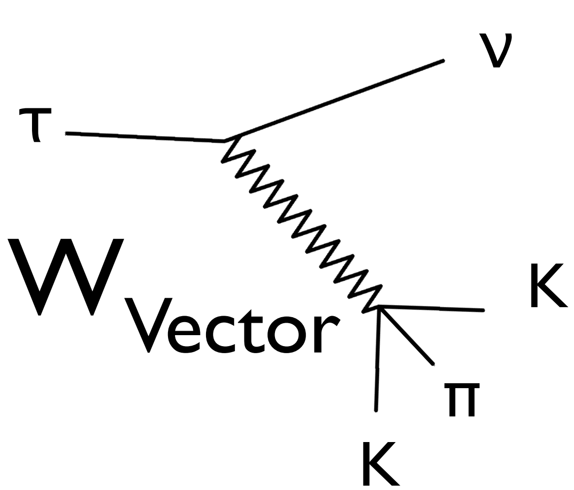
Consider the QCD chiral lagrangian for low-energy pion interactions.
Field space is SU(3)xSU(3)/SU(3) = SU(3): $U = e^{i\pi^a t^a}$

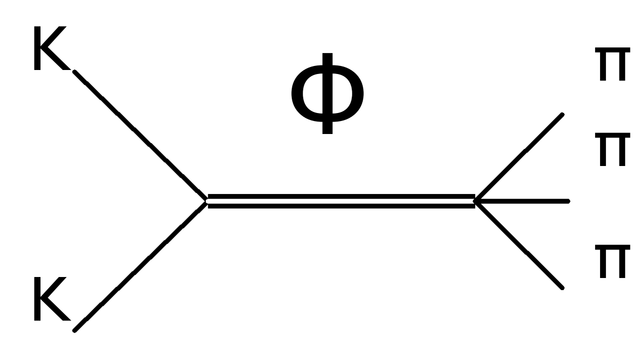
At first sight, it appears that the effective action conserves the internal parity $U \leftrightarrow U^\dagger$

$$\Gamma \sim \int d^4x \, \mathrm{Tr}\left[|D_\mu U|^2 + c_1 |D_\mu U|^4 + c_2 D_\mu U D_\nu U^\dagger D_\mu U D_\nu U^\dagger + \dots \right]$$

This would forbid interactions involving odd numbers of mesons, e.g. $\pi_0 \rightarrow \gamma\gamma$







Need general action that is

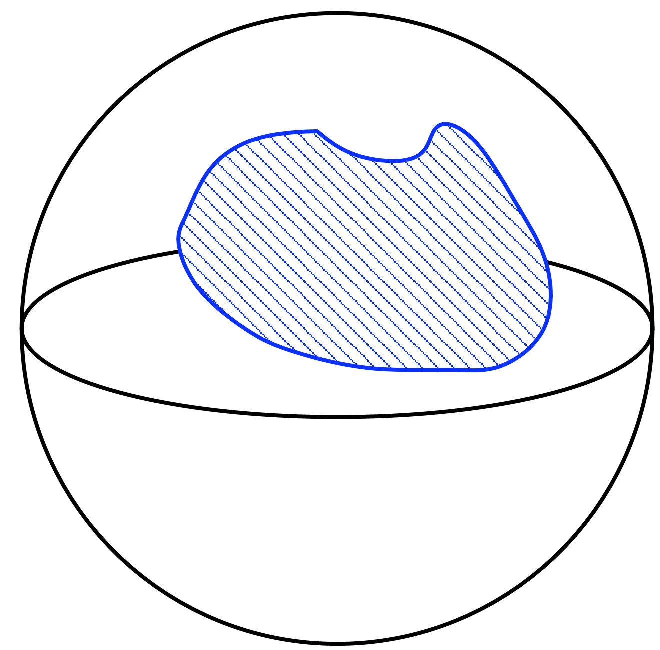
- *globally SU(3)xSU(3) invariant*
- *four dimensional*
- *local*

Just like before, but harder to visualize

$$\Gamma'(U) = \text{number} \times \text{“area bounded by the image of spacetime on } SU(N)\text{”}$$

$$\Gamma'(U) = \text{"integer"} \times \frac{-i}{240\pi^2} \int_{M^5} \mathrm{Tr}(\alpha^5)$$

$$\alpha = (dU)\, U^\dagger$$



Any candidate UV completion theory is labeled by an integer: 0, 1, 2, 3, ...

QCD is a “3” theory: # colors=3

"The QCD chiral lagrangian has a well-known Z_2 symmetry: the parity which exchanges the chirality L↔R "

False !!!

In QCD, there is only one parity:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + A\!\!\!/_V + A\!\!\!/_A \gamma_5)\psi$$

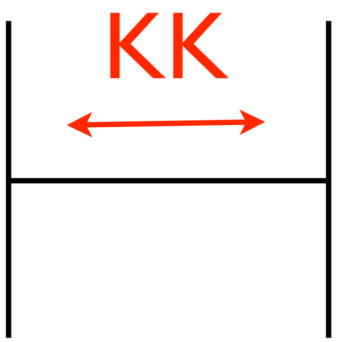
$$\begin{aligned} \psi &\to \gamma^0\psi \\ A_V &\to +A_V \\ A_A &\to -A_A \end{aligned} \qquad \textit{and} \qquad \vec{x} \to -\vec{x}$$

- leading term in chiral Lagrangian respects two parities: $\pi \to -\pi$, $x \to -x$
- WZW term breaks both parities, preserving only the combination

similar story with extra dimensions and "mooses"

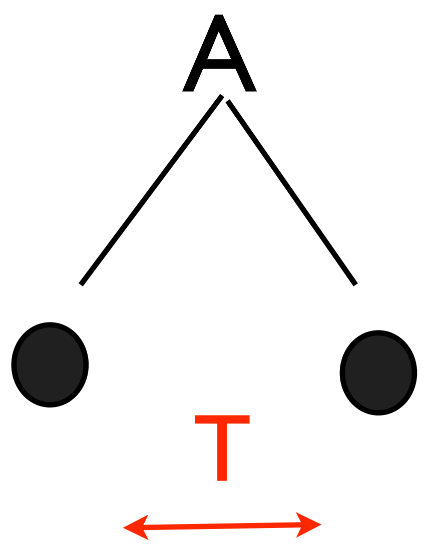
"… KK-parity corresponds to the symmetry of reflection about the midpoint in the extra dimension … KK-parity conservation implies that the lightest KK particle is stable."

Unless we leave out an operator (the Chern Simons term), this parity is violated



"… the minimal moose model has a reflection symmetry which exchanges the two sites …"

Unless we leave out an operator (the topological, or WZW term), this parity is violated



- it is fascinating that π^0 decays to $\gamma\gamma$, but this is established physics
- T parity an interesting concept: it is generally violated, but in an interesting and constrained way (predictive)

To do

This is a theory talk. Lots of phenomenology to explore

- investigate production modes of B'

$$e^+e^- \text{ or } q\bar{q} \text{ or } \mu^+\mu^- \ \to \ (\gamma^*, Z^*) \to \tilde{B} + Z;\ \tilde{B} + \gamma;\ \tilde{B} + WW$$

$$e^+e^- \text{ or } q\bar{q} \text{ or } \mu^+\mu^- \ \to \ (\gamma^*, Z^*) \to \tilde{B} + h^0;\ \tilde{B} + 2h^0$$

$$q\bar{q} \ \to \ W^* \to \tilde{B} + W;\ \tilde{B} + W + h^0;\ \tilde{B} + W + 2h^0$$

- decays of B'

$$e^+e^- \text{ or } q\bar{q} \text{ or } \mu^+\mu^- \ \to \ (\gamma^*, Z^*, \tilde{B}^*) \to$$

$$\tilde{Z} + Z + (0,1,2)h^0;\ \tilde{Z} + \gamma + (0,1,2)h^0;\ WW + (0,1,2)h^0$$

- An example application:

$$\Gamma(\tilde{B} \to ZZ) \approx N_c^2 \left[\frac{1}{2\pi} \left(\frac{\tilde{g}^3}{144\pi^2} \right)^2 \frac{m_Z^2}{m_{\tilde{B}}} \right]$$

- Not likely a displaced vertex (or missing energy or dark matter!!)

- May be interesting to consider angular distribution

Summary

- anomalies not just a nuisance - a low energy probe of UV completion physics
- implications are contrary to much of the current literature
- this is nothing subtle or complicated, just an application of effective field theory
- lots of interesting directions to explore