The Three Site Model collider phenomenology

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In collaboration with
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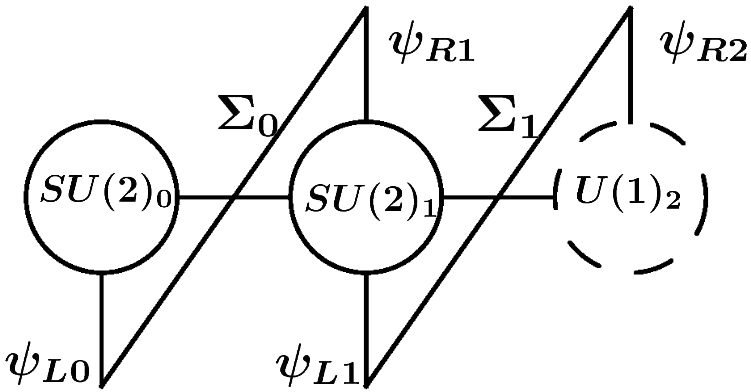
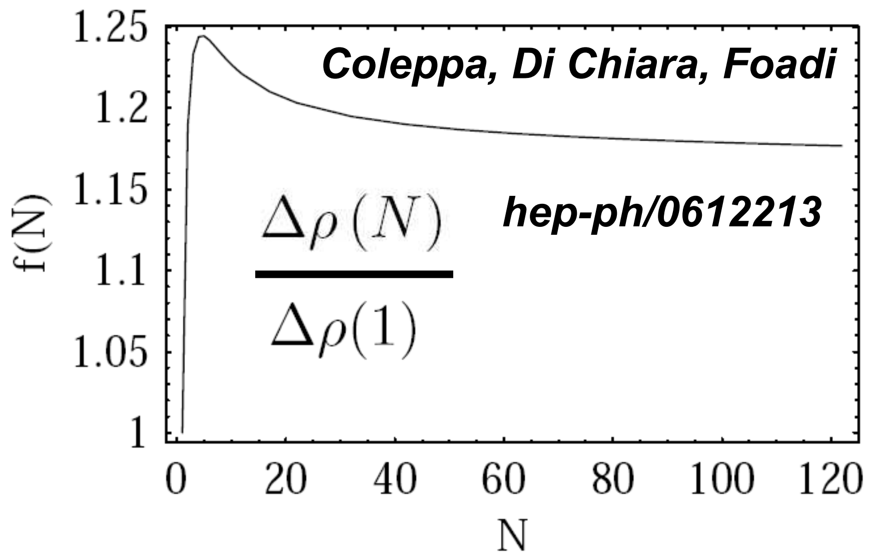
TSM: Representative of a Higgsless Extra Dimension

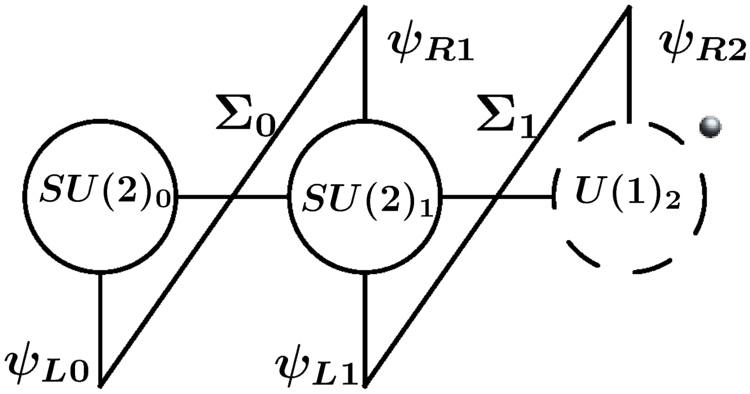


- Low energy phenomenology of a Higgsless ED is dominated by the 1st KK mode.



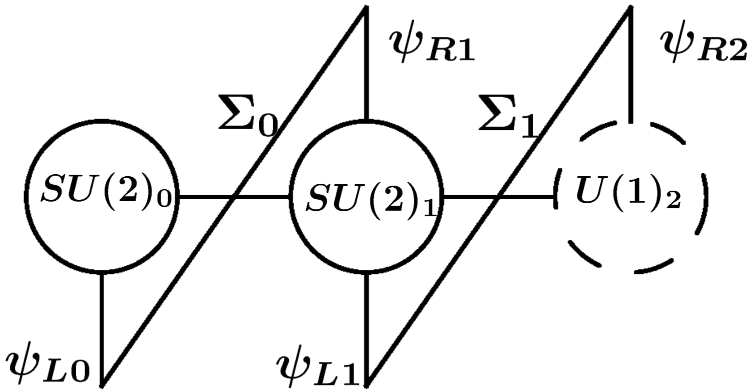
- The Three Site Model consistently implements the 1st KK mode in a gauge invariant way.

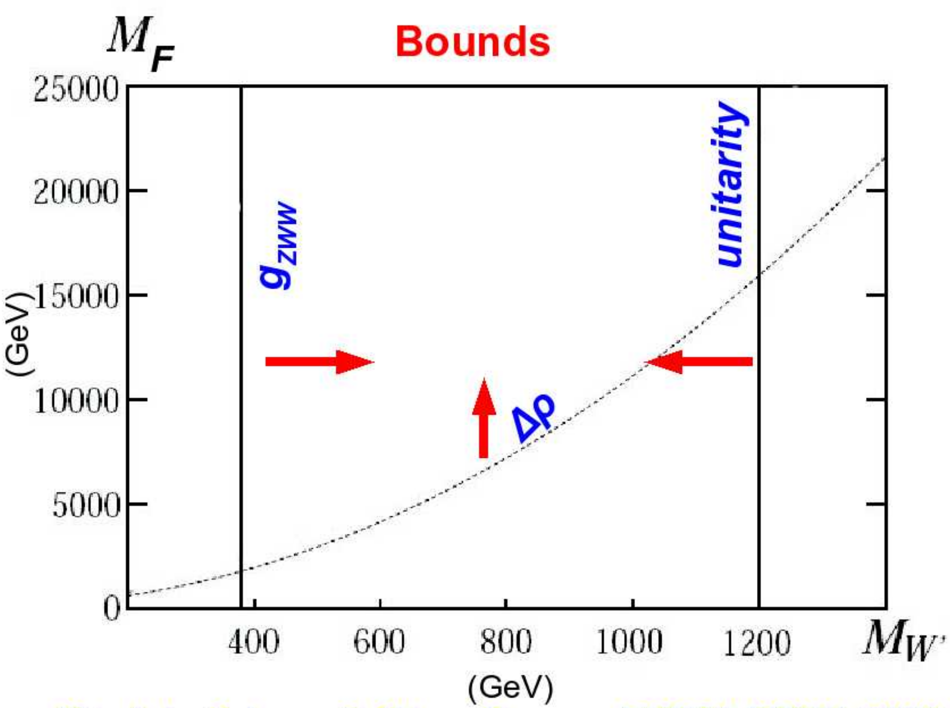
TSM: Representative of Dynamical EWSB



- Warped Higgsless ED is conjectured to be dual to a walking technicolor theory.
- The Three Site Model consistently implements the vector resonances (TC) in a gauge invariant way.
- Satisfies precision electroweak measurements (S=0).

The Three Site Model is testable

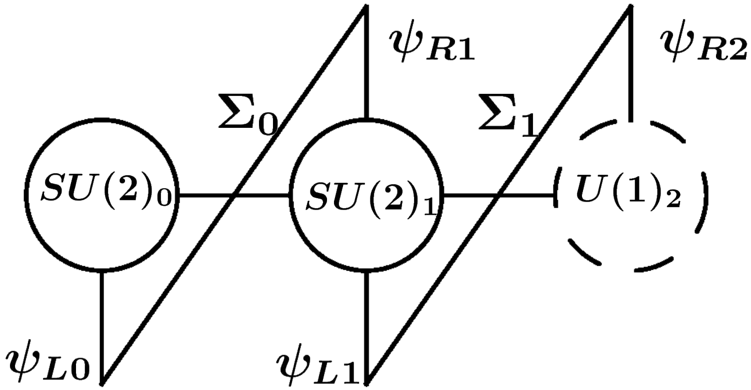




Chivukula, Coleppa, Di Chiara, Simmons: PRD **74**, 075011 (2006)

The parameter space is:

- Simple
- Bounded
  - from below by experiment
  - from above by unitarity
- **Can be tested at the LHC**
  - this talk, work in progress



Gauge Sector

$$\mathcal{L}_{F^2} = -\tfrac{1}{2}\mathrm{Tr}\left[F_0^2 + F_1^2 + F_2^2\right]$$

Casalbuoni, De Curtis, Dominici, Gatto (BESS) PLB 155 (1985) 95

Gauge - Goldstone Sector

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{2}\mathrm{Tr}\left[(D_\mu\Sigma_0)^\dagger D^\mu\Sigma_0 + (D_\mu\Sigma_1)^\dagger D^\mu\Sigma_1\right]$$

$$x = \frac{2M_W}{M_{W'}} \qquad M_W = g_1 f\,\frac{\sqrt{2+x^2-\sqrt{4+x^4}}}{2\sqrt{2}}$$

$$\frac{g_0}{g_1} = x \ll 1$$

$$\frac{g_2}{g_0} = s/c = t$$

$$\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2}$$

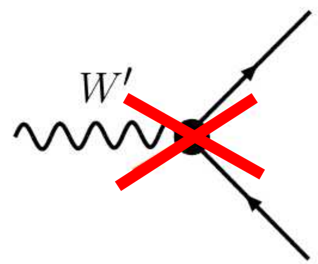
$$\Sigma_j = e^{i\frac{2\pi_j}{f}}$$

Fermion - Goldstone Sector

$$\mathcal{L}_{\Sigma\psi} = -M_F\left(\epsilon_L\bar{\psi}_{L0}\Sigma_0\psi_{R1} + \bar{\psi}_{L1}\psi_{R1} + \bar{\psi}_{L1}\Sigma_1\epsilon_R\psi_{R2}\right)$$

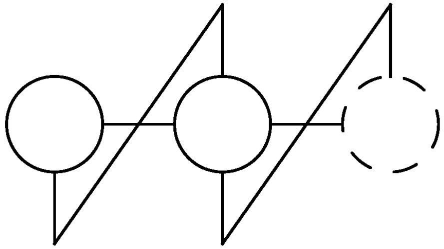
ideal delocalization (IDEL)

$$\frac{g_0(\psi_{L0}^f)^2}{g_1(\psi_{L1}^f)^2} = \frac{v_W^0}{v_W^1} \rightarrow \epsilon_L^2 = \frac{2x^2}{2 - x^2 + \sqrt{4+x^4}}$$



$$g_W^{TSM} = g_W^{SM} + O(x^4)$$

Independent parameters: M_W, s_W, $M_{W'}$, M_F



Allowed deviation from IDEL

$$-0.33 < S < 0.07 \text{ at } 95\%\text{C.L.}$$

$$M_H^{ref} = 117\text{GeV}$$

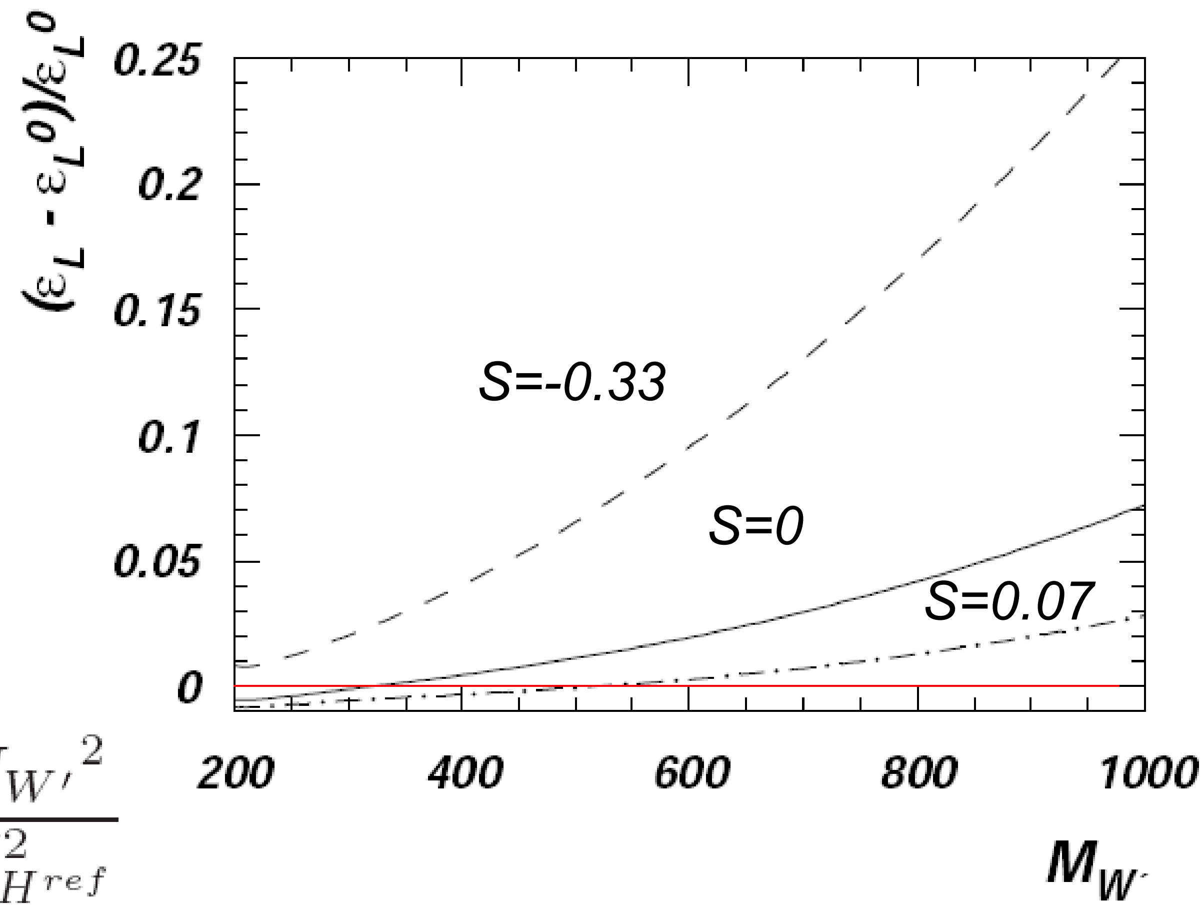
$$g_{We\nu} = \frac{e}{s_M}\left(1 + \frac{\alpha_{em}}{4s_M^2}S^0\right)$$

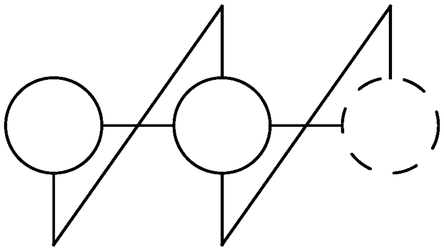
$$g_{We\nu} = \frac{e}{s_M}\left(1 + \frac{x^2}{4} - \frac{\epsilon_L^2}{2}\right)$$

$$\epsilon_L^2 = \frac{1}{2}\left[x^2 - \frac{\alpha_{em}(1+x^2)}{s_M^2}S^0\right]$$

$$S = S^0 + \delta S = S^0 + \frac{1}{12\pi}\log\frac{{M_{W'}}^2}{M_{H^{ref}}^2}$$

(see E.Simmons and C.Jackson talks)



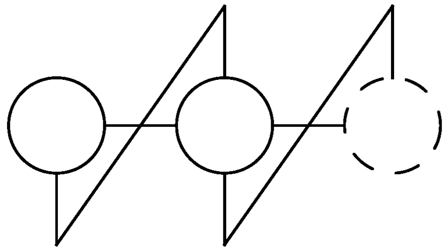


Particle Content

$$\gamma, G \qquad Z, W^{\pm} \qquad Z', W'^{\pm}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{pmatrix} u' \\ d' \end{pmatrix} \begin{pmatrix} c' \\ s' \end{pmatrix} \begin{pmatrix} t' \\ b' \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \qquad \begin{pmatrix} \nu'_e \\ e' \end{pmatrix} \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix} \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}$$



CalcHEP (by Alexander Pukhov)

- **User friendly graphical interface.**
  - Batch mode also available.
- **Easy implementation of new models.**
  - Especially using LanHEP (by Andrei Semenov).
- **Feynman gauge and unitary gauge.**
  - Important cross check.
- **Interface with Pythia**
- **Many other new features**

LanHEP (by Andrei Semenov)

- ***Automatic Feynman rules from Lagrangian***
- ***Has checks for***
  - *Hermiticity*
  - *BRST invariance*
  - *EM charge conservation*
  - *Particle mixings, mass terms, and mass matrices*

Example of model Implementation using LanHEP

LanHEP

$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left(F_0^2 + F_1^2 + F_2^2\right) \quad \textbf{where} \quad F_j^{\mu\nu} = \partial^\mu W_j^\mu - \partial^\nu W_j^\mu + i g_j \left[W_j^\mu, W_j^\nu\right]$$

```
%%%%%%%%%%  Kinetic and self interaction Lagrangian terms.
lterm  -F**2/4  where  F=deriv^mu*W23^nu-deriv^nu*W23^mu.
lterm  -F**2/4  where  F=deriv^mu*W0^nu^a-deriv^nu*W0^mu^a-g*eps^a^b^c*W0^mu^b*W0^nu^c.
lterm  -F**2/4  where  F=deriv^mu*W1^nu^a-deriv^nu*W1^mu^a-g/x*eps^a^b^c*W1^mu^b*W1^nu^c.
```

(gauge kinetic term as an example)

lhep 3-site.mdl

CalcHEP

Lagrangian

P1	P2	P3	P4	> Factor
A	W+	W-		-g*v0g
A	~W+	~W-		-g*v0g
W+	W-	Z		-g/x
W+	W-	~Z		-g/x
W+	Z	~W-		-g/x
W+	~W-	~Z		-g/x
W-	Z	~W+		-g/x
W-	~W+	~Z		-g/x
Z	~W+	~W-		-g/x
~W+	~W-	~Z		-g/x
A	A	W+	W-	-g**2*v0g**2
A	A	~W+	~W-	-g**2*v0g**2
A	W+	W-	Z	-g**2*v0g/x
A	W+	W-	~Z	-g**2*v0g/x
A	W+	Z	~W-	-g**2*v0g/x
A	W+	~W-	~Z	-g**2*v0g/x
A	W-	Z	~W+	-g**2*v0g/x
A	W-	~W+	~Z	-g**2*v0g/x
A	Z	~W+	~W-	-g**2*v0g/x
A	~W+	~W-	~Z	-g**2*v0g/x
W+	W+	W-	W-	g**2/x**2
W+	W+	W-	~W-	g**2/x**2
W+	W+	~W-	~W-	g**2/x**2
W+	W-	W-	~W+	g**2/x**2
W+	W-	Z	Z	-g**2/x**2
W+	W-	Z	~Z	-g**2/x**2
W+	W-	~W+	~W-	g**2/x**2
W+	W-	~Z	~Z	-g**2/x**2
W+	Z	Z	~W-	-g**2/x**2
W+	Z	~W-	~Z	-g**2/x**2
W+	~W+	~W-	~W-	g**2/x**2
W+	~W-	~Z	~Z	-g**2/x**2
W-	W-	~W+	~W+	g**2/x**2
W-	Z	Z	~W+	-g**2/x**2
W-	Z	~W+	~Z	-g**2/x**2
W-	~W+	~W+	~W-	g**2/x**2
W-	~W+	~Z	~Z	-g**2/x**2
Z	Z	~W+	~W-	-g**2/x**2
Z	~W+	~W-	~Z	-g**2/x**2
~W+	~W+	~W-	~W-	g**2/x**2
~W+	~W-	~Z	~Z	-g**2/x**2

Example of model Implementation using LanHEP

LanHEP

$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left(F_0^2 + F_1^2 + F_2^2\right) \quad \textbf{where} \quad F_j^{\mu\nu} = \partial^\mu W_j^\mu - \partial^\nu W_j^\mu + i g_j \left[W_j^\mu, W_j^\nu\right]$$

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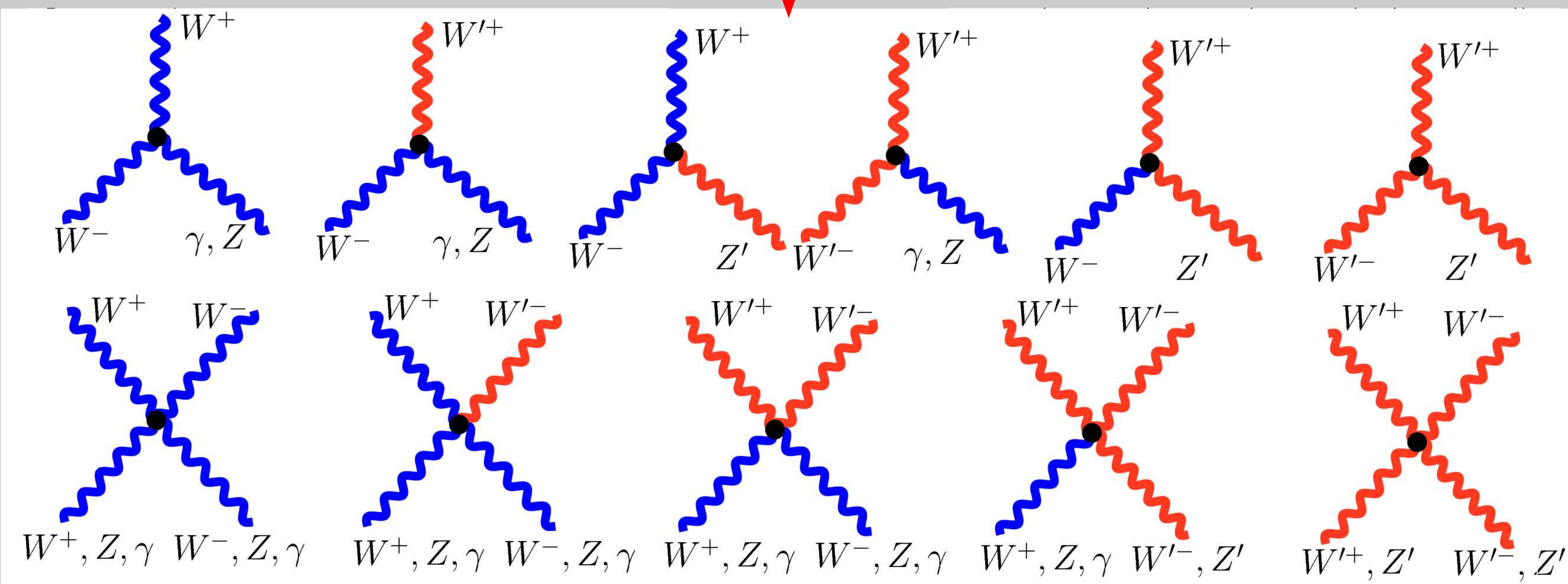
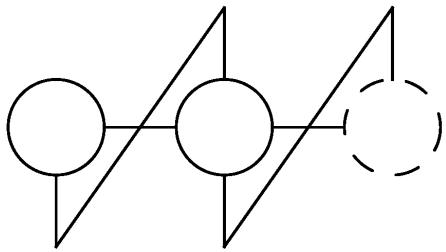
lterm  -F**2/4  where  F=deriv^mu*W1^nu^a-deriv^nu*W1^mu^a-g/x*eps^a^b^c*W1^mu^b*W1^nu^c.
```

(gauge kinetic term as an example)

lhep 3-site.mdl

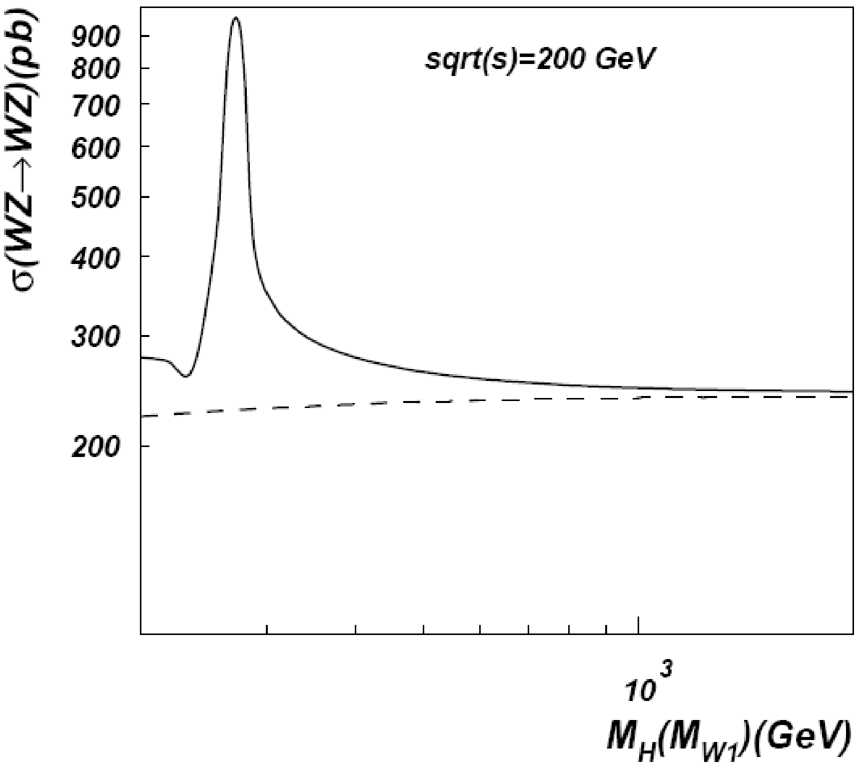
CalcHEP





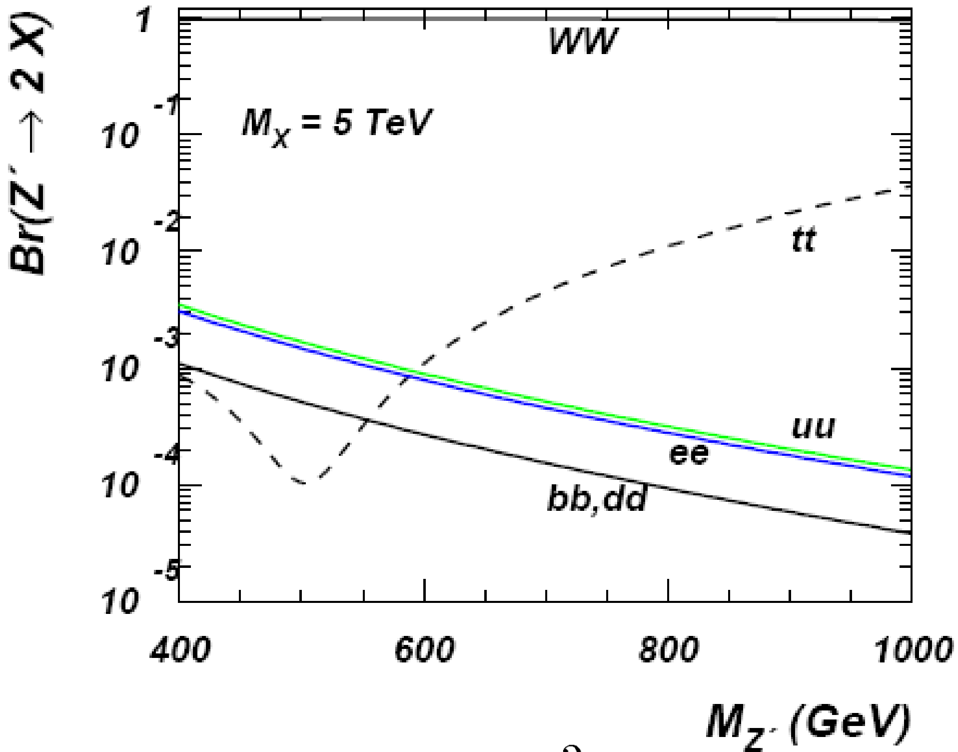
Checks:

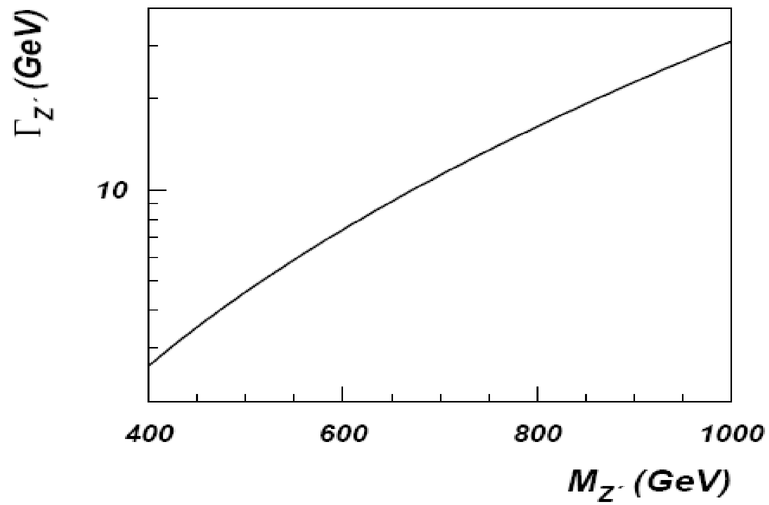
- **Feynman vs. Unitary gauge.**
- **Decoupling of heavy fields.**
- **Masses and mixings (LanHEP).**
- **Hermiticity (LanHEP).**



Gauge boson widths and branchings

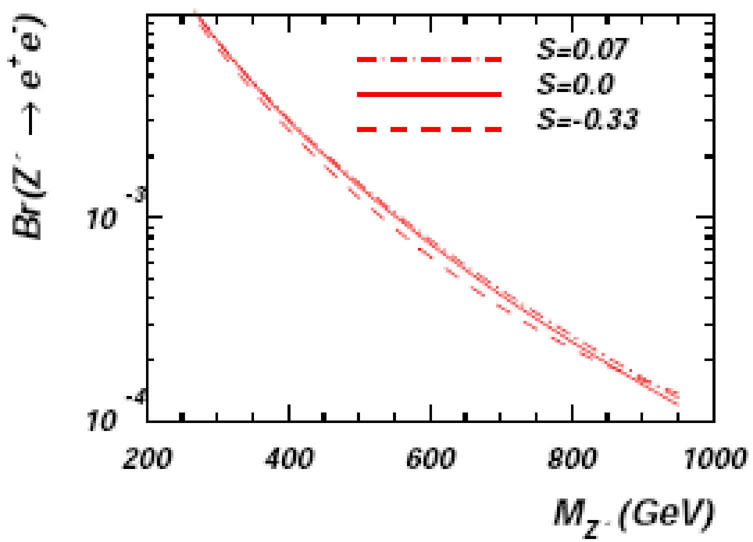
- **Fermiophobic nature of the gauge bosons**
- **Dominant decay into WW and WZ pairs**
- **Z' Br does not depend much on deviation from ideal delocalization**





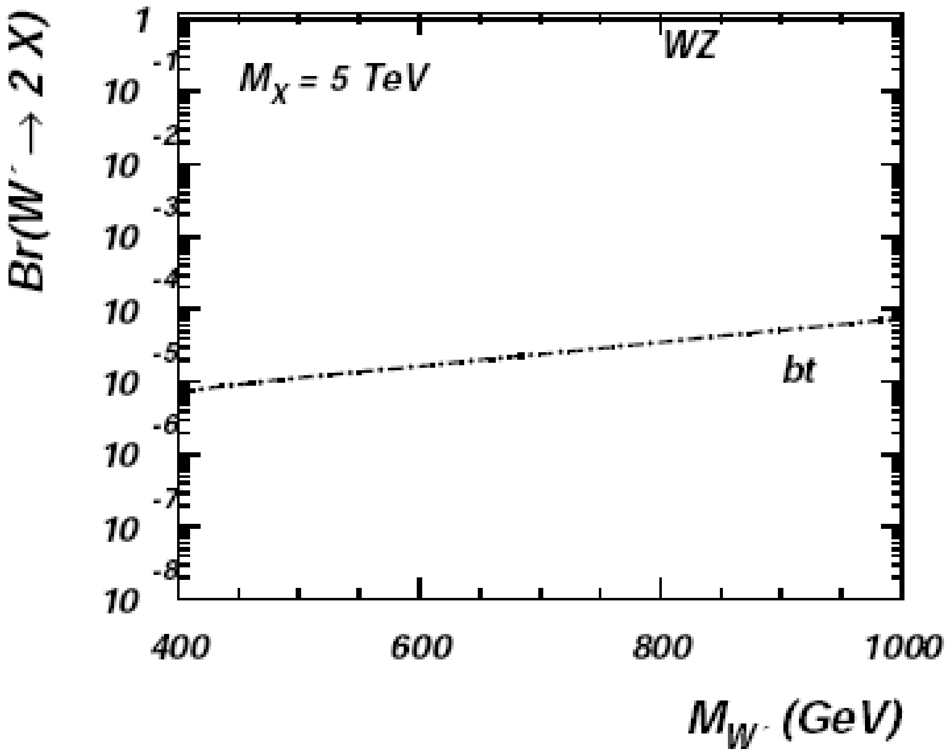
$$\Gamma(Z' \to W^+W^-) = \frac{e^2 M_{W'}}{192\pi x^2 s_w^2}$$

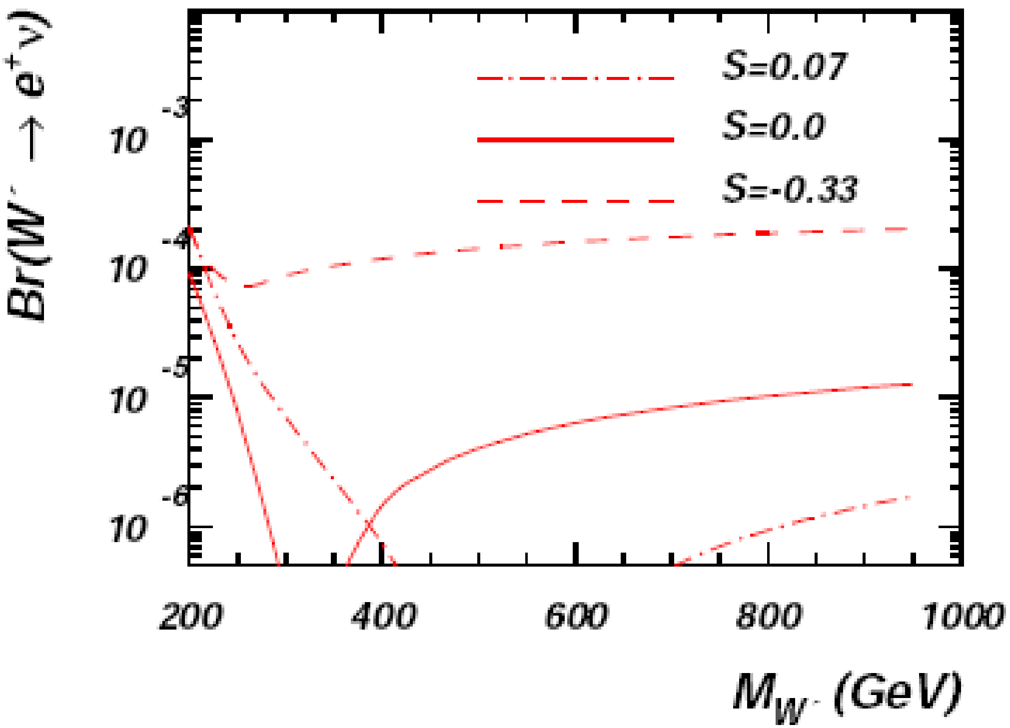
$$\Gamma(Z' \to e^+e^-) = \frac{5e^2 M_{W'} x^2 s_w^2}{384\pi c_w^4}$$



W' decays

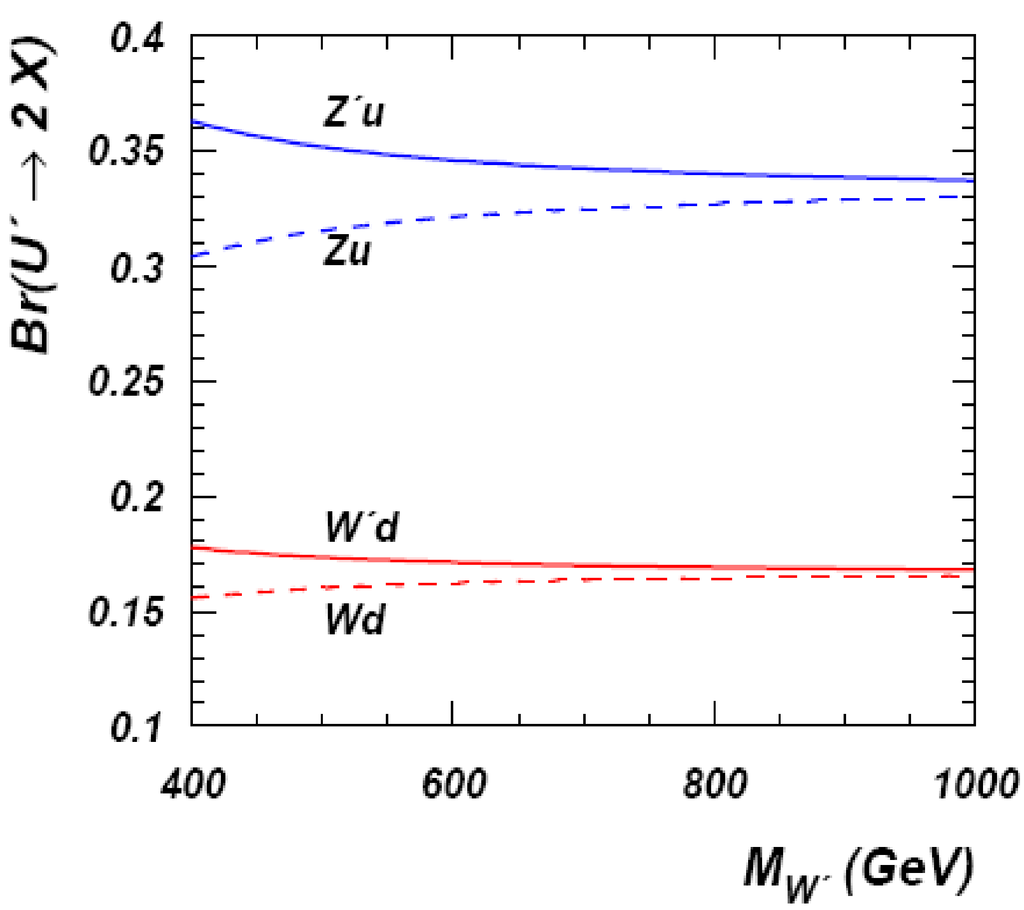
- **decays into fermions strongly depend on delocalization**

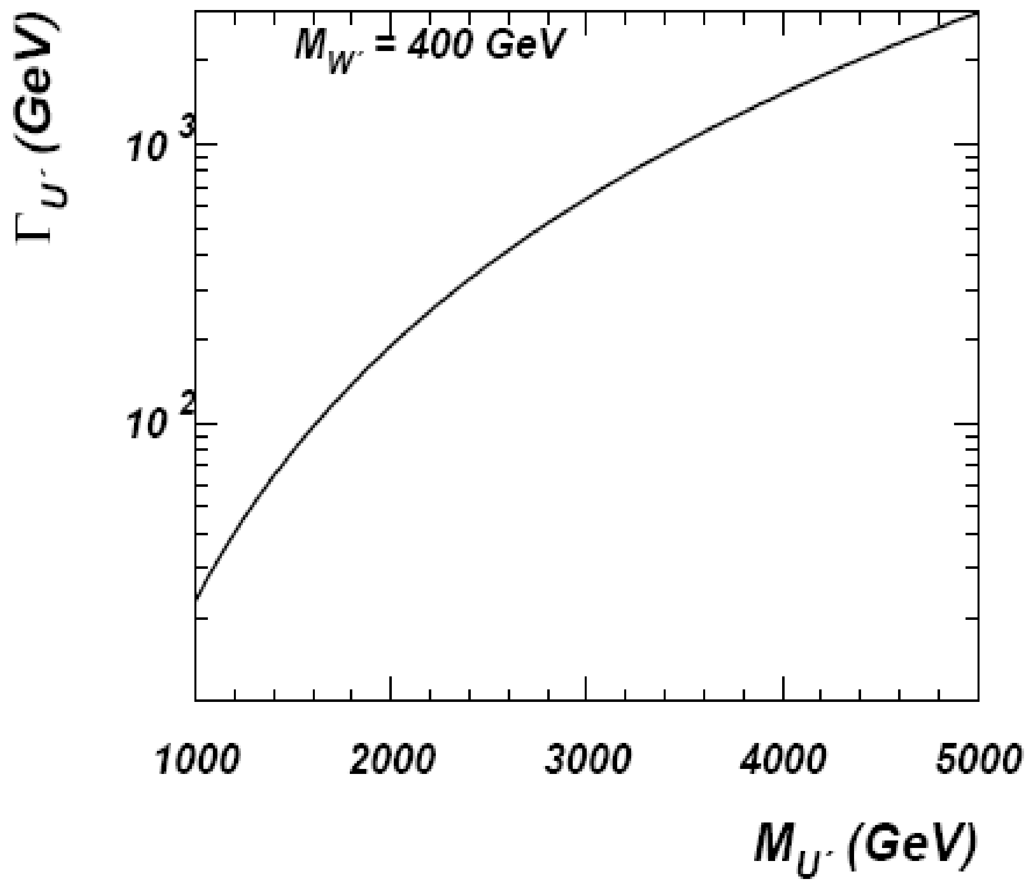




$$\Gamma(W' \to e^+e^-) = \frac{e^2 M_{W'} x^2 \left(1 - \frac{2\epsilon_L^2}{x^2}\right)^2}{192\pi s_w^2}$$

Heavy fermions



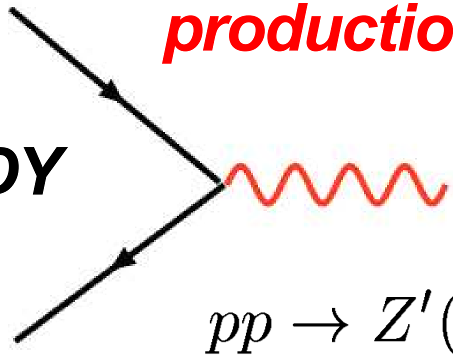


- ***crucial ingredient of the model, in particular, provide unitarity (see Neil Christensen's talk)***
- ***but are too heavy to be observed even in strong pair production processes***

Three Site Model Signatures

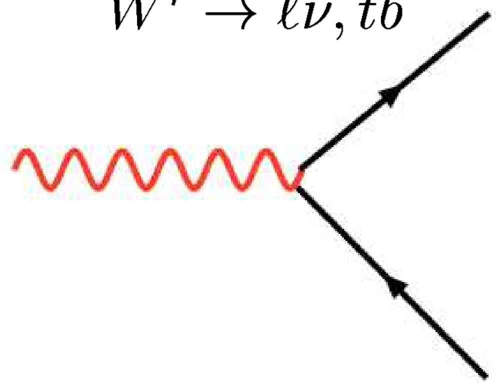
production | **decay**

DY

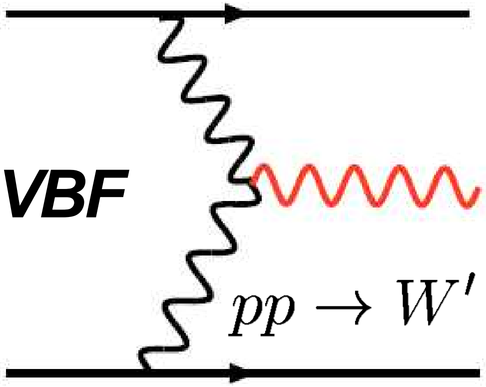


$pp \to Z'(W')$

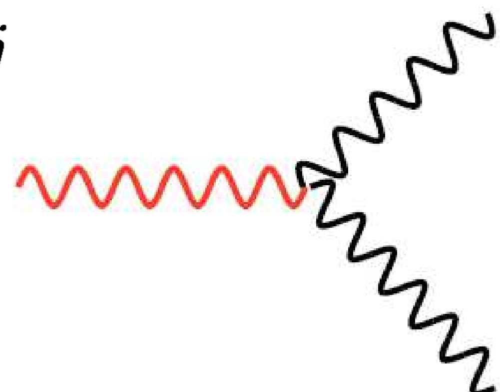
$Z' \to \ell^+\ell^-, t\bar{t}$
$W' \to \ell\nu, tb$



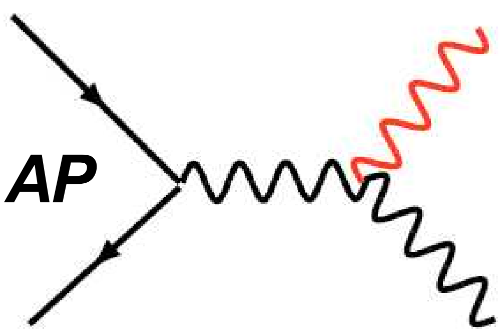
VBF



$pp \to W'(Z')jj$

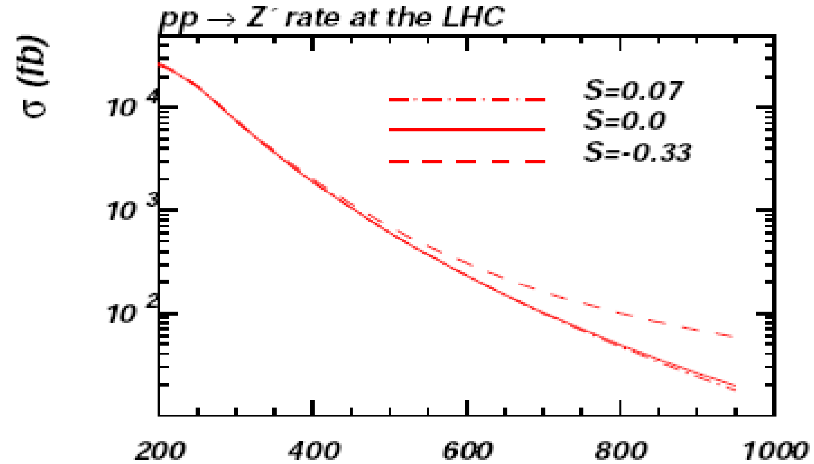


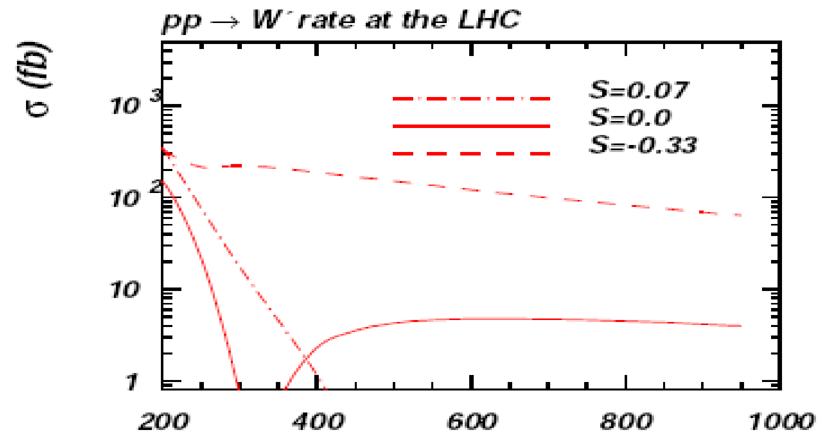
AP

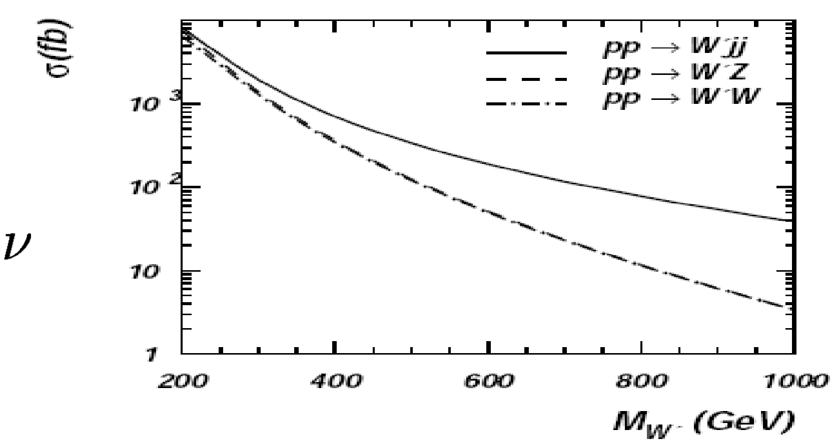


$pp \to WZ', WW', \ldots$

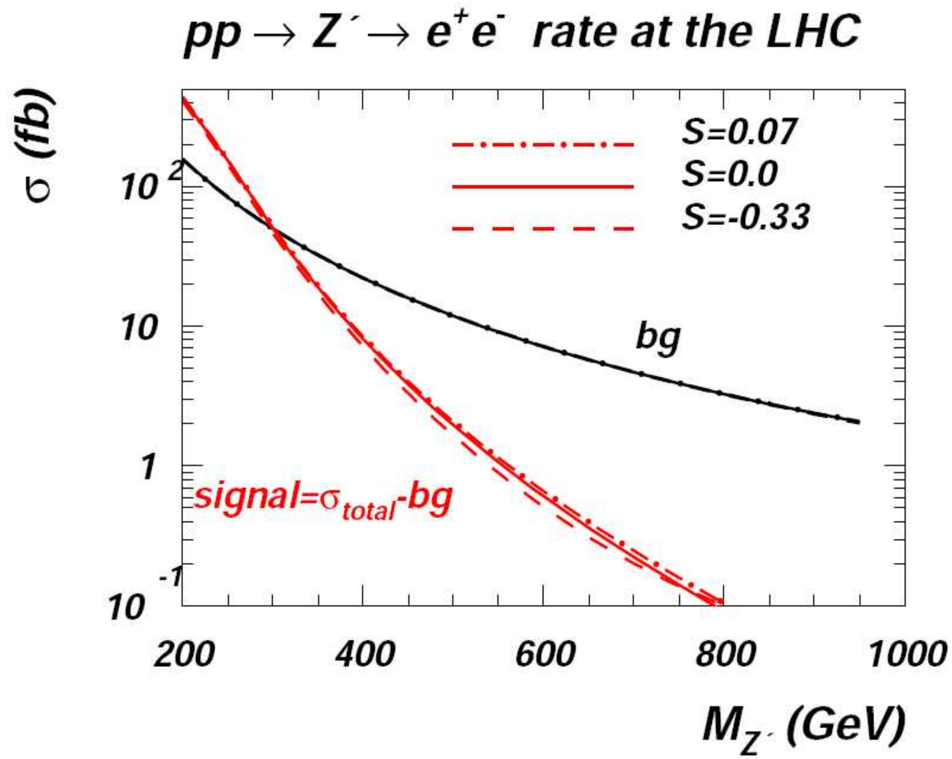
$Z' \to W^+W^- \to \ell^+\ell^-\nu$
$W' \to WZ \to \ell\ell\ell\nu$

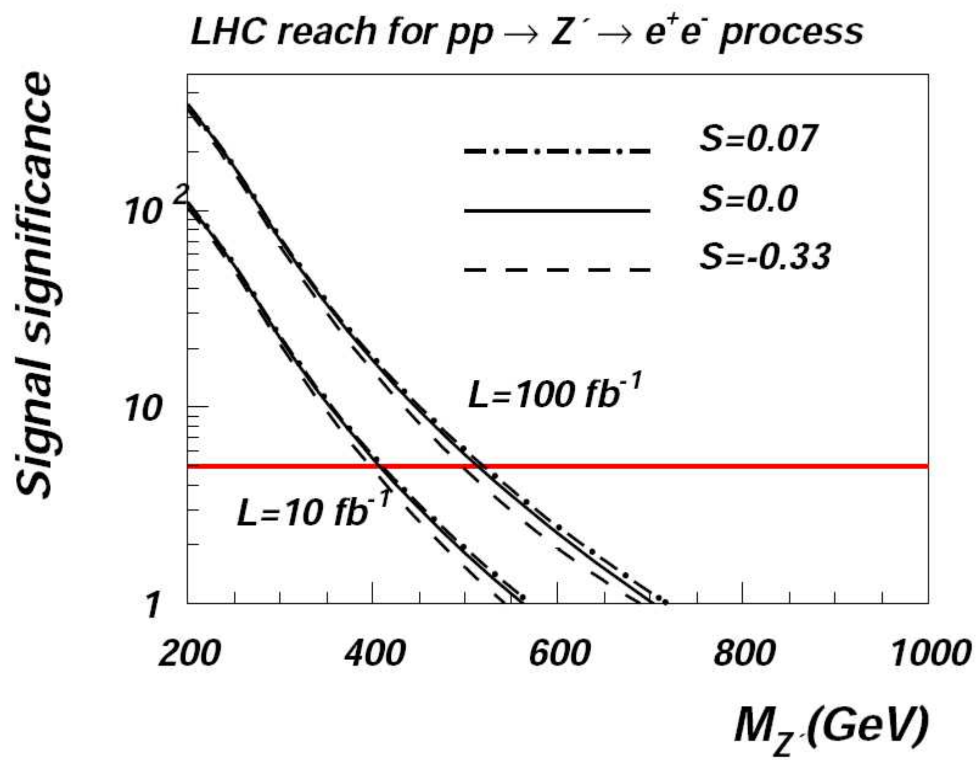




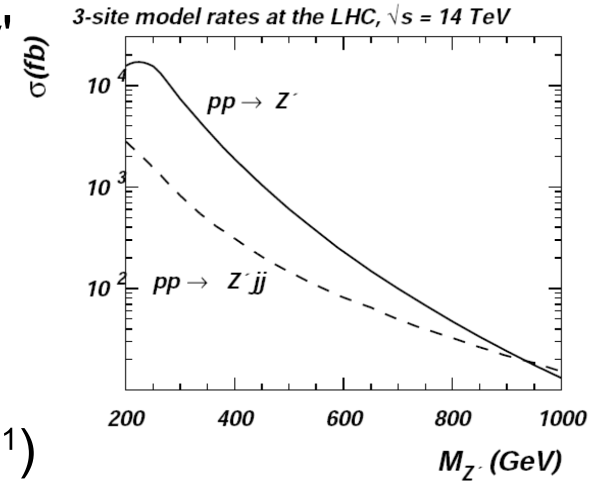


LHC reach for DY di-lepton signature





- Decay and production are suppressed by x^4 compared to 'toy' PYTHIA Z' model
- The realistic (e.g. TSM) is very different from the toy one!
  - Discovery range drops from ~3-5 TeV down to ~0.5 TeV
  - fermiophobic Z' required by EW data
  - Z'WW coupling is non-vanishing to provide unitarity
- $e^+e^- + \mu^+\mu^-$ + VBF will extend limit up to ~0.6 TeV (100 fb^{-1})

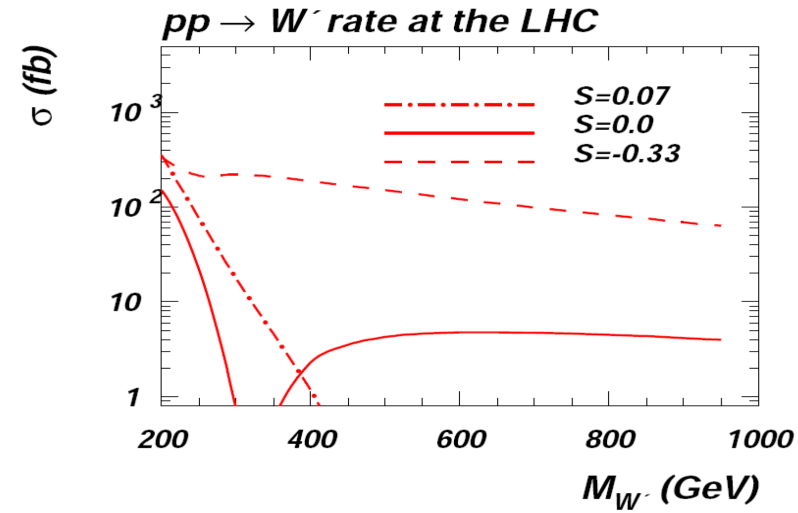


LHC reach for DY tri-lepton signature

In case of maximal deviation from idea delocalization

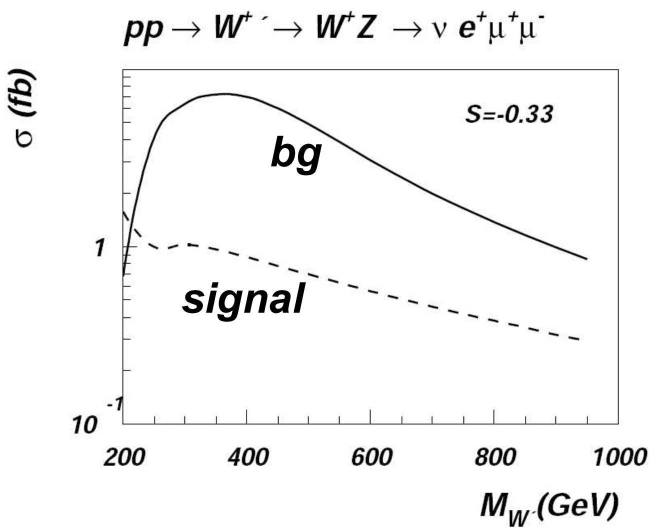
$$pp \to W' \to WZ \to 3\ell + \nu$$

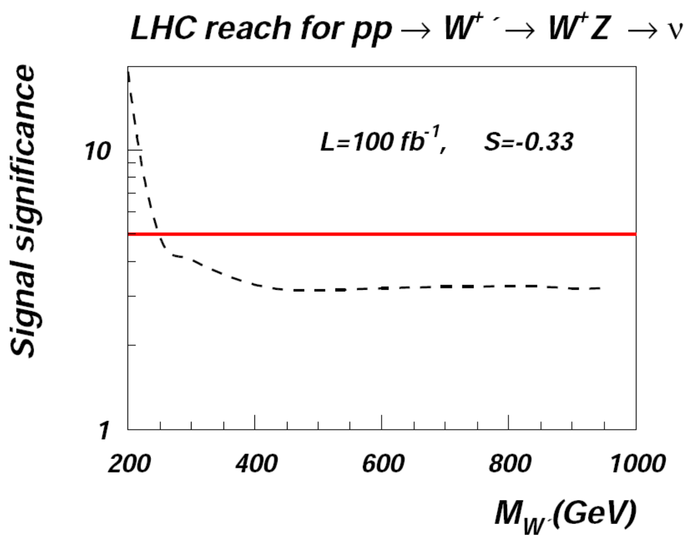
process can become important

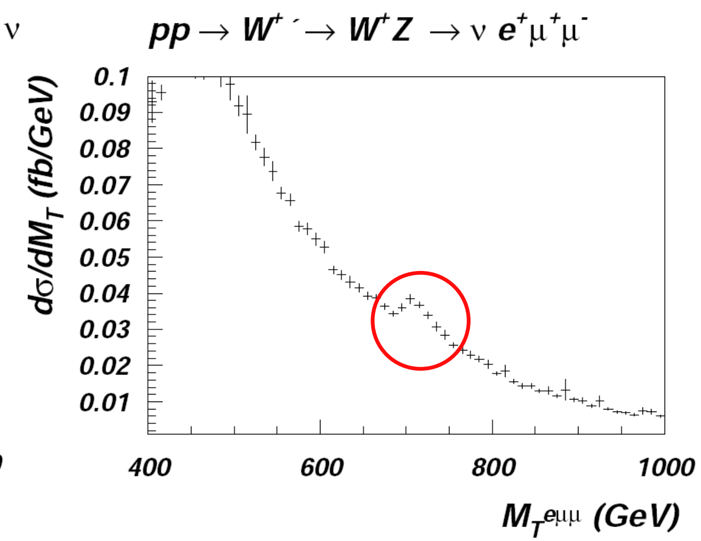


cuts:
$|M_{\mu^+\mu^-} - M_Z| < 10$ GeV
$P_T^\ell > 20$ GeV
$M_{W'} + 5\Gamma_{W'} > M_T^{e\mu\mu} > M_{W'} - \Gamma_{W'}$

~ 1 fb for $M_{W'}$=700 GeV but
further BG reduction is necessary: work in progress



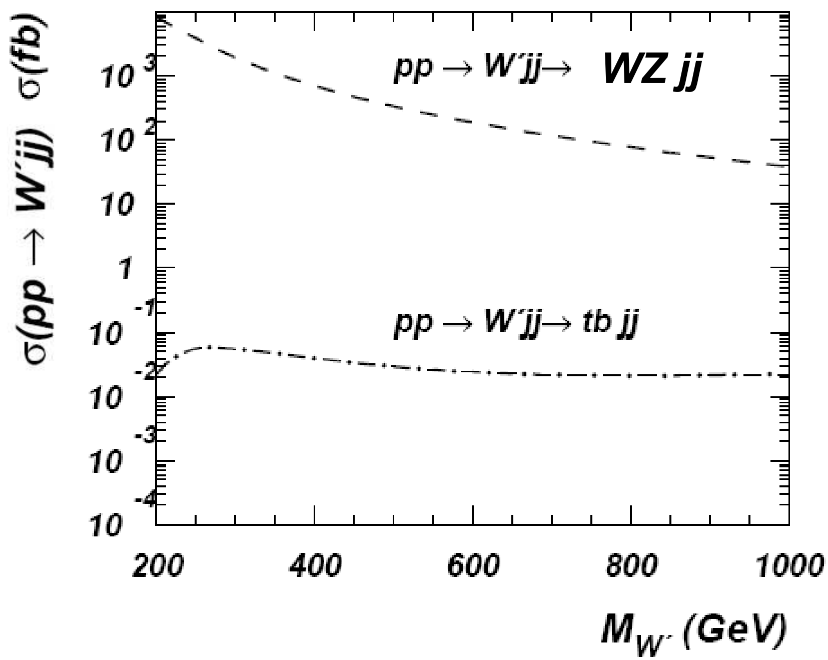


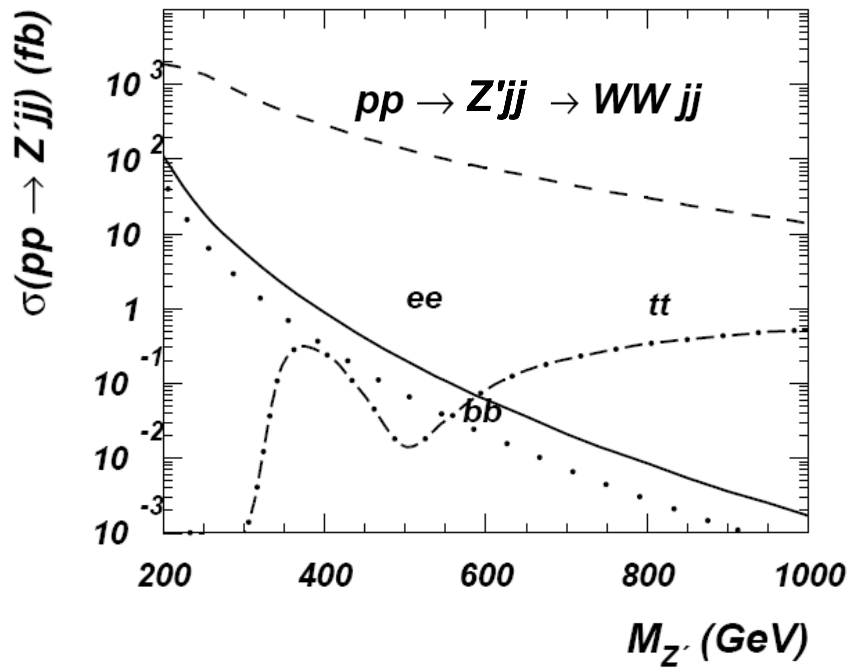


WW → Z' and WZ → W' Fusion

next promising step

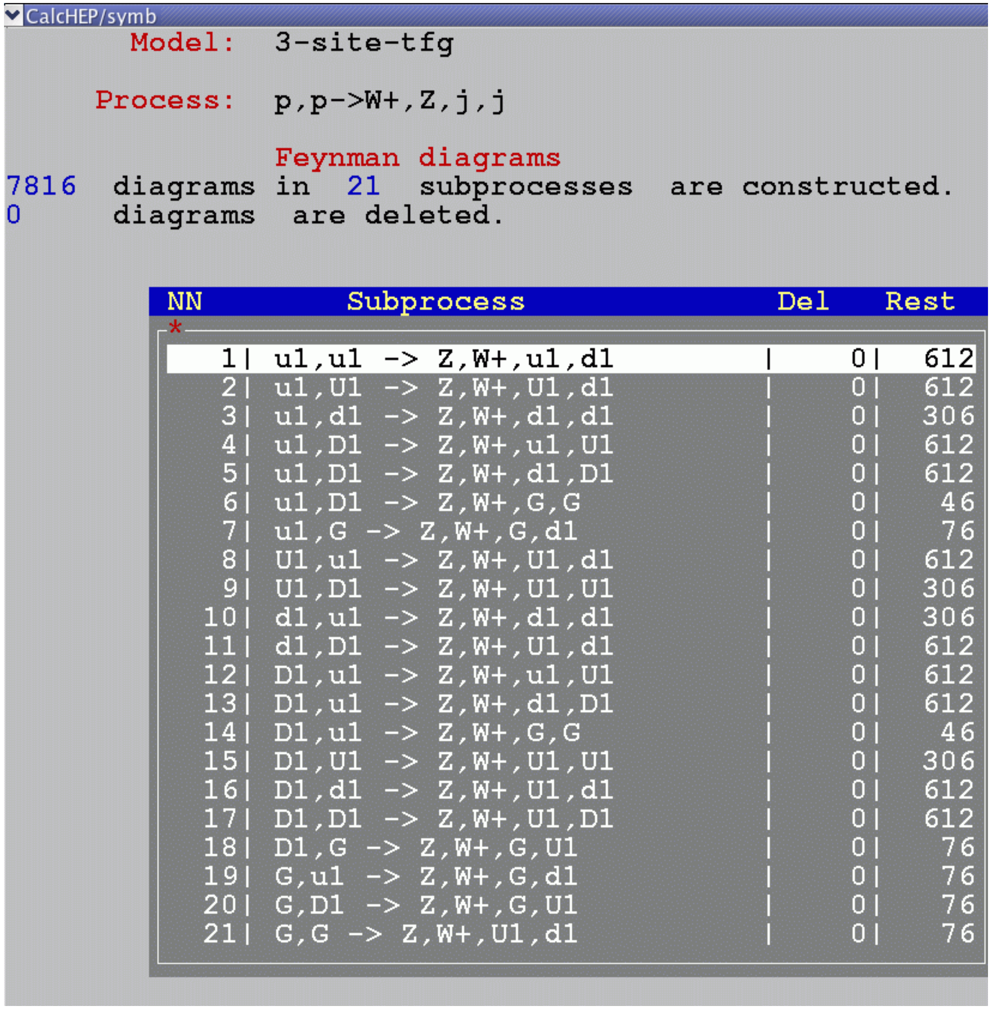
- ~ 1fb for tri-lepton signature for $M_{W'}$ ~ 1TeV
- lower/more reducible background as compared to DY bg
- manageable with CalcHEP!





$pp \to W^+ Zjj$

- ***No effective WZ approximation.***
- ***Complete set of signal and background diagrams including interference.***



CalcHEP/symb

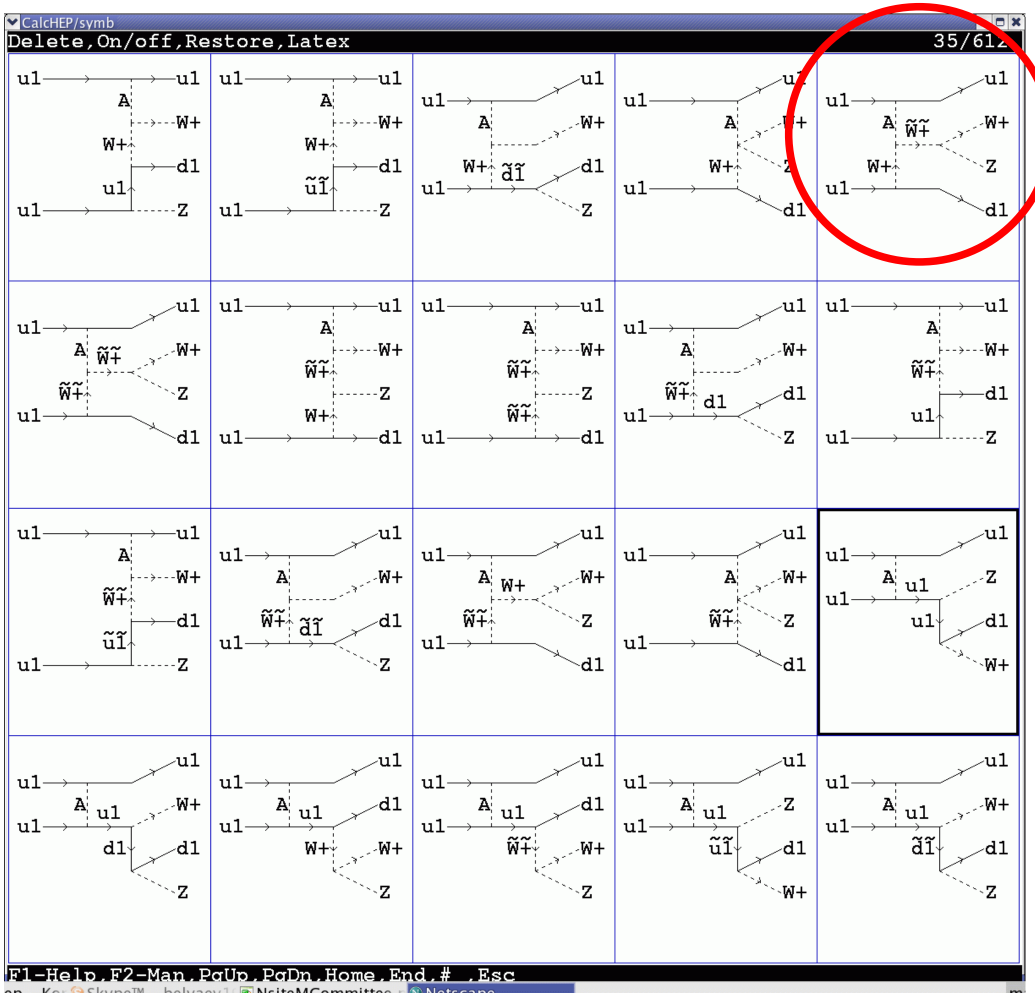
Model: 3-site-tfg

Process: p,p->W+,Z,j,j

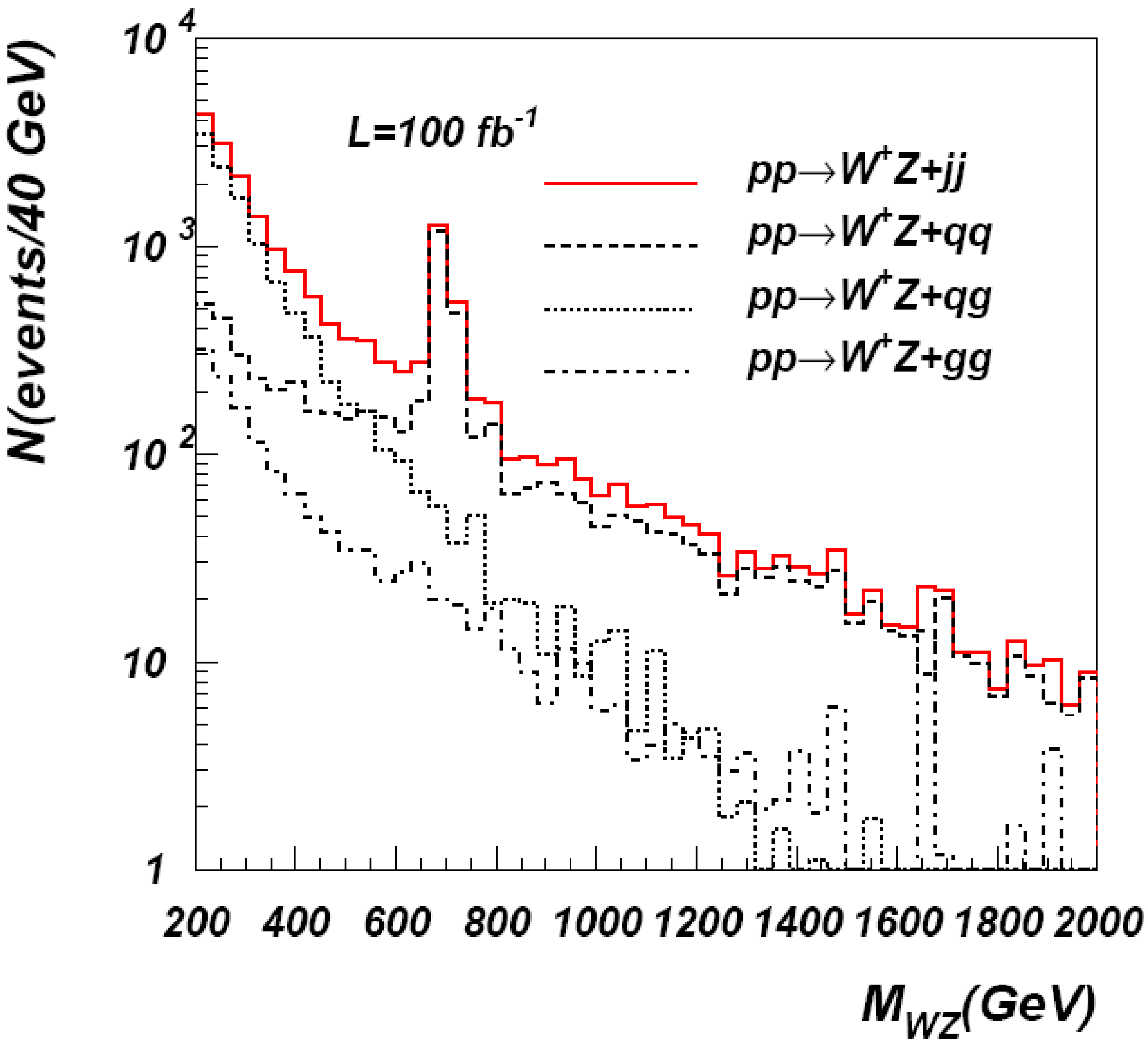
Feynman diagrams

7816 diagrams in 21 subprocesses are constructed.
0 diagrams are deleted.

NN	Subprocess	Del	Rest
1	u1,u1 -> Z,W+,u1,d1	0	612
2	u1,U1 -> Z,W+,U1,d1	0	612
3	u1,d1 -> Z,W+,d1,d1	0	306
4	u1,D1 -> Z,W+,u1,U1	0	612
5	u1,D1 -> Z,W+,d1,D1	0	612
6	u1,D1 -> Z,W+,G,G	0	46
7	u1,G -> Z,W+,G,d1	0	76
8	U1,u1 -> Z,W+,U1,d1	0	612
9	U1,D1 -> Z,W+,U1,U1	0	306
10	d1,u1 -> Z,W+,d1,d1	0	306
11	d1,D1 -> Z,W+,U1,d1	0	612
12	D1,u1 -> Z,W+,u1,U1	0	612
13	D1,u1 -> Z,W+,d1,D1	0	612
14	D1,u1 -> Z,W+,G,G	0	46
15	D1,U1 -> Z,W+,U1,U1	0	306
16	D1,d1 -> Z,W+,U1,d1	0	612
17	D1,D1 -> Z,W+,U1,D1	0	612
18	D1,G -> Z,W+,G,U1	0	76
19	G,u1 -> Z,W+,G,d1	0	76
20	G,D1 -> Z,W+,G,U1	0	76
21	G,G -> Z,W+,U1,d1	0	76



Preliminary $pp \to W^+ Zjj$



$p_T^j > 30$ GeV

$2 < |\eta^j| < 4.5$

$E^j > 300$ GeV

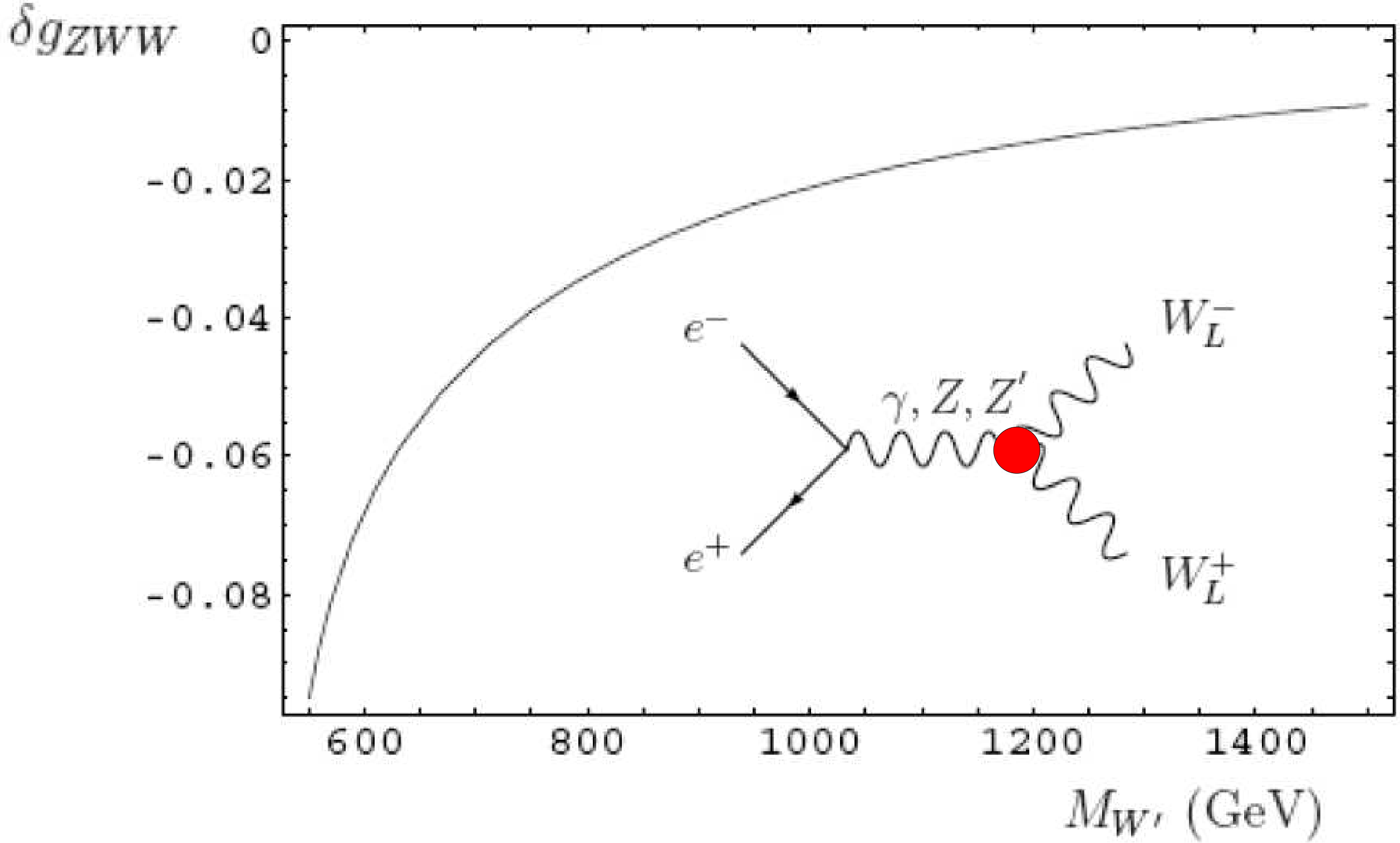
$E^{W,Z} > 200$ GeV

$\Delta Rjj > 0.5.$

the complete WZqq BG is factor 4 bigger then PYTHIA effective V-boson approximation!

To be compared with Birkedal, Matchev, Perelstein: PRL 94, 191803 (2005).

Prospects for ILC@ 0.5 TeV: g_{WWZ}



$$\delta g_{ZWW} = \frac{g_{\chi Zee} g_{ZWW}}{g_{\chi Zee_{SM}} g_{ZWW_{SM}}} + \frac{g_{\chi Z'ee} g_{Z'WW}}{g_{\chi Zee_{SM}} g_{ZWW_{SM}}} \frac{s - M_Z^2}{s - M_{Z'}^2} - 1$$

ILC sensitivity is ~ 4 x 10^{-4} with 500 fb^{-1}

hep-ex/0106057 American LC Working Group

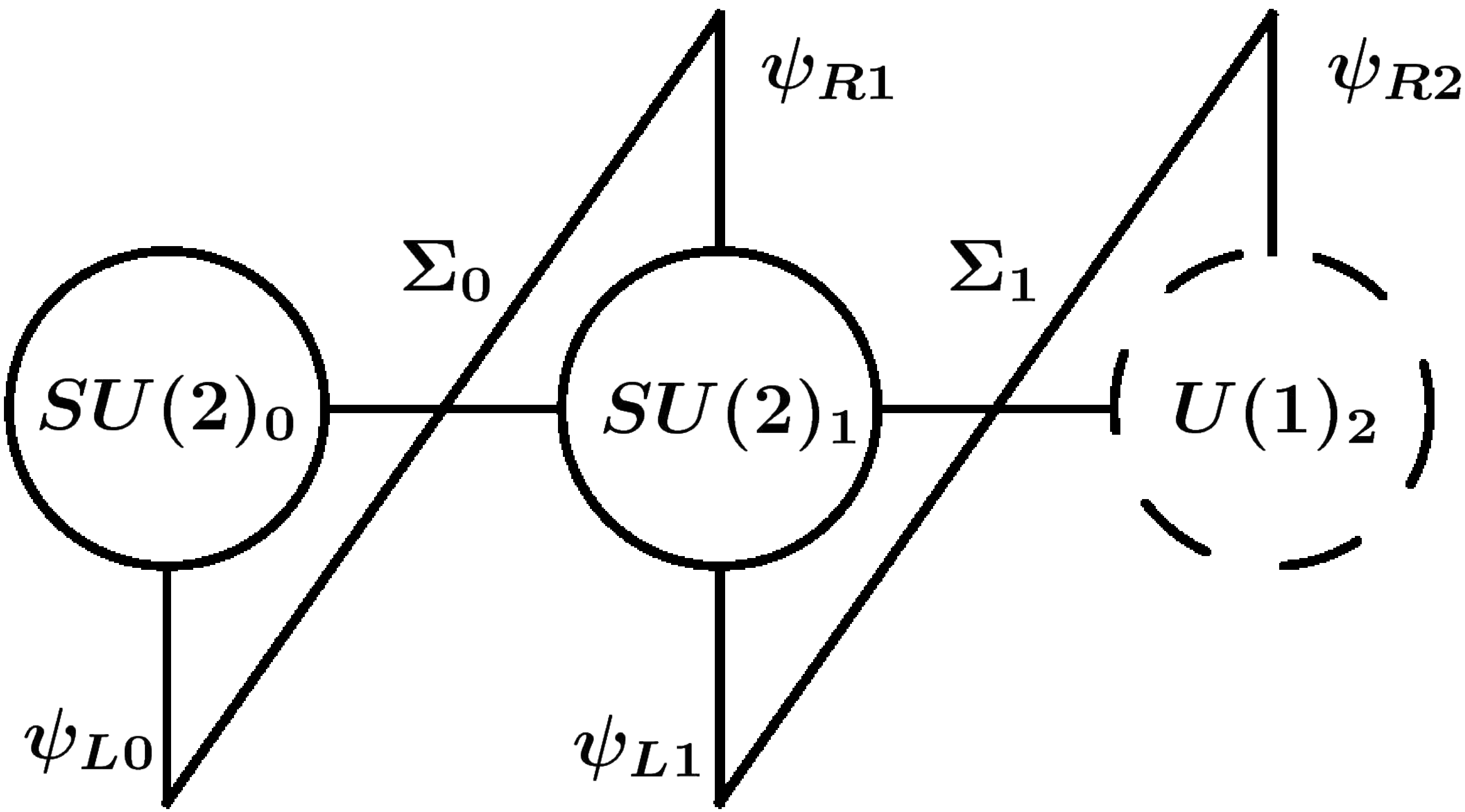
Conclusions and outlook

- Three site model is compelling
  - Is simple, yet consistently implements the 1st KK mode of a Higgsless ED
  - Is representative of Higgsless models and their duals – dynamical symmetry breaking models
  - Is consistent with precision electroweak observables (IDEL)
  - Has a simple parameter space (M_F , $M_{W'}$)
- Implemented into ClacHEP – powerful tool for pheno studies
  - model is complete and tested in both gauges
  - public: hep.pa.msu.edu/people/belyaev/public/3-site/
- Offers distinctive exiting phenomenology
  - fermiophobic Z',W': di-lepton DY discovery range is up to $M_{W'}$ ~0.6 TeV

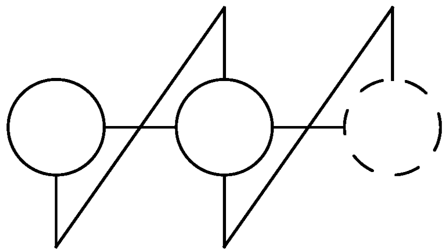
    – very different from 'toy' models
  - resonances in WZ scattering
  - WZ tri-lepton signatures
    - DY: W' production in case of large deviation from IDEL
    - VBF: could test $M_{W'} \sim$ 1TeV : work in progress!
  - 0.5 TeV ILC can test $M_{W'}$ beyond 1 TeV with g_{WWZ} coupling measurement

Appendix

The Three Site Model



Chivukula, Coleppa, Di Chiara, Simmons
PRD **74**, 075011 (2006)



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

$$W_j = \begin{pmatrix} \frac{1}{2}W_j^0 & \frac{1}{\sqrt{2}}W_j^+ \\ \frac{1}{\sqrt{2}}W_j^- & -\frac{1}{2}W_j^0 \end{pmatrix}$$

where *j=0,1*

$$W_2 = \begin{pmatrix} \frac{1}{2}W_2^0 & 0 \\ 0 & -\frac{1}{2}W_2^0 \end{pmatrix}$$

Gauge Sector

$$g_0 = g, \quad g_1 = \tilde{g}, \quad g_2 = g'$$
$$\tilde{g} \gg g, g'$$
$$\Rightarrow g/\tilde{g} = x \ll 1, g'/g = s/c = t$$

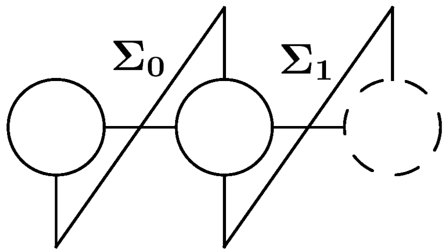
$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g'^2}$$

$$\mathcal{L}_{F^2} = -\frac{1}{2}\mathrm{Tr}\left[F_0^2 + F_1^2 + F_2^2\right]$$

where

$$F_j^{\mu\nu} = \partial^\mu W_j^\mu - \partial^\nu W_j^\mu + i g_j \left[W_j^\mu, W_j^\nu\right]$$

Casalbuoni, De Curtis, Dominici, Gatto
(BESS) Phys. Lett. B155 (1985) 95



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Gauge - Goldstone Sector

$$\mathcal{L}_{D\Sigma} = \frac{f^2}{2}\mathrm{Tr}\Big[\,(D_\mu\Sigma_0)^\dagger\, D^\mu\Sigma_0 + (D_\mu\Sigma_1)^\dagger\, D^\mu\Sigma_1\Big]$$

$$\Sigma_j = e^{i\frac{2\pi_j}{f}}$$

where

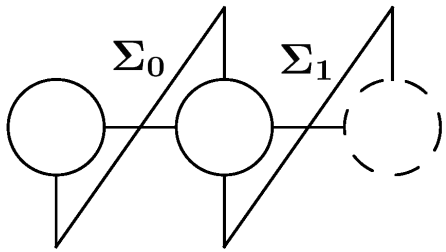
$$D_\mu\Sigma_j = \partial_\mu\Sigma_j + ig_jW_j\Sigma_j - ig_{j+1}\Sigma_jW_{j+1}$$

$$\pi_j = \begin{pmatrix} \frac{1}{2}\pi_j^0 & \frac{1}{\sqrt{2}}\pi_j^+ \\ \frac{1}{\sqrt{2}}\pi_j^- & -\frac{1}{2}\pi_j^0 \end{pmatrix}$$

This gives the gauge boson mass matrices:

$$M_\pm^2 = \frac{f^2}{4}\begin{pmatrix} g_0^2 & -g_0g_1 \\ -g_0g_1 & 2g_1^2 \end{pmatrix}$$

$$M_N^2 = \frac{f^2}{4}\begin{pmatrix} g_0^2 & -g_0g_1 & 0 \\ -g_0g_1 & 2g_1^2 & -g_1g_2 \\ 0 & -g_1g_2 & g_2^2 \end{pmatrix}$$



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Independent parameters: M_W , M_Z , e , $M_{W'}$

Dependent parameters: g_0 , g_1 , g_2 , f

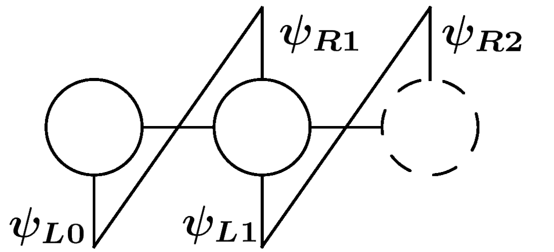
$$x = \frac{g_0}{g_1} \qquad t = \frac{g_2}{g_0}$$

$$\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2}$$

$$\frac{M_W^2}{M_{W'}^2} = \frac{2+x^2-\sqrt{4+x^4}}{2+x^2+\sqrt{4+x^4}}$$

$$\frac{M_W^2}{M_Z^2} = \frac{2+x^2-\sqrt{4+x^4}}{2+x^2(1+t^2)-\sqrt{4+x^4(1-t^2)^2}}$$

$$M_W = g_1 f \frac{\sqrt{2+x^2-\sqrt{4+x^4}}}{2\sqrt{2}}$$



Fermion - Gauge Sector

$$\mathcal{L}_{D\psi} = \bar{\psi}_{L0} \not{D} \psi_{L0} + \bar{\psi}_1 \not{D} \psi_1 + \bar{\psi}_{R2} \not{D} \psi_{R2}$$

$Y_{0,1Q} = 1/6 \quad Y_{0,1L} = -1/2$

$Y_{2u} = 2/3$

$Y_{2d} = -1/3 \quad Y_{2e} = -1$

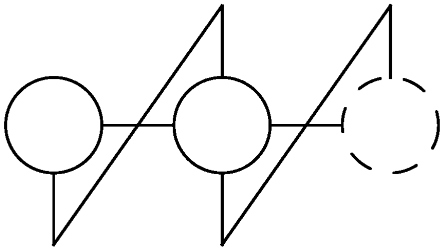
where

$$D_\mu \psi_j = \partial_\mu \psi_j + i g_j W_j \psi_j + i g_2 Y_{jf} W_2 \psi_j$$

for *j=1,2*
and

$$D_\mu \psi_2 = \partial_\mu \psi_2 + i g_2 Y_{2f} W_2 \psi_2$$

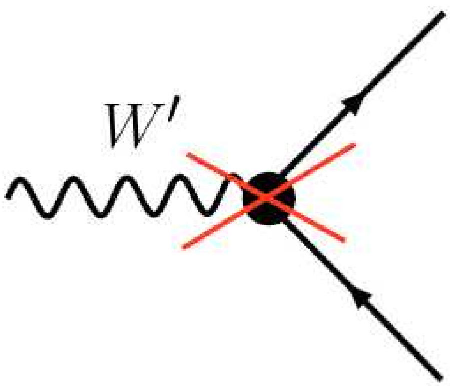
Ideal Delocalization (IDEL)



$$g_i v^i_{Le} v^i_{L\nu} = [g_{W_{SM}} + O(x^4)] v^i_W$$

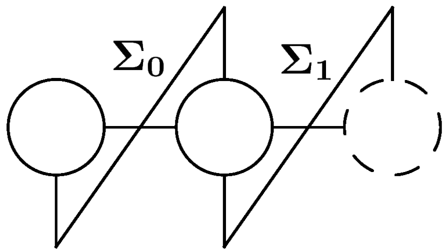
$$\begin{aligned} g_{W_{TSM}} &= g_0 v^0_{Le} v^0_{L\nu} v^0_W + g_1 v^1_{Le} v^1_{L\nu} v^1_W \\ &= [g_{W_{SM}} + O(x^4)] \left(v^0_W v^0_W + v^1_W v^1_W \right) \\ &= g_{W_{SM}} + O(x^4) \end{aligned}$$

$$\epsilon_L^2 = \frac{2x^2}{2 - x^2 + \sqrt{4 + x^4}}$$



$$\begin{aligned} g_{W'_{TSM}} &= g_0 v^0_{Le} v^0_{L\nu} v^0_{W'} + g_1 v^1_{Le} v^1_{L\nu} v^1_{W'} \\ &= g_{W_{SM}} \left(v^0_W v^0_{W'} + v^1_W v^1_{W'} \right) \\ &= 0 \end{aligned}$$

Chivukula, Simmons, He, Kurachi, Tanabashi: PRD 72, 015008 (2005)
Casalbuoni, Deandrea, De Curtis, Dominici, Gatto, Grazzini, : PRD 53, 5201 (1996)



Gauge Fixing Sector

$$\mathcal{L}_{GF} = -\mathrm{Tr}\left[G_0^2 + G_1^2 + G_2^2\right]$$

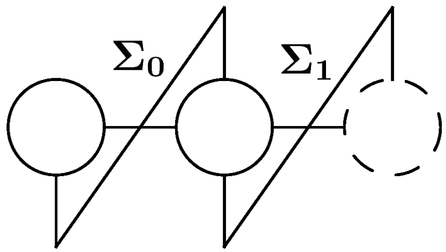
where

$$G_0 = \partial \cdot W_0 - \tfrac{1}{2} g_0 f(\pi_0 \quad)$$

$$G_1 = \partial \cdot W_1 - \tfrac{1}{2} g_1 f(\pi_1 - \pi_0)$$

$$G_2 = \partial \cdot W_2 - \tfrac{1}{2} g_2 f(\quad - \pi_1^{ns})$$

$$\pi_1^{ns} = \begin{pmatrix} \frac{1}{2}\pi_j^0 & 0 \\ 0 & -\frac{1}{2}\pi_j^0 \end{pmatrix}$$



$SU(2)_0 \times SU(2)_1 \times U(1)_2$

Ghost Sector

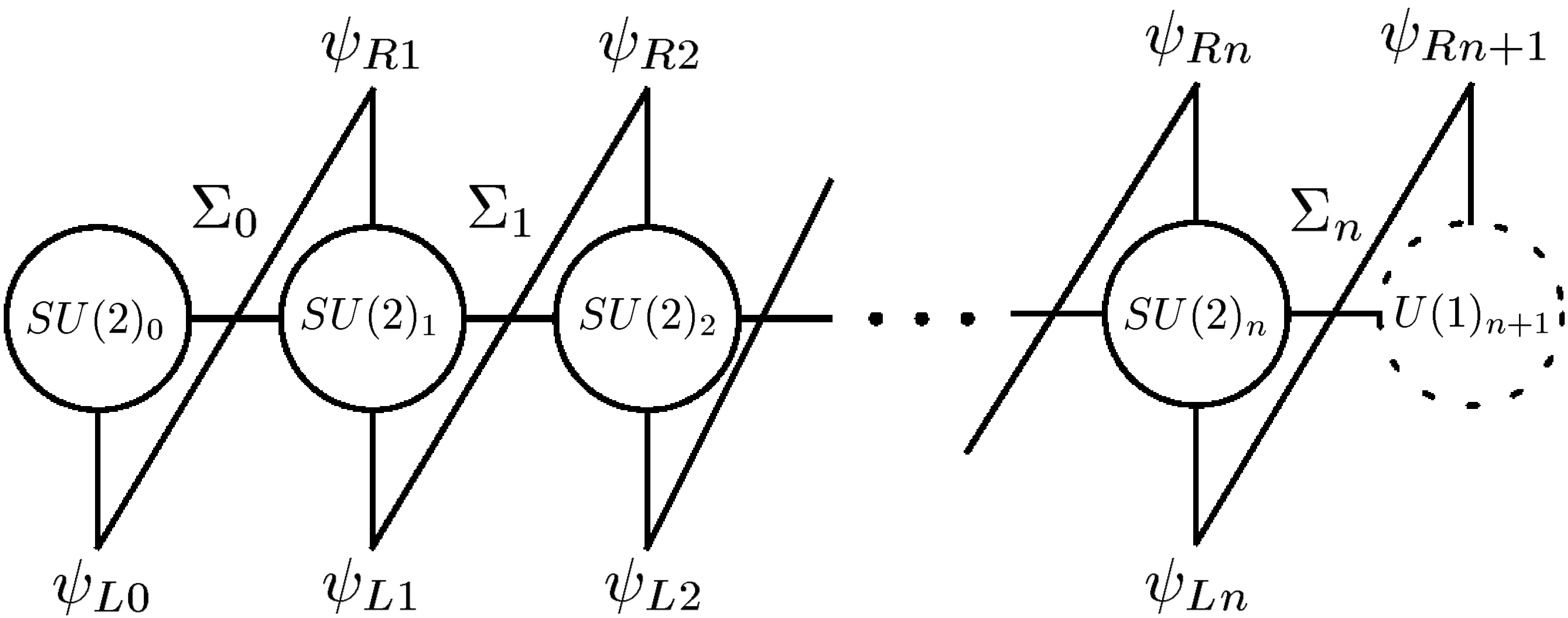
$$\mathcal{L}_{\bar{c}c} = -\mathrm{Tr}\Big[\bar{c}_0 \delta_{BRST} G_0 + \bar{c}_1 \delta_{BRST} G_1 + \bar{c}_2 \delta_{BRST} G_2\Big]$$

where

$$\delta_{BRST} W_{\mu j} = -\Big(\partial_\mu c_j + i g_j \,[\, W_{\mu j} \,,\, c_j \,]\Big)$$

$$\delta_{BRST} \pi_j = \tfrac{1}{2} f \big(g_j c_j - g_{j+1} c_{j+1}\big) + \tfrac{i}{2} \,[\, g_j c_j + g_{j+1} c_{j+1} \,,\, \pi_j \,]$$

$$- \tfrac{1}{6f} \big[\, \pi_j \,,\, [\, \pi_j \,,\, g_j c_j - g_{j+1} c_{j+1} \,] \,\big] + \cdots$$


ψ_{R1}
ψ_{R2}
ψ_{Rn}
ψ_{Rn+1}
Σ_0
Σ_1
Σ_n
$SU(2)_0$
$SU(2)_1$
$SU(2)_2$
$SU(2)_n$
$U(1)_{n+1}$
ψ_{L0}
ψ_{L1}
ψ_{L2}
ψ_{Ln}