Unitarity
and
Bounds on the scale of Fermion Mass Generation
in
Deconstructed Higgsless Models

Michigan State University
Neil Christensen

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R. Sekhar Chivukula
Neil D. Christensen
Baradhwaj Coleppa
Elizabeth H. Simmons

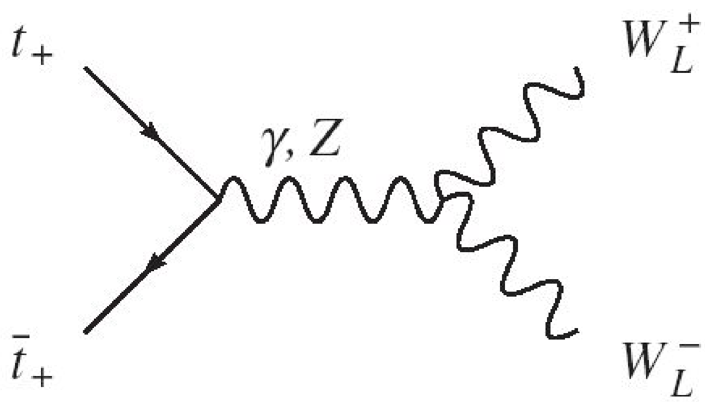
What is the scale of fermion mass generation?

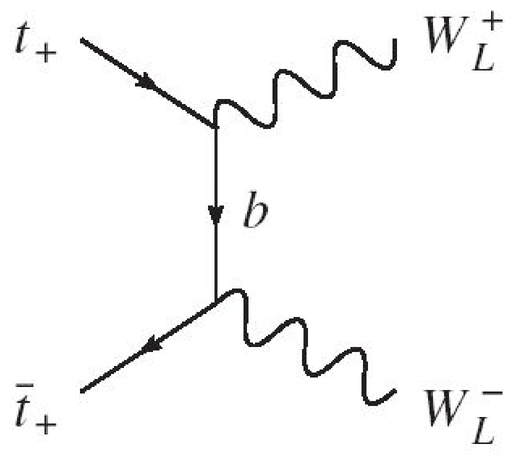
- Is it the same as the scale of EW gauge boson mass generation?

Can we find an upper bound on this scale?

- Yes – PRL **59**, 2405 (1987) Appelquist and Chanowitz did this for the SM without a Higgs.
- How is this bound modified in a Higgsless model?
- How does this bound depend on the parameters in a Higgsless model?

AC Bound: $t_+\bar{t}_+ \to W_L^+ W_L^-$



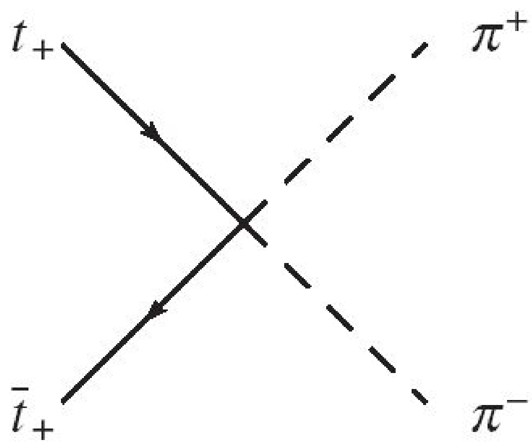


- The bound is obtained in the helicity nonconserving channel of the process $t\bar{t} \to W_L^+ W_L^-$.
- The amplitude was calculated at leading order in $m_t, M_W/\sqrt{s}$.
- The colors and helicities were summed over.
- $2g_{tt\gamma}g_{\gamma WW} + g_{LttZ}g_{ZWW} + g_{RttZ}g_{ZWW} - g_{LtbW}^2 = 0$
- Only the contribution in the T channel is left at leading order in $\sqrt{s}$.

$$\mathcal{M}(\psi \to W_L W_L) = \frac{\sqrt{6}m_t\sqrt{s}\cos\theta}{2M_W^2}(2g_{tt\gamma}g_{\gamma WW} + g_{LttZ}g_{ZWW} + g_{RttZ}g_{ZWW} - g_{LtbW}^2) + \frac{\sqrt{6}m_t\sqrt{s}}{2M_W^2}g_{LtbW}^2$$

$$\mathcal{M} = \frac{\sqrt{6}m_t\sqrt{s}}{v^2}$$

AC Bound: Equivalence Theorem



$$\mathcal{M} = \frac{\sqrt{6} m_t \sqrt{s}}{v^2}$$

- The calculation can also be done using the equivalence theorem giving the same result.
- Only the 4 point vertex contributes at order $\sqrt{s}$.
- The J=0 partial wave amplitude was calculated and set less than ½ giving a bound for $\sqrt{s}$.

$$a_0 = \frac{1}{32\pi} \int_{-1}^{1} d\cos\theta\, \mathcal{M} = \frac{m_t \sqrt{6s}}{16\pi v^2}$$

$$\sqrt{s} \lesssim \frac{8\pi v^2}{m_t \sqrt{6}} \approx 3.5 \text{ TeV}$$

AC Bound: Thoughts

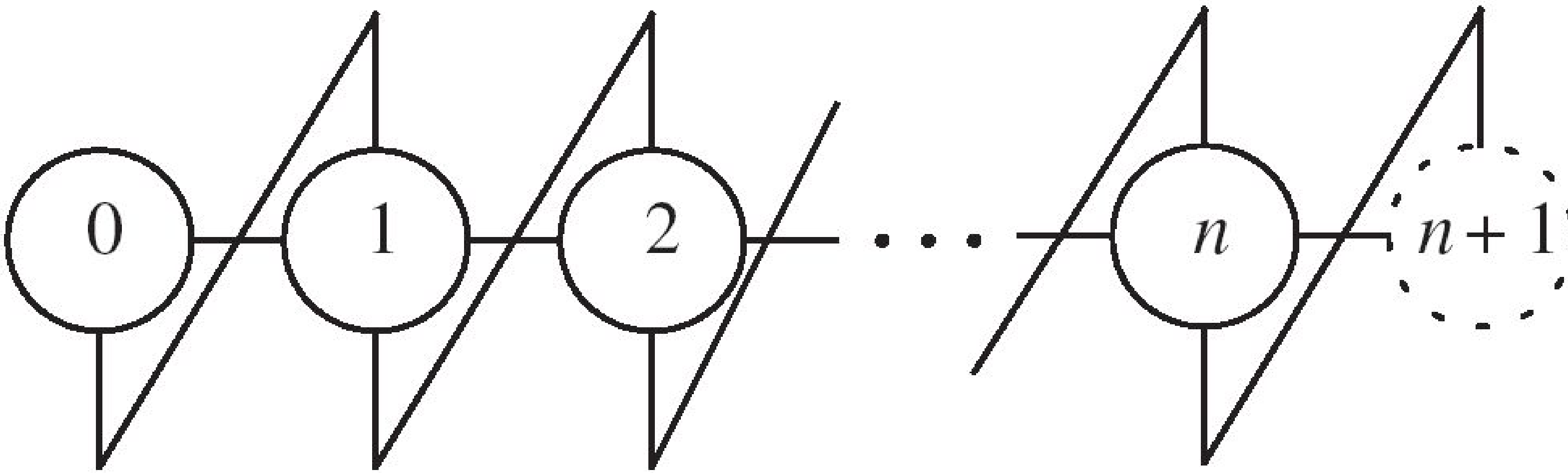
M. Golden: PLB **338**, 295 (1994)

- Is the AC bound really unique?
  Won't the fields that unitarize $W_L^+W_L^- \to W_L^+W_L^-$
  also unitarize $t_+\bar{t}_+ \to W_L^+W_L^-$?
- Consider the Higgs:
  It unitarizes both processes.

Higgsless Models:

- PLB **525**, 175 (2002), PLB **532**, 121 (2002), PLB **562**, 109 (2003), IJMPA **20**, 3362 (2005).
  $W_L^+W_L^- \to W_L^+W_L^-$ is unitarized by the exchange of the KK modes of the W gauge boson.
- Phys. Rev. D **75**, 073018 (2007):
- $t_+\bar{t}_+ \to W_L^+W_L^-$ is unitarized by the exchange of the KK modes of the bottom quark.

n(+2) Site Model: Introduction



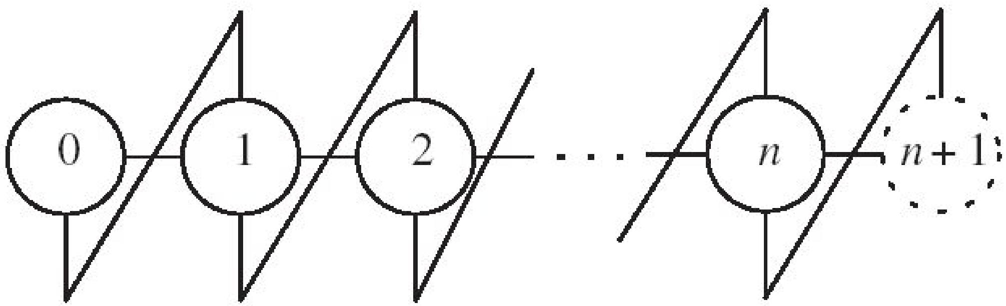
•The n(+2) site model is a deconstructed Higgsless model with n extra SU(2) gauge groups.
•The circles represent the gauge groups at each lattice points.
•The horizontal lines represent the interactions of the gauge bosons and the nonlinear Sigma fields $\Sigma_j = e^{i\frac{2\pi_j}{f}}$.
•The vertical lines represent the fermions.
•The diagonal lines represent the interactions of the fermions and the Sigma fields.

$$G = SU(2)_0 \times \prod_{j=1}^{n} SU(2)_j \times U(1)_{n+1} \rightarrow U(1)_{em}$$

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4} \mathrm{Tr}\Big[\sum_j (D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j\Big]$$

$$\mathcal{L}_{\psi\Sigma} = -M_F\Big[\epsilon_L \bar{\psi}_{L0}\Sigma_0\psi_{R1} - \sum_j \bar{\psi}_{Lj}\psi_{Rj} + \sum_j \bar{\psi}_{Lj}\Sigma_j\psi_{R,j+1} + \bar{\psi}_{Ln}\epsilon_R\Sigma_n\psi_{R,n+1} + \text{H.c.}\Big]$$

n(+2) Site Model: Gauge Bosons



$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4}\,\mathrm{Tr}\Big[\sum_j (D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j\Big]$$

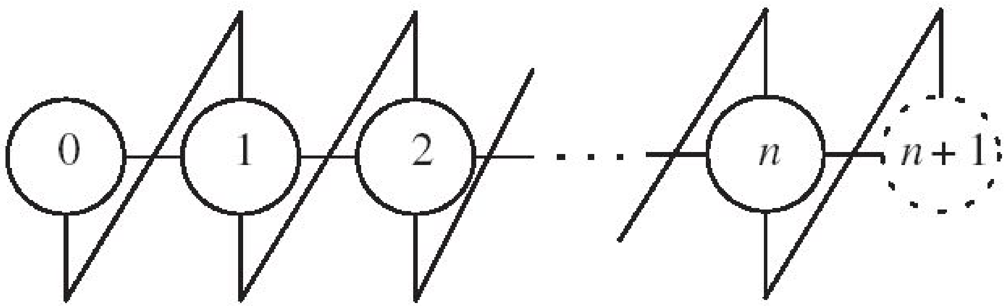
- The masses of the gauge bosons are obtained by expanding the Sigma field to leading order.
- The mass matrices are then diagonalized giving the mass eigenstates.
- The W and Z gauge bosons are mostly localized at the ends (the branes) with a small presence in the interior (bulk).
- $x \propto \frac{M_W}{M_{W1}}$

$$M_n^2 = \frac{\tilde{g}^2 f^2}{4}\begin{pmatrix} x^2 & -x & 0 & 0 & \cdot & 0 & 0 \\ -x & 2 & -1 & 0 & \cdot & 0 & 0 \\ 0 & -1 & 2 & -1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 & 0 \\ 0 & 0 & 0 & \cdot & -1 & 2 & -xt \\ 0 & 0 & 0 & \cdot & 0 & -xt & x^2t^2 \end{pmatrix}$$

$$v_{W0}^0 = 1$$

$$v_{W0}^j = \frac{n-j+1}{n+1}x$$

n(+2) Site Model: Fermions



$$\mathcal{L}_{\psi\Sigma} = -M_F\Big[\epsilon_L \bar{\psi}_{L0}\Sigma_0\psi_{R1} - \sum_j \bar{\psi}_{Lj}\psi_{Rj} + \sum_j \bar{\psi}_{Lj}\Sigma_j\psi_{R,j+1} + \bar{\psi}_{Ln}\epsilon_R\Sigma_n\psi_{R,n+1} + \text{H.c.}\Big]$$

- The fermions at the same site are vectorial and allowed Dirac masses.
- The fermions at adjacent sites are coupled via Σ.
- The mass matrix comes from the leading order expansion of Σ.
- The mass matrices were diagonalized and the mass eigenstates used in calculations.
- ϵ_L , ϵ_{Rf} $\ll 1$
- ϵ_L is flavor universal.

$$M_{F_0} = M_F\epsilon_L\epsilon_{R_f}$$

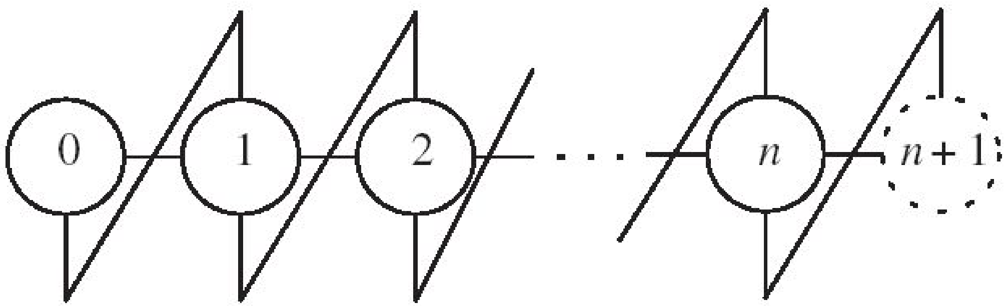
$$v^0_{LF_0} = 1$$

$$v^j_{LF_0} = \epsilon_L$$

$$v^j_{RF_0} = \epsilon_{R_f}$$

$$v^{n+1}_{RF_0} = 1$$

n(+2) Site Model: S



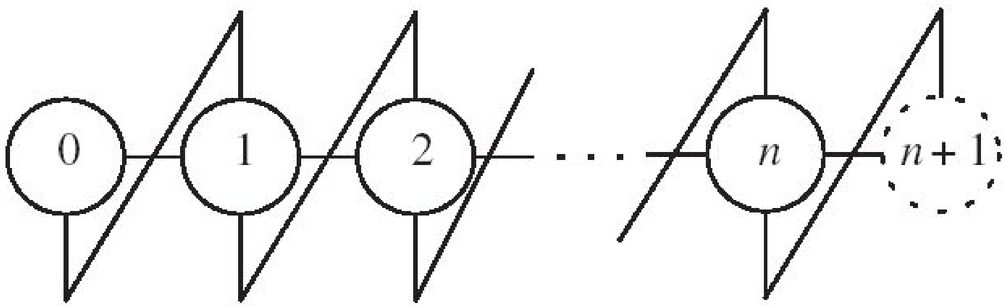
$$g_{We\nu} = \frac{e}{s_M}\left(1 + \frac{\alpha}{4s_M^2}S\right)$$

•Alterations of $g_{We\nu}$ can be parametrized by S.
•S can be calculated in the n(+2) site model and set to zero.

$$g_{We\nu} = \frac{e}{s_M}\left(1 + \frac{n(n+2)}{6(n+1)}x^2 - \frac{n}{2}\epsilon_L^2\right)$$

$$S = 0 \implies \epsilon_L^2 = \frac{n+2}{3(n+1)}x^2$$

n(+2) Site Model: Goldstone Bosons



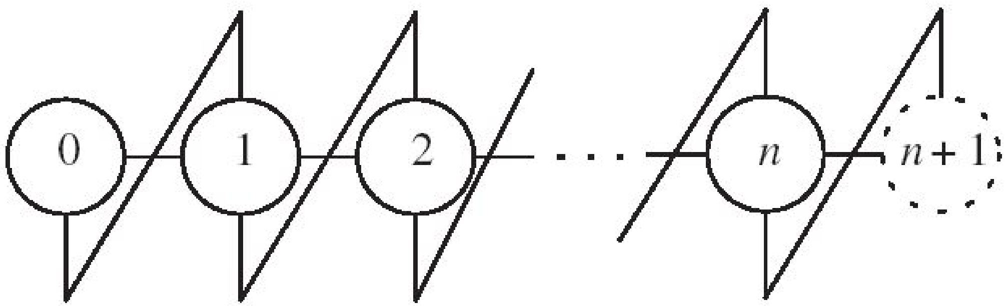
$$\mathcal{L}_{D\Sigma} = \frac{f^2}{4}\,\mathrm{Tr}\left[\sum_j (D_\mu \Sigma_j)^\dagger D^\mu \Sigma_j\right]$$

•The Goldstone bosons are determined by their mixing with the gauge bosons that eat them.
•The Goldstone bosons eaten by the W and Z are particularly simple.

$$\mathcal{L}_{\pi W} = -i\frac{\tilde{g}f}{2}\Big[\{\partial_\mu \pi_0, xW_0^\mu - W_1^\mu\} + \sum_{j=1}^{n-1}\{\partial_\mu \pi_j, W_j^\mu - W_{j+1}^\mu\} + \{\partial_\mu \pi_n, W_n^\mu - xtW_{n+1}^\mu\}\Big]$$

$$v_{\pi_0^\pm}^{[l]} = \frac{1}{\sqrt{n+1}} = v_{\pi_0^0}^{[l]}$$

n(+2) Site Model: Couplings



•The coupling are determined by expanding the Σ and replacing the site states with mass eigenstates.

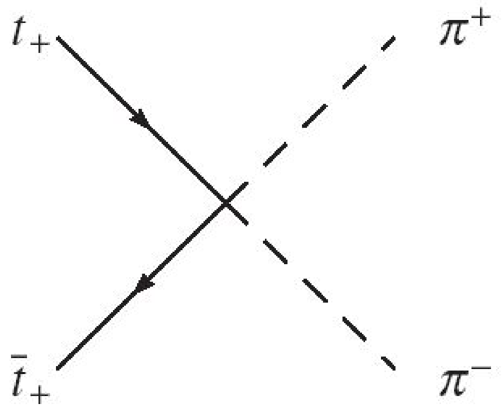
•The 4 point vertex $g_{tt\pi^+\pi^-}$ is not only suppressed by m_t/v but is also suppressed by $1/(n+1)$.

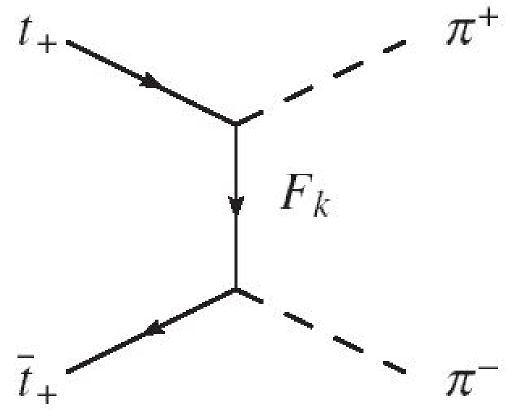
• $g_{tt\pi^+\pi^-}$ vanishes in the continuum limit!

$$g_{RtF_k\pi} = -i\frac{\sqrt{2}M_F}{f}\Big[\epsilon_L v^0_{LF_k} v^1_{Rt} v^{[0]}_\pi + \sum_i v^i_{LF_k} v^{i+1}_{Rt} v^{[i]}_\pi + \epsilon_{Rt} v^n_{LF_k} v^{n+1}_{Rt} v^{[n]}_\pi\Big] = \frac{i\sqrt{2}M_F\epsilon_R}{\sqrt{2n+1}(n+1)v}\tan\Big[\frac{(n-k+1)\pi}{2n+1}\Big]$$

$$g_{tt\pi^+\pi^-} = \frac{M_F}{f^2}\Big[\epsilon_L v^0_{Lt} v^1_{Rt} (v^{[0]}_\pi)^2 + \sum_i v^i_{Lt} v^{i+1}_{Rt} (v^{[i]}_\pi)^2 + \epsilon_{Rt} v^n_{Lt} v^{n+1}_{Rt} (v^{[n]}_\pi)^2\Big] = \frac{m_t}{(n+1)v^2}$$

n(+2) Site Model: Calculation





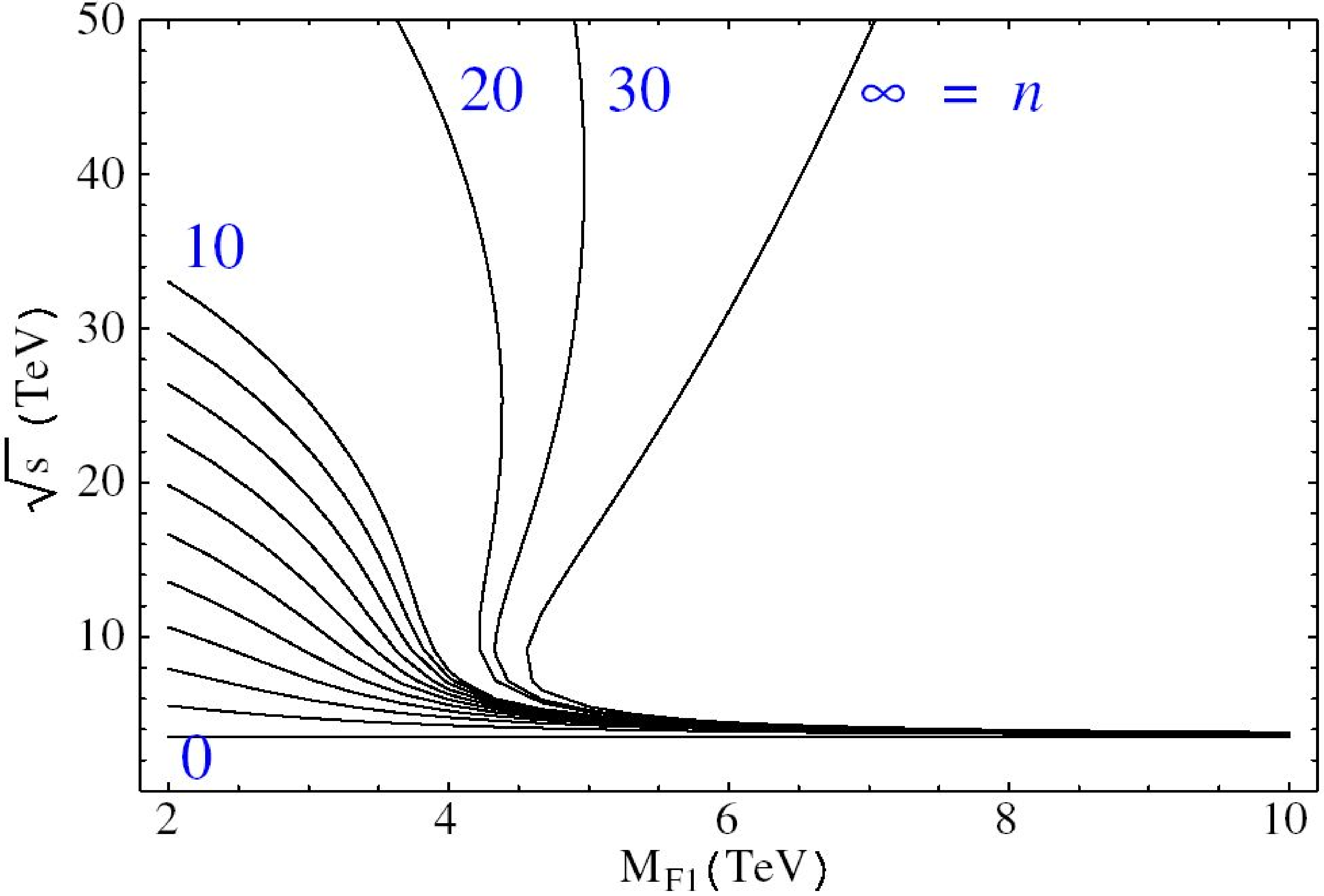
- The 4 point diagram grows like $\sqrt{s}$ for all energies.
- The T channel diagrams grow like $\sqrt{s}$ up to M_{F_k} .
- It is the F_k that unitarize this process and not the W_k !

$$\mathcal{M} = \sqrt{6s}\left(g_{tt\pi^+\pi^-} - \sum_k \frac{M_{F_k} g_{LtF_k\pi} g_{RtF_k\pi}}{t - M_{F_k}^2}\right)$$

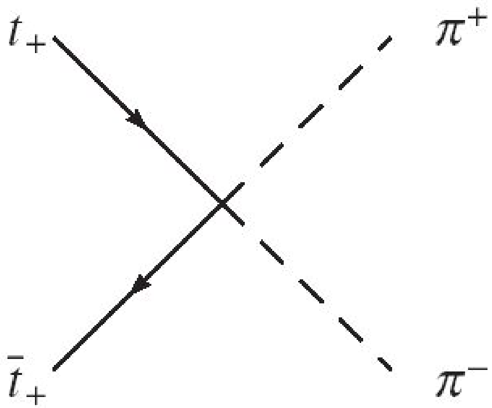
$$a_0 = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\,\mathcal{M}$$
$$= \frac{\sqrt{6}}{16\pi}\left[g_{tt\pi^+\pi^-}\sqrt{s} + \sum_k g_{LtF_k\pi} g_{RtF_k\pi}\, g\left(\frac{\sqrt{s}}{M_{F_k}}\right)\right]$$

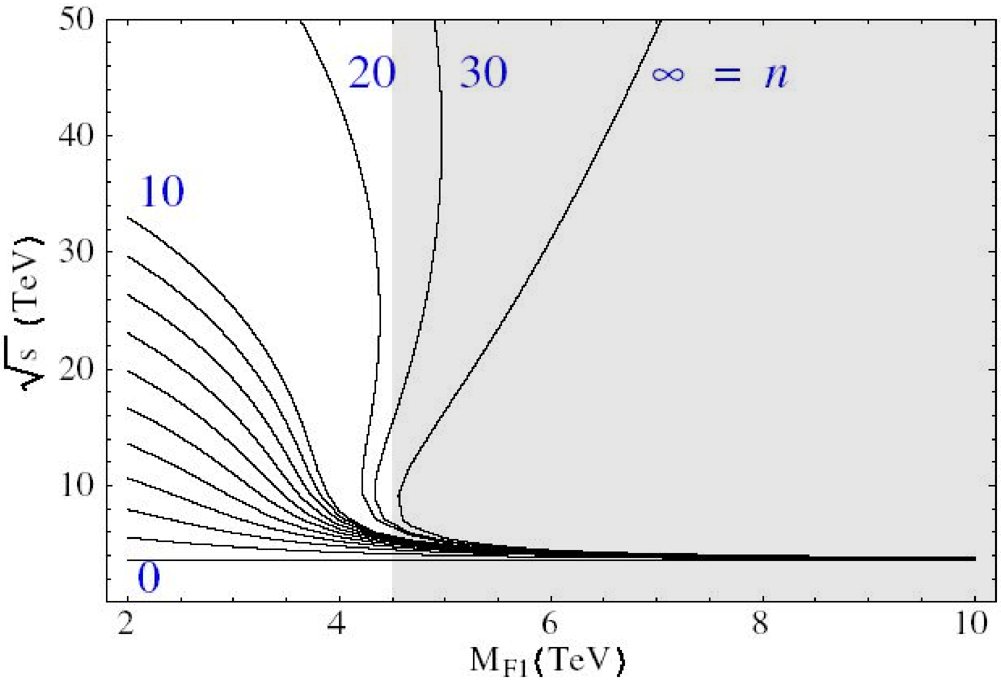
$$g(x) = \frac{1}{x}\ln(1 + x^2)$$

n(+2) Site Model: Unitarity Bound



n(+2) Site Model: $M_{F1} \ll 4.5\text{TeV}$





•For $M_{F1} \ll 4.5\text{TeV}$, the bound is determined by the 4 point vertex.

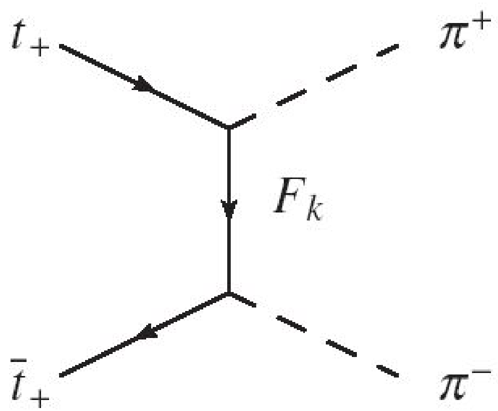
•In that limit, the bound is just a multiple of the AC bound.

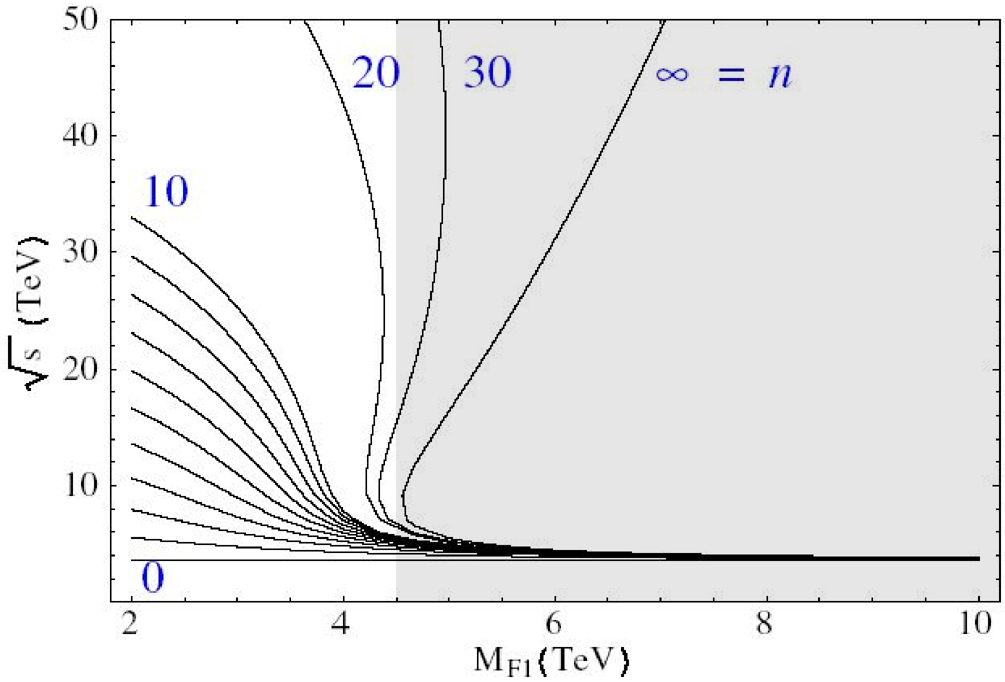
•The bound disappears in the continuum limit.

$$a_0 \simeq \frac{\sqrt{6s}m_t}{16\pi v^2(n+1)} \lesssim \frac{1}{2}$$

$$\sqrt{s} \lesssim (n+1)3.5\ \text{TeV}$$

n(+2) Site Model: $n \to \infty$





- The edge can be determined in the $n \to \infty$ limit where the 4 point vertex disappears
- The T channel is dominated by the first KK mode.

$$\lim_{n\to\infty} a_0 = \frac{2\sqrt{6}M_{F_1}m_t}{\pi^4 v^2}\sum_k \frac{(-1)^{k+1}}{(2k-1)^2}\, g\left(\frac{\sqrt{s}}{(2k-1)M_{F_1}}\right)$$

$$\lim_{n\to\infty} a_0(k=1) \approx \frac{2\sqrt{6}M_{F_1}m_t}{\pi^4 v^2}\, g\left(\frac{\sqrt{s}}{M_{F_1}}\right)$$

$$M_{F_1} \lesssim \frac{\pi^4 v^2}{2\sqrt{6}m_t \ln(5)} \sim 4.25 \text{ TeV}$$

Summary

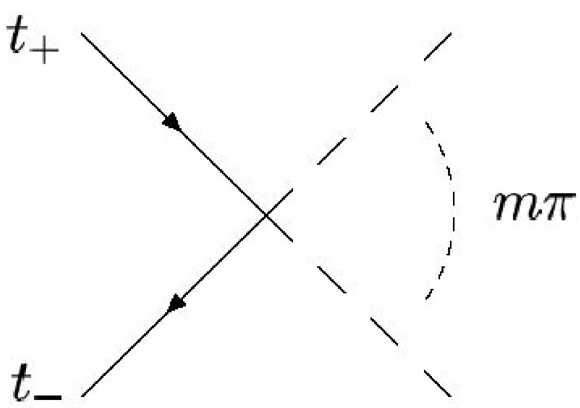
In this Higgsless model:

- The process $t_+\bar{t}_+ \to W_L^+ W_L^-$ is unitarized by a different set of fields than unitarize $W_L^+ W_L^- \to W_L^+ W_L^-$.

- The bound on the scale of fermion mass generation is independent of the scale of gauge boson mass generation.

Appendix

$2 \to m$: MNW



$$\sim \frac{m_t}{v^m}$$

$$\mathcal{L} = -m_t \left(\bar{t}_L \; , \; 0 \right) e^{i \frac{2\pi}{v}} \begin{pmatrix} t_R \\ 0 \end{pmatrix}$$

$$g_{tt\pi^m} \sim \frac{m_t}{v^m}$$

$$\sigma \sim \left(\frac{m_t}{v^m} \right)^2 s^{m-2} \lesssim \frac{4\pi}{s}$$

$$\sqrt{s} \lesssim \left(\frac{v^m}{m_t} \right)^{\frac{1}{m-1}} \xrightarrow[m \to \infty]{} v$$

•PRD **65**, 033004 (2002)
Maltoni, Niczyporuk, and Willenbrock

• $2 \to m$ may give a stronger bound than $2 \to 2$.

•Estimated $g_{tt\pi^m}$.

•Estimated the phase space as s^{m-2} .

•Showed that based on these estimates the bound could be reduced to v!

$2 \to m:$ DH

$$\mathcal{I}_m = \int \frac{d^3k_1 \cdots d^3k_m}{2E_1 \cdots 2E_m} \delta^{(4)}(P - k_1 - \cdots - k_m)$$

$$= \left(\frac{\pi}{2}\right)^{m-1} \frac{s^{m-2}}{(m-1)!(m-2)!}$$

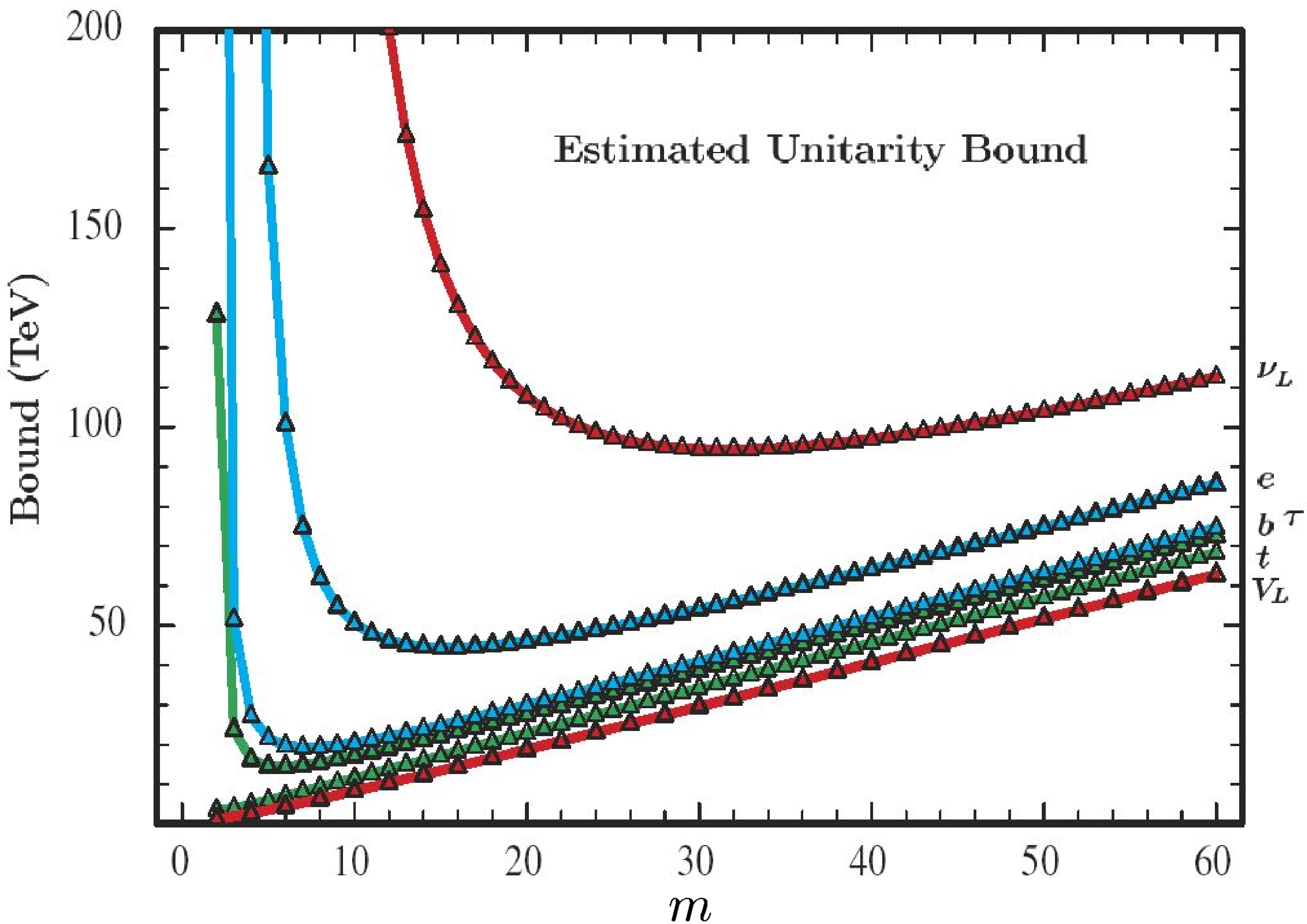
- PRD **71**, 093009 (2005)
  Dicus and He
- Strange, shouldn't $\sqrt{s} \geq mM_W$?
- They carefully recalculated the phase space.

$$\sigma \sim \left(\frac{m_t}{v^m}\right)^2 \frac{s^{m-2}}{(m-1)!(m-2)!} \lesssim \frac{4\pi}{s}$$

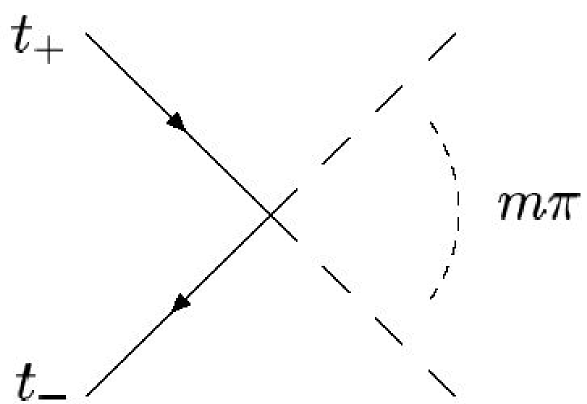
$$\sqrt{s} \lesssim \left(\frac{v^m}{m_t}\right)^{\frac{1}{m-1}} \Big((m-1)!(m-2)!\Big)^{\frac{1}{2(m-1)}}$$

$$\overrightarrow{m \to \infty} \; \sqrt{s} \lesssim \frac{m}{3} v$$

$2 \to m:$ DH


Estimated Unitarity Bound
Bound (TeV)
200
150
100
50
0
10
20
30
40
50
60
m
ν_L
e
b
τ
t
V_L

$2 \to m:$ n(+2) site



$$\sim \frac{m_t}{(n+1)^{m-1} v^m} \xrightarrow[n\to\infty]{} 0$$

- These vertices are further suppressed by $1/n^m$.
- These vertices disappear as $n \to \infty$.

$$g_{tt\pi^m} = \frac{2^m M_F}{\left(\sqrt{2}\right)^m m! f^m} \Big[\epsilon_L v_{Lt}^0 v_{Rt}^1 \left(v_\pi^0\right)^m + \sum_j v_{Lt}^j v_{Rt}^{j+1} \left(v_\pi^j\right)^m + \epsilon_{Rt} v_{Lt}^n v_{Rt}^{n+1} \left(v_\pi^n\right)^m \Big]$$

$$g_{tt\pi^m} = \frac{\left(\sqrt{2}\right)^m m_t}{m!(n+1)^{m-1} v^m}$$