CKM and Tri-bimaximal MNS Matrices in a SU(5) × $^{(d)}$T Model

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Introduction

- Neutrino Oscillation Parameters [Circa 2006]

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sin^2\theta_{12} = 0.30\ (0.25 - 0.34), \quad \sin^2\theta_{23} = 0.5\ (0.38 - 0.64), \quad \sin^2\theta_{13} = 0\ (< 0.028)\ .$$

- Tri-bimaximal neutrino mixing:

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{\rm atm,\,TBM} = 1/2 \qquad \sin\theta_{13,\rm TBM} = 0.$$

$$\sin^2\theta_{\odot,\rm TBM} = 1/3$$

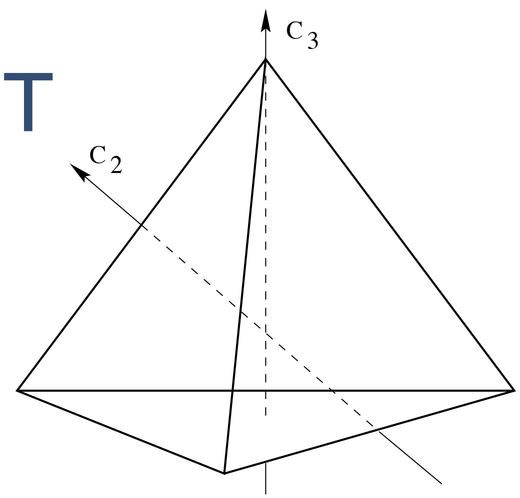
$$\tan^2\theta_{\odot,\rm exp} = 0.429 \qquad \tan^2\theta_{\odot,\rm TBM} = 1/2$$

Tri-bimaximal Neutrino Mixing from A4

- even permutations of 4 objects (invariance group of tetrahedron)
- generated by two basic permutations: (1234)

  S=(4321) ; T=(2314)
- 4! / 2 = 12 elements
- defining relations: $S^2 = T^3 = (ST)^3 = 1$
- four in-equivalent representations: 1, 1', 1'', 3
- successfully give rise to tri-bimaximal leptonic mixing:
  - ⋆ lepton doublets ~ 3
  - ⋆ RH charged leptons: 1, 1', 1''
- generalization to quark sector:
  - ⋆ no CKM mixing -- lack of doublet representations
  - ⋆ mass hierarchy -- need additional U(1) symmetry

Group Theory of $^{(d)}T$



- Double covering of A4
- in-equivalent representations:

A4: 1, 1', 1'', 3
other: 2, 2', 2''

- generators:

$$S^2 = R,\ T^3 = 1,\ (ST)^3 = 1,\ R^2 = 1$$

R=1: 1, 1', 1'', 3
R= -1: 2, 2', 2''

- product rules:

$$1^0 \ \equiv\ 1,\ 1^1 \ \equiv\ 1',\ 1^{-1} \ \equiv\ 1''$$

$$1^a \otimes r^b = r^b \otimes 1^a = r^{a+b} \qquad \text{for } r = 1, 2 \qquad a, b \ =\ 0, \pm 1$$

$$1^a \otimes 3 = 3 \otimes 1^a = 3$$

$$2^a \otimes 2^b = 3 \oplus 1^{a+b}$$

$$2^a \otimes 3 = 3 \otimes 2^a = 2 \oplus 2' \oplus 2''$$

$$3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1''$$

The Model

- Symmetry: SU(5) x $^{(d)}T$
- Particle Content $10(Q,\, u^c,\, e^c)_L \qquad \overline{5}(d^c, \ell)_L$

	T_3	T_a	$\overline{F}$	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
SU(5)	10	10	$\overline{5}$	5	$\overline{5}$	45	1	1	1	1	1	1	1	1
$^{(d)}T$	1	2	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1
Z_{12}	ω^5	ω^2	ω^5	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1

$\omega = e^{i\pi/6}$.

- additional $Z_{12} \times Z'_{12}$ symmetry:
  - ★ predictive model: number of operators as small as possible
  - ★ vacuum misalignment: neutrino sector vs charged fermion sector

The Model

$H_5T_3T_a$	$\psi',\ \psi$ $\psi\phi,\ \psi\phi',\ \psi'\phi,\ \psi'\phi',\ \psi'\zeta,\ \psi'N,\ \psi N$ $\psi^3,\ \psi\psi'^2,\ \psi\phi^2,\ \psi\phi'^2,\ \psi\phi\zeta,\ \psi\phi'\zeta,\ \psi'^3,\ \psi'\psi^2,\ \psi'\phi^2,\ \psi'\phi'^2,\ \psi'\phi\zeta,\ \psi'\phi'\zeta,$ $\psi\phi N, \psi\phi' N,\ \psi'\phi N, \psi'\phi' N$
	$\psi\xi,\ \psi'\xi,\ \psi\xi^2,\ \psi\xi\phi,\ \psi\xi\phi',\ \psi\xi\zeta,\ \psi'\xi^2\ \psi'\xi\phi,\ \psi'\xi\phi',\ \psi'\xi\zeta,\ \psi\xi N,\ \psi'\xi N,\ \psi'\eta,\ \psi\phi\eta,\ \psi\phi'\eta,\ \psi\xi\eta,$ $\psi'\phi\eta,\ \psi'\phi'\eta,\ \psi'\xi\eta,\ \psi\eta,\ \psi\phi\eta,\ \psi\phi'\eta,\ \psi'\phi\eta,\ \psi'\phi'\eta,\ \psi\phi\eta,\ \psi\phi'\eta,\ \psi'\phi\eta,\ \psi'\phi'\eta$
$H_5T_aT_a$	$\phi,\ \ \phi'$ $\phi'^2,\ \psi^2,\ \psi'^2,\ \phi\phi',\ \psi\psi'$ $\phi^3,\ \phi^2\zeta,\ \phi\zeta^2,\ \phi'^2\zeta,\ \phi'\zeta^2, \phi\phi'\zeta,\ \phi\phi'^2,\ \phi'\phi^2,\ \phi N^2,\ \phi' N^2,\ \phi'^2 N,\ \phi\phi' N,\ \phi N\zeta,\ \phi' N\zeta$
	$\xi,\ \xi^2,\ \xi\zeta,\ \xi N,\ \xi\eta,\ \xi^2,\ \xi\phi,\ \xi\phi',\ \xi^3,\ \xi^2\zeta,\ \xi^2\eta, \xi^2\zeta,\ \xi N\zeta,\ \xi N\eta,\ \xi\zeta\eta,\ \xi\phi^2,\ \xi\phi'^2,\ \xi\phi\phi',$ $\xi^2\phi,\ \xi^2\phi',\ \xi\phi N,\ \xi\phi\eta,\ \xi\phi\zeta,\ \xi\phi' N,\ \xi\phi'\eta,\ \xi\phi'\zeta,\ \phi^2\eta,\ \phi\eta^2,\ \phi\eta N,\ \phi\eta\zeta,\ \phi'\eta^2,\ \phi'\eta N,$ $\phi'\eta\zeta, \phi\eta, \phi'\eta,\ \xi N^2,\ \xi\eta^2,\ \xi\zeta^2$
$H'_{\overline{5}}\overline{F}T_3$	$\phi,\ \phi'$ $\psi^2,\ \phi^2,\ \phi'^2,\ \phi'\phi,\ \psi'^2,\ \psi\psi',\ \phi'\zeta,\ \phi' N,\ \phi N$ $\phi^3,\ \phi'^3,\ \phi^2\phi',\ \phi\phi'^2,\ \phi\zeta^2,\ \phi'\zeta^2, \phi\psi^2,\ \phi'\psi'^2,\ \zeta\psi^2,\ \zeta\psi'^2,\ \phi'\psi^2,\ \phi\psi^2,$ $\phi N^2,\ \phi' N^2,\ \phi N\zeta,\ \phi' N\zeta,\ N\psi^2,\ \zeta\psi^2,\ \zeta\psi\psi',\ N\psi\psi'$
	$\xi,\ \xi^2,\ \xi N,\ \xi\zeta,\ \xi\eta,\ \xi\phi,\ \xi\phi',\ \xi^3,\ \xi^2 N,\ \xi^2\zeta,\ \xi^2\eta,\ \xi^2\phi,\ \xi^2\phi',\ \xi\phi^2,$ $\xi\phi'^2,\ \xi\phi\phi',\ \xi\phi N,\ \xi\phi\zeta,\ \xi\phi\eta,\ \xi\phi' N,\ \xi\phi'\zeta,\ \xi\phi'\eta,\ \phi'\eta,\ \phi\eta^2,\ \phi\eta N,\ \phi\eta\zeta,\ \phi'\eta^2,\ \phi'\eta N,\ \phi'\eta\zeta,\ \eta\psi^2,$ $\eta\psi'^2,\ \phi\eta,\ \phi\eta N,\ \phi\eta\zeta,\ \phi'\eta^2,\ \phi'\eta N,\ \eta\psi\psi'$
$H'_{\overline{5}}\overline{F}T_a$	$\psi,\ \psi'$ $\psi\phi',\ \psi'\phi,\ \psi'\phi',\ \phi\psi$ $\psi\phi^2,\ \psi\phi\zeta,\ \psi'\phi\zeta,\ \psi\phi'^2,\ \psi'\phi'^2,\ \psi\phi\phi',\ \psi'\phi\phi',\ \psi\phi'\zeta,\ \psi'\phi'\zeta,\ \psi\phi N,\ \psi'\phi N,\ \psi\phi' N,\ \psi'\phi' N$
	$\psi\xi,\ \psi'\xi,\ \psi\xi^2,\ \psi'\xi^2,\ \psi\xi\phi,\ \psi\xi\phi',\ \psi'\xi\phi,\ \psi'\xi\phi',$ $\psi\xi N,\ \psi\xi\eta, \psi\xi\zeta,\ \psi'\xi\zeta,\ \psi'\xi\eta,\ \psi'\xi N,\ \psi\phi\eta,\ \psi'\phi\eta,\ \psi'\phi'\eta,\ \psi\phi'\eta,\ \psi'\phi'\eta,\ \psi\phi\eta,\ \psi'\phi\eta$

The Model

- Abelian subgroups of $^{(d)}T$:

$$Z_3: \quad G_{\mathrm{T}} \qquad Z_4: \quad G_{\mathrm{TST^2}}$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

$$TST^2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

- $^{(d)}T$ breaking:

$$^{(d)}T \longrightarrow G_{\mathrm{TST^2}}: \quad \langle\xi\rangle = \xi_0\Lambda\begin{pmatrix}1\\1\\1\end{pmatrix}, \quad \langle\phi'\rangle = \phi'_0\Lambda\begin{pmatrix}1\\1\\1\end{pmatrix},$$

$$^{(d)}T \longrightarrow G_{\mathrm{T}}: \quad \langle\phi\rangle = \phi_0\Lambda\begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\psi\rangle = \psi_0\Lambda\begin{pmatrix}1\\0\end{pmatrix}$$

$$^{(d)}T \longrightarrow \text{nothing}: \quad \langle\psi'\rangle = \psi'_0\Lambda\begin{pmatrix}1\\1\end{pmatrix}$$

$$^{(d)}T \longrightarrow G_{\mathrm{S}}: \quad \langle\zeta\rangle = \zeta_0, \quad \langle N\rangle = N_0$$

$$^{(d)}T - \text{invariant}: \quad \langle\eta\rangle = u$$

The Model

- Lagrangian: only 9 operators allowed!!

$$
\begin{aligned}
\mathcal{L}_{\rm Yuk} &= \mathcal{L}_{\rm TT} + \mathcal{L}_{\rm TF} + \mathcal{L}_{\rm FF} \\
\mathcal{L}_{\rm TT} &= y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi \zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \\
\mathcal{L}_{\rm TF} &= \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi N + y_d H'_{\overline{5}} \overline{F} T_a \phi^2 \psi' \right] \\
\mathcal{L}_{\rm FF} &= \frac{1}{M_x \Lambda} \left[\lambda_1 H_5 H_5 \overline{F}\, \overline{F} \xi + \lambda_2 H_5 H_5 \overline{F}\, \overline{F} \eta \right] ,
\end{aligned}
$$

Neutrino Sector

- Operators: $$\mathcal{L}_{\rm FF} = \frac{1}{M_x\Lambda}\left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F}\xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F}\eta\right]$$
- Symmetry breaking:

$${}^{(d)}T \longrightarrow G_{\rm TST^2}: \quad \langle\xi\rangle = \xi_0\Lambda\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad\qquad {}^{(d)}T - \text{invariant}: \quad \langle\eta\rangle = u$$

- Resulting mass matrix:

$$M_\nu = \frac{\lambda v^2}{M_x}\begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u-\xi_0 \\ -\xi_0 & u-\xi_0 & 2\xi_0 \end{pmatrix}$$

$$V_\nu^{\rm T} M_\nu V_\nu = {\rm diag}(u+3\xi_0,\, u,\, -u+3\xi_0)\frac{v_u^2}{M_x} \qquad U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Up Quark Sector

- Operators: $\mathcal{L}_{\rm TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi\zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3$
- top mass: allowed by $^{(d)}$T
- lighter family acquire masses thru operators with higher dimensionality

➡ dynamical origin of mass hierarchy

- symmetry breaking:

$$^{(d)}T \longrightarrow G_{\rm T} : \quad \langle \phi \rangle = \phi_0 \Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad ^{(d)}T \longrightarrow G_{\rm TST^2} : \quad \langle \phi' \rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- Mass matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

Down Quark Sector

- operators: $$\mathcal{L}_{\rm TF} = \frac{1}{\Lambda^2} y_b H_{\overline{5}}' \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi N + y_d H_{\overline{5}}' \overline{F} T_a \phi^2 \psi' \right]$$
- generation of b-quark mass: breaking of ${}^{(d)}T$: dynamical origin for hierarchy between m_b and m_t
- lighter family acquire masses thru operators with higher dimensionality

➡ dynamical origin of mass hierarchy

- Georgi-Jarlskog relations: $m_\mu \simeq 3 m_s \qquad m_d \simeq 3 m_e$
- symmetry breaking:

$${}^{(d)}T \longrightarrow G_{\rm T} : \quad \langle \phi \rangle = \phi_0 \Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \psi \rangle = \psi_0 \Lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad {}^{(d)}T \longrightarrow \text{nothing} : \ \langle \psi' \rangle = \psi_0' \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- mass matrix:

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0, \qquad M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

Quark and Lepton Mixing Matrices

- CKM mixing matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u \qquad M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

V_{cb} V_{ub}

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \implies \theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

$$U_{\mathrm{MNS}} = V_{e,L}^\dagger U_{\mathrm{TBM}} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\mathrm{TBM}} - \frac{1}{2}\theta_c \cos\beta \qquad \theta_{13} \simeq \theta_c/3\sqrt{2}$$

leptonic CPV

Numerical Results

- **Experimentally:** $m_u : m_c : m_t = \epsilon_u^2 : \epsilon_u : 1, \quad m_d : m_s : m_b = \epsilon_d^2 : \epsilon_d : 1$

  $\epsilon_u \simeq (1/200) = 0.005 \qquad \epsilon_d \simeq (1/20) = 0.05$

- **Model Parameters:**

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

$$k \equiv y'\psi_0\zeta_0 = -0.032$$
$$h \equiv \psi_0^2 = 0.0053$$
$$g \equiv \phi_0'^3 = -2.25 \times 10^{-5}$$

7 parameters in charged fermion sector

$$\frac{M_d}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & c & 0 \\ b & b & 1 \end{pmatrix}$$

$$y_b\phi_0\zeta_0 \simeq m_b/m_t \simeq (0.011)$$
$$c \equiv \psi_0 N_0/\zeta_0 = 0.0474$$
$$b \equiv \phi_0\psi_0'/\zeta_0 = 0.00789$$

- **Mixing Matrices:**

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.976 & 0.217 & 0.00778 \\ 0.216 & 0.975 & 0.040 \\ 0.015 & 0.0378 & 0.999 \end{pmatrix}$$

$$|U_{\mathrm{MNS}}| = |V_{e,L}^{\dagger} U_{\mathrm{TBM}}| = \begin{pmatrix} 0.838 & 0.545 & 0.0550 \\ 0.364 & 0.608 & 0.706 \\ 0.409 & 0.578 & 0.706 \end{pmatrix}$$

- **neutrino masses:**

$$u = -1.87 \times 10^{-2}, \quad \xi_0 = 1.15 \times 10^{-2}, \quad M_x \sim 10^{14}\ \mathrm{GeV}$$

Conclusions

- SU(5) x $^{(d)}$T symmetry: tri-bimaximal lepton mixing & realistic CKM matrix
- Z_{12} x Z_{12}' symmetry: only 9 operators present (only 9 parameters in Yukawa sector)
  - ⋆ forbid proton decay
  - ⋆ likely linked to orbifold compactification
- dynamical origin of mass hierarchy (including m_b vs m_t)
- interesting sum rules:

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\mathrm{TBM}} - \frac{1}{2}\theta_c\cos\beta$$

right amount to account for discrepancy bt exp best fit value and TBM prediction

$$\theta_{13} \simeq \theta_c/3\sqrt{2} \sim 0.05$$