Statistical Analysis of future Neutrino Mass Experiments including Neutrino-less Double Beta Decay

MAX-PLANCK-GESELLSCHAFT

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für Kernphysik

ν

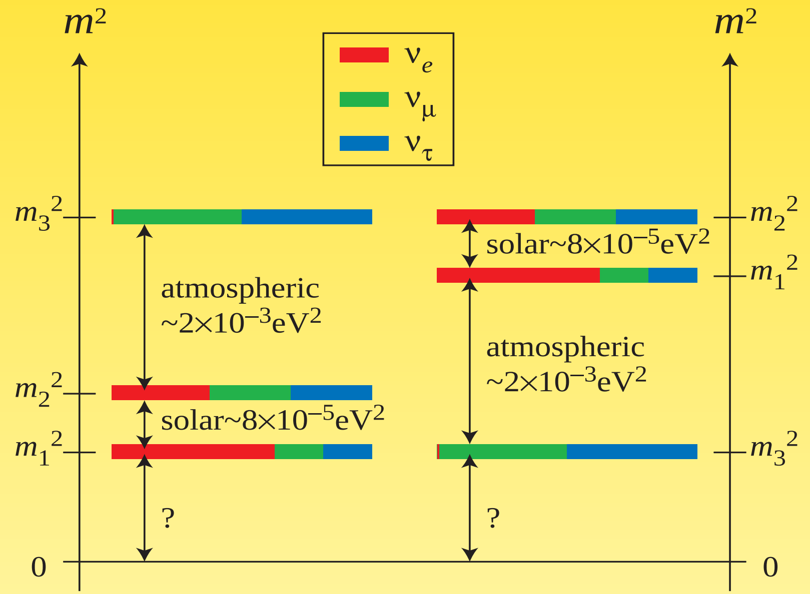
$m_\nu = m_L - m_D^T M_R^{-1} m_D$

erc

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

W. Maneschg, A. Merle, W.R., Europhys. Lett. **85**, 51002 (2009) [arXiv:0812.0479 [hep-ph]]

Before I forget. . .

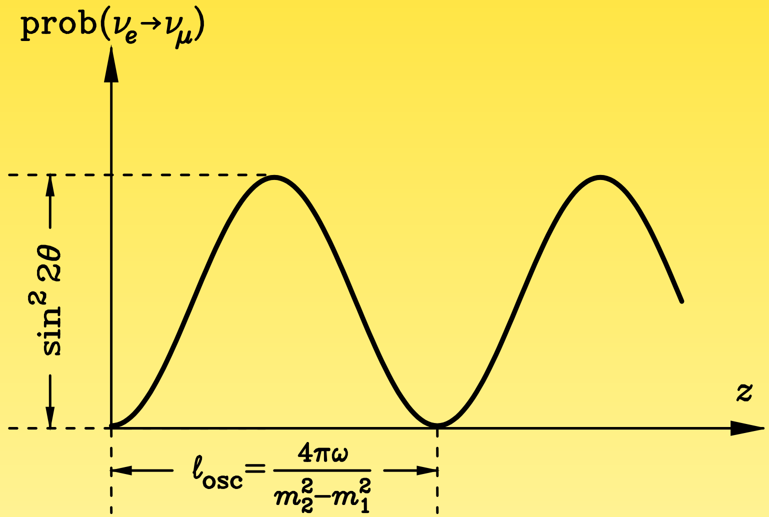


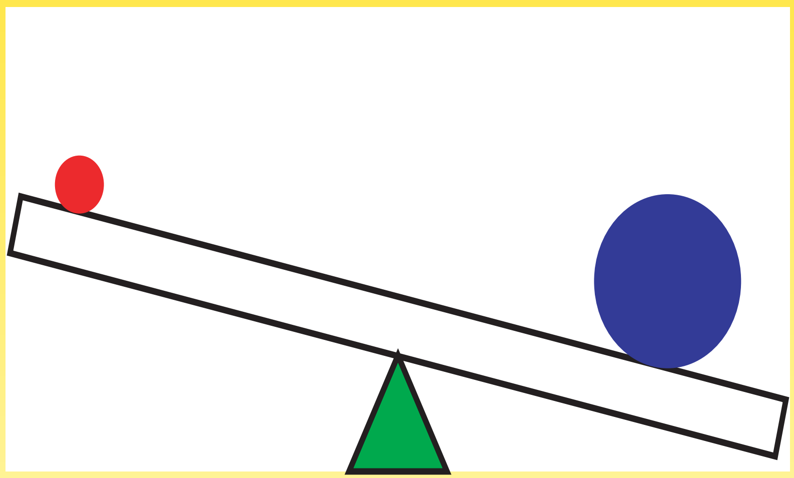
Status and goal of neutrino physics

understand form and origin of

fundamental object in low energy Lagrangian:

$$\mathcal{L} = \frac{1}{2}\,\overline{\nu_\alpha^c}\,(m_\nu)_{\alpha\beta}\,\nu_\beta + h.c. \text{ with } m_\nu = U^*\,\mathrm{diag}(m_1, m_2, m_3)\,U^\dagger$$





$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta\, \sin^2 \frac{\Delta m^2}{4E}\, L$$

Oscillations!

Masses?

$$m_\nu = -m_D^T\, M_R^{-1}\, m_D$$

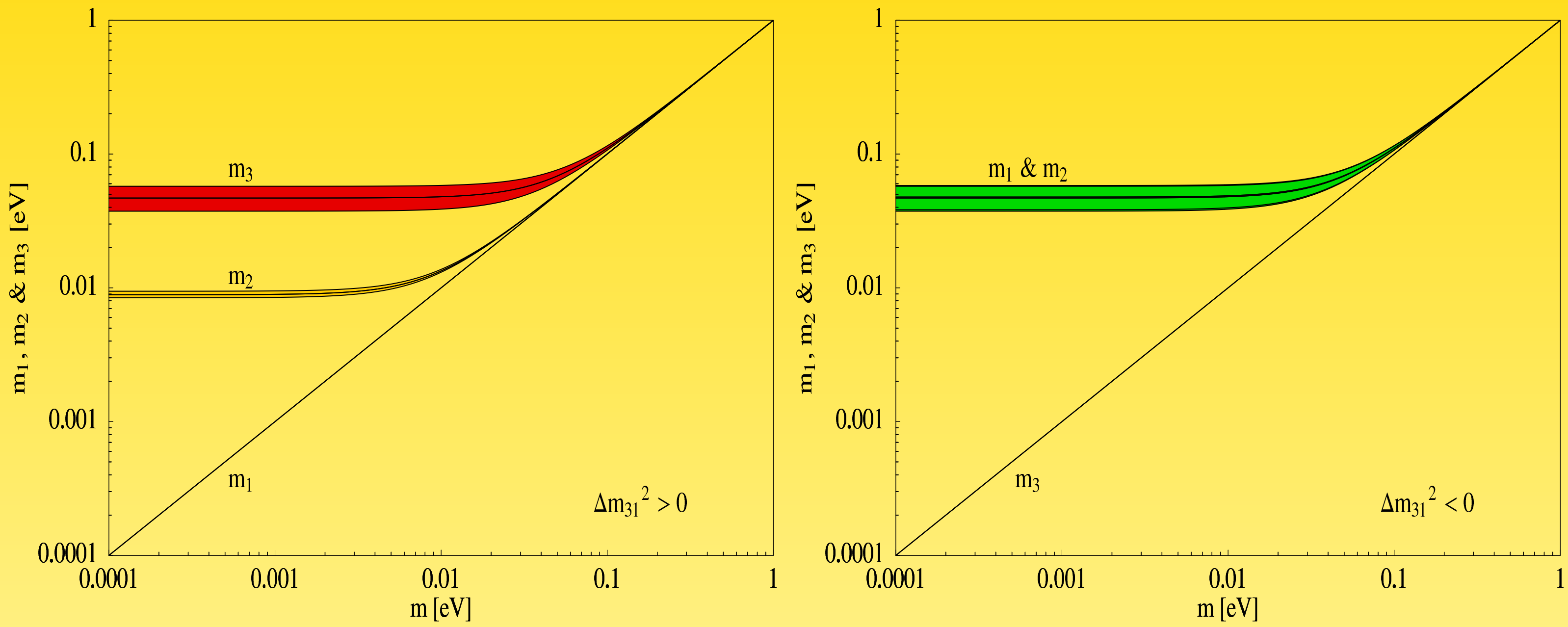
See-saw!

Lepton Number Violation?

Why do we need neutrino mass?

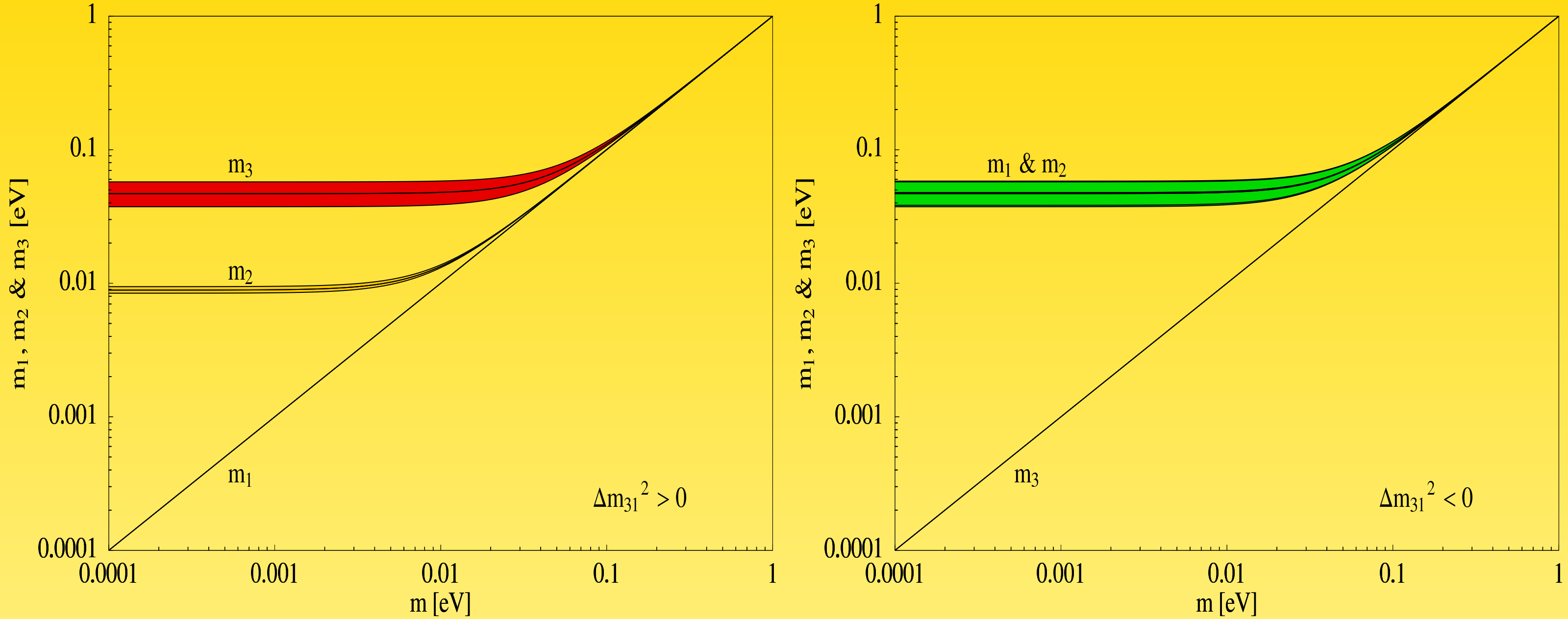
- in all seesaws (type I, II, III): neutrino mass inverse proportional to its origin
- GUTs: normal hierarchy...
- IH and QD neutrinos: special flavor symmetries required...
- QD: HDM, strong RG effects, leptogenesis,...

Neutrino masses



- $m_3^2 \simeq \Delta m_{\rm A}^2 \gg m_2^2 \simeq \Delta m_\odot^2 \gg m_1^2$: normal hierarchy (NH)
- $m_2^2 \simeq |\Delta m_{\rm A}^2| \simeq m_1^2 \gg m_3^2$: inverted hierarchy (IH)
- $m_3^2 \simeq m_2^2 \simeq m_1^2 \equiv m_0^2 \gg \Delta m_{\rm A}^2$: quasi-degeneracy (QD)

Neutrino masses



Neutrino mass hierarchy is moderate!

$$\text{NH}: \qquad \frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m_\odot^2}{\Delta m_\text{A}^2}} \simeq \frac{1}{5}$$

$$\text{IH}: \qquad \frac{m_1}{m_2} \gtrsim 1 - \frac{1}{2}\,\frac{\Delta m_\odot^2}{\Delta m_\text{A}^2} \simeq 0.98$$

Small and Large Neutrino Masses

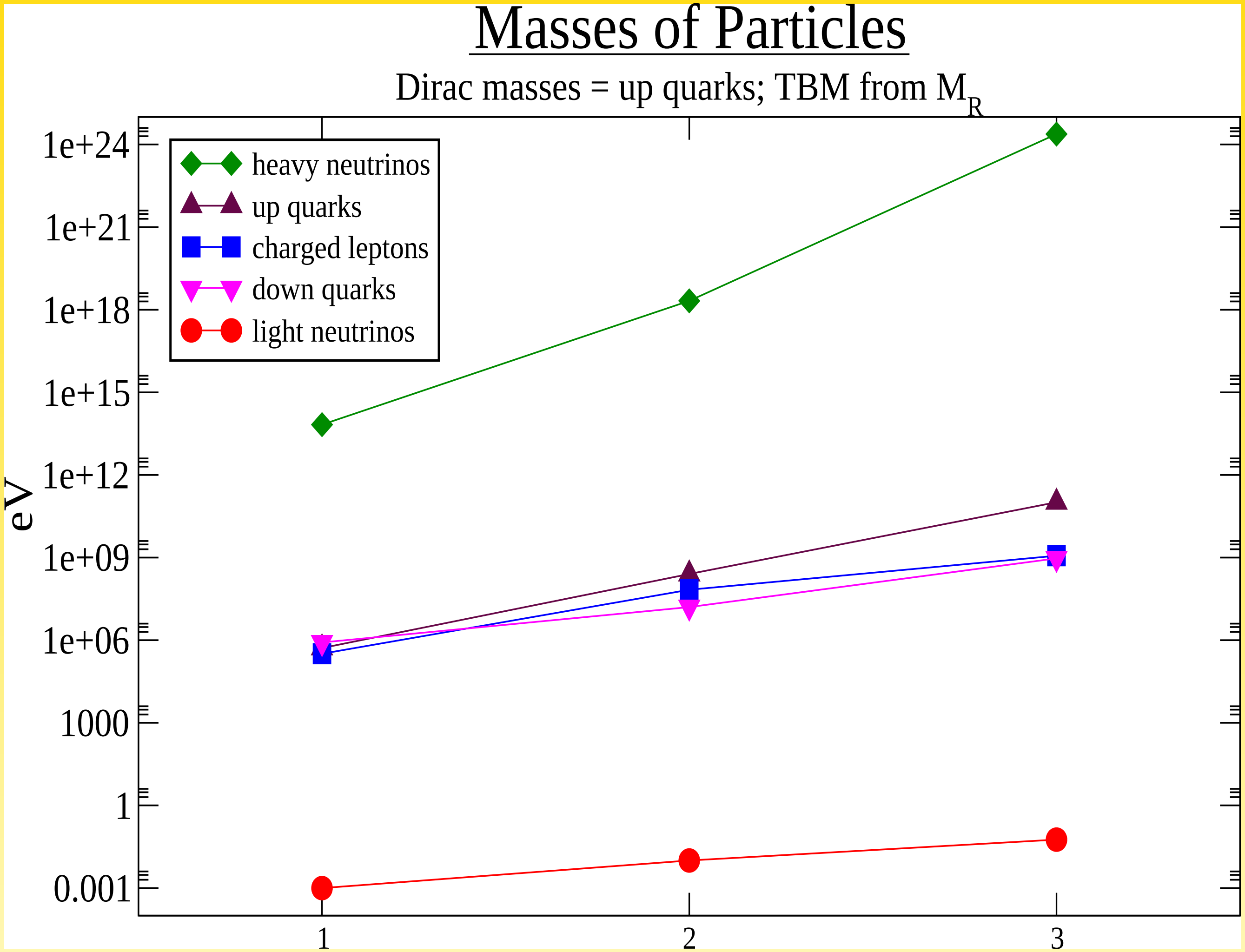
$SO(10)$-like relation $(\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120})$

symmetric $m_D = m_{\rm up} \Rightarrow m_\nu = m_D^T \, M_R^{-1} \, m_D$ gives $M_R = -m_{\rm up} \, m_\nu^{-1} \, m_{\rm up}$

if diagonal, tri-bimaximal neutrino mixing and NH $(m_3 \gg m_2 \gg m_1)$

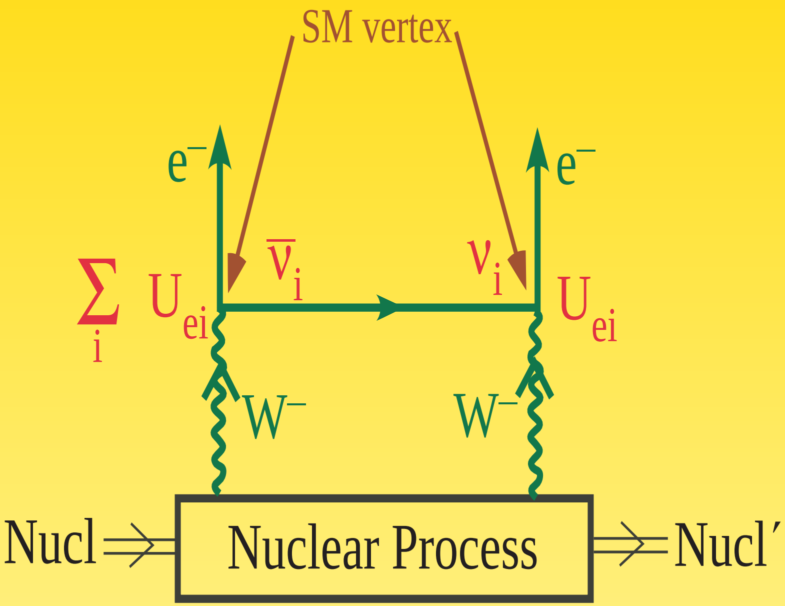
$$M_1 \simeq 3 \, \frac{2 \, m_u^2}{m_2} \; , \quad M_2 \simeq \frac{2 \, m_c^2}{m_3} \; , \quad M_3 \simeq \frac{1}{3} \, \frac{m_t^2}{2 \, m_1}$$

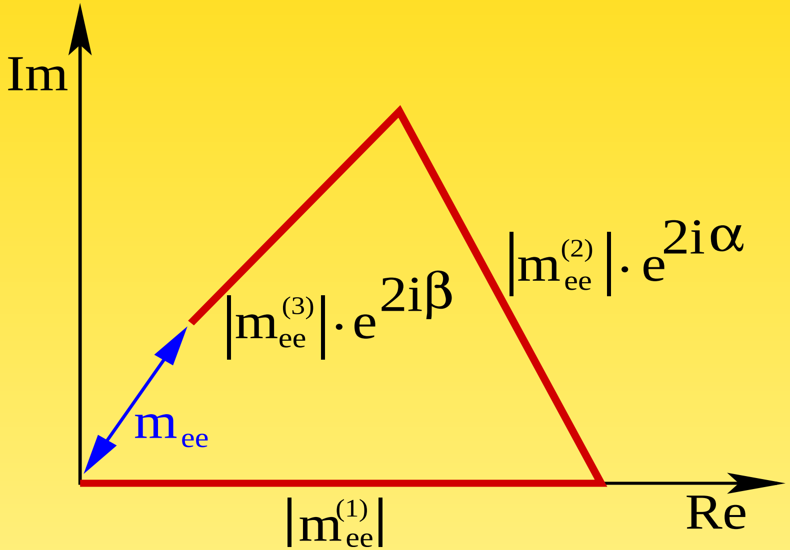
- $m_1 \propto m_t^2$, $m_2 \propto m_u^2$, $m_3 \propto m_c^2$ due to large neutrino mixing
- $M_3 : M_2 : M_1 = m_t^2 : m_c^2 : m_u^2$ due to moderate light neutrino mass hierarchy $\Rightarrow$ stronger hierarchy in M_R
- $M_1 \simeq 10^5$ GeV, $M_2 \simeq 10^9$ GeV, $M_3 \simeq 10^{15}$ GeV



Masses of Particles
Dirac masses = up quarks; TBM from M_R
heavy neutrinos
up quarks
charged leptons
down quarks
light neutrinos
eV
1e+24
1e+21
1e+18
1e+15
1e+12
1e+09
1e+06
1000
1
0.001
1
2
3

$\Delta L \neq 0$: Neutrinoless Double Beta Decay





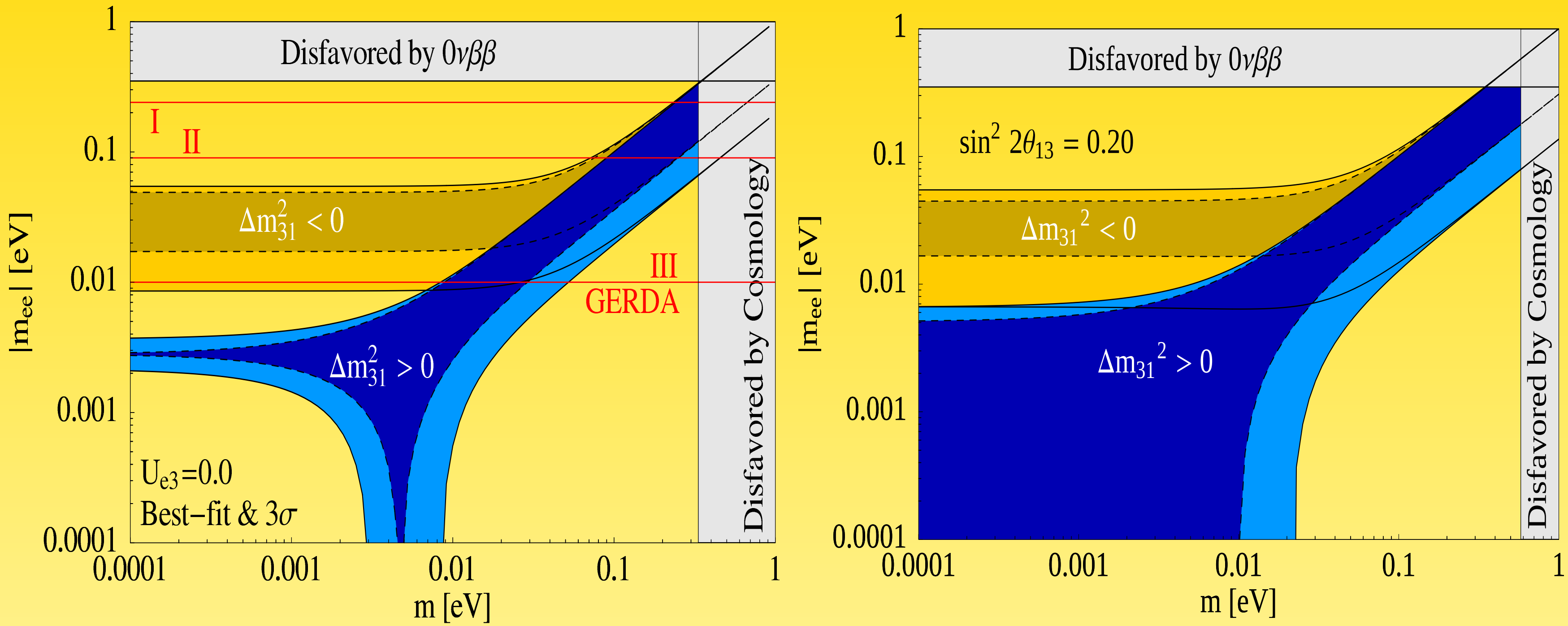
Amplitude proportional to coherent sum

("Effective mass"):

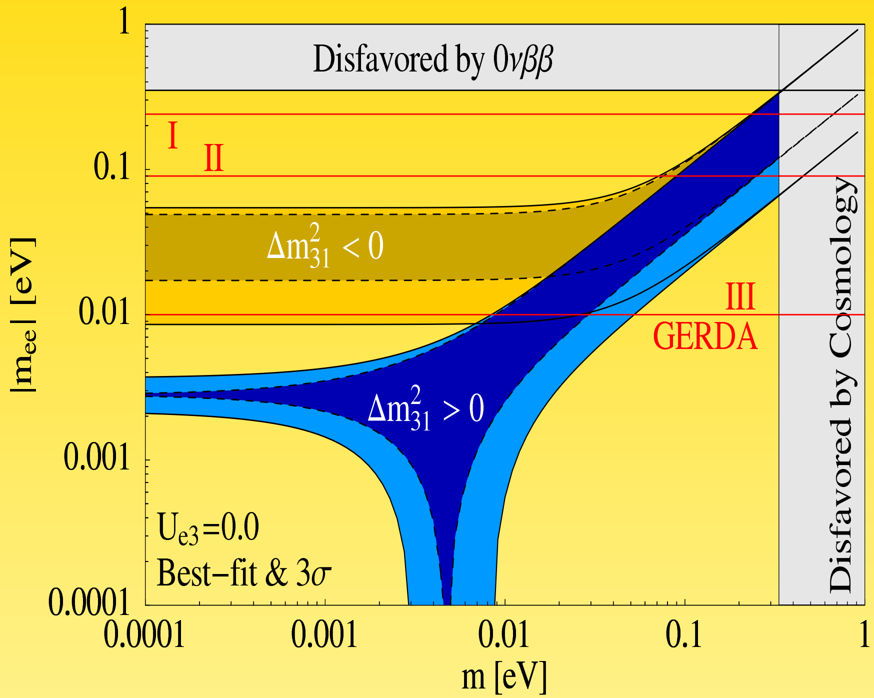
$$|m_{ee}| \equiv \left| \sum U_{ei}^2 \, m_i \right| = f\left(\theta_{12}, m_i, |U_{e3}|, \mathrm{sgn}(\Delta m_{\mathrm{A}}^2), \alpha, \beta\right)$$

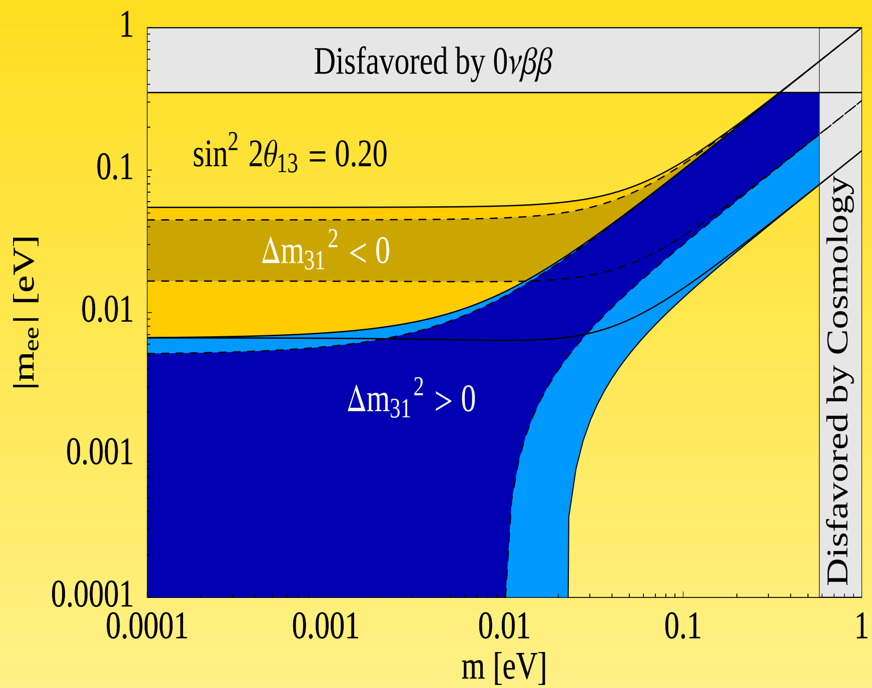
7 out of 9 parameters of $m_\nu \ldots$

Kim, 1996; Minakata & Yasuda, 1996; Hirsch & Klapdor-Kleingrothaus, 1997; Bilenky, Giunti & Monteno, 1997; Fukuyama, Matsuda & Nishiura, 1997; Bilenky, Giunti, Kim & Monteno, 1998; Fukuyama, Matsuda & Nishiura, 1998; Vissani, 1999; Giunti, 1999; Bilenky, Giunti, Grimus, Kayser & Petcov, 1999; Ma, 1999; Wodecki & Kaminsky, 2000; Kalliomaki & Maalampi, 2000; Rodejohann, 2000; Matsuda, Takeda, Fukuyama & Nishiura, 2000; Klapdor-Kleingrothaus, Päs & Smirnov, 2001; Falcone & Tramontano, 2001; Bilenky, Pascoli & Petcov, 2001; Xing, 2001; Osland & Vigdel, 2001; Pascoli & Petcov, 2001; Barger, Glashow, Marfatia & Whisnant, 2002; Hambye, 2002; Minakata & Sugiyama, 2002; Klapdor-Kleingrothaus & Sarkar, 2002; Xing, 2002; Haba & Suzuki, 2002; Pakvasa & Roy, 2002; Rodejohann, 2002; Haba, Nakamura & Suzuki, 2002; Päs & Weiler, 2002; Barger, Glashow , Langacker, Marfatia, 2002; Civitarese & Suhonen, 2002; Pascoli, Petcov & Rodejohann, 2002; Sugiyama, 2002; Avignone & King, 2002; Minakata & Sugiyama, 2002; Cheung, 2003; Abada & Bhattacharyya, 2003; Giunti, 2003; Pascoli & Petcov, 2003; Elliott, 2003; Stoica, 2004; Brahmachari, 2004; Bilenky, Fäßler & Simkovic, 2004; Pascoli & Petcov; 2004; Deppisch, Päs & Suhonen, 2004; Joniec & Zralek, 2004; Pascoli & Petcov, 2005; Pascoli, Petcov & Schwetz, 2005; Goswami & Rodejohann, 2005; Choubey & Rodejohann, 2005; Bilenky, Fäßler, Gutsche, & Simkovic, 2005; Lindner, Merle & Rodejohann, 2005;

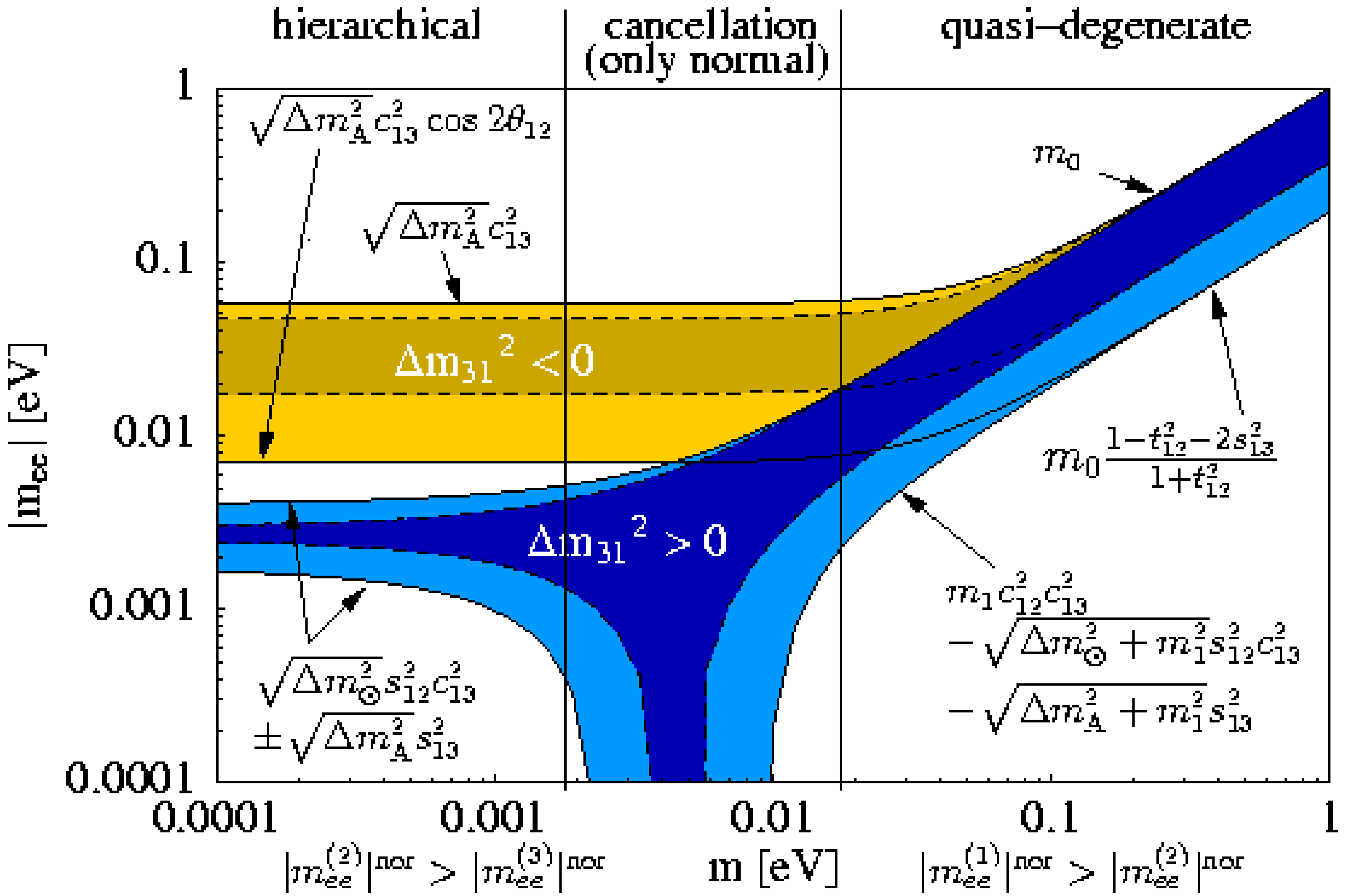


Our plots are blue and yellow...

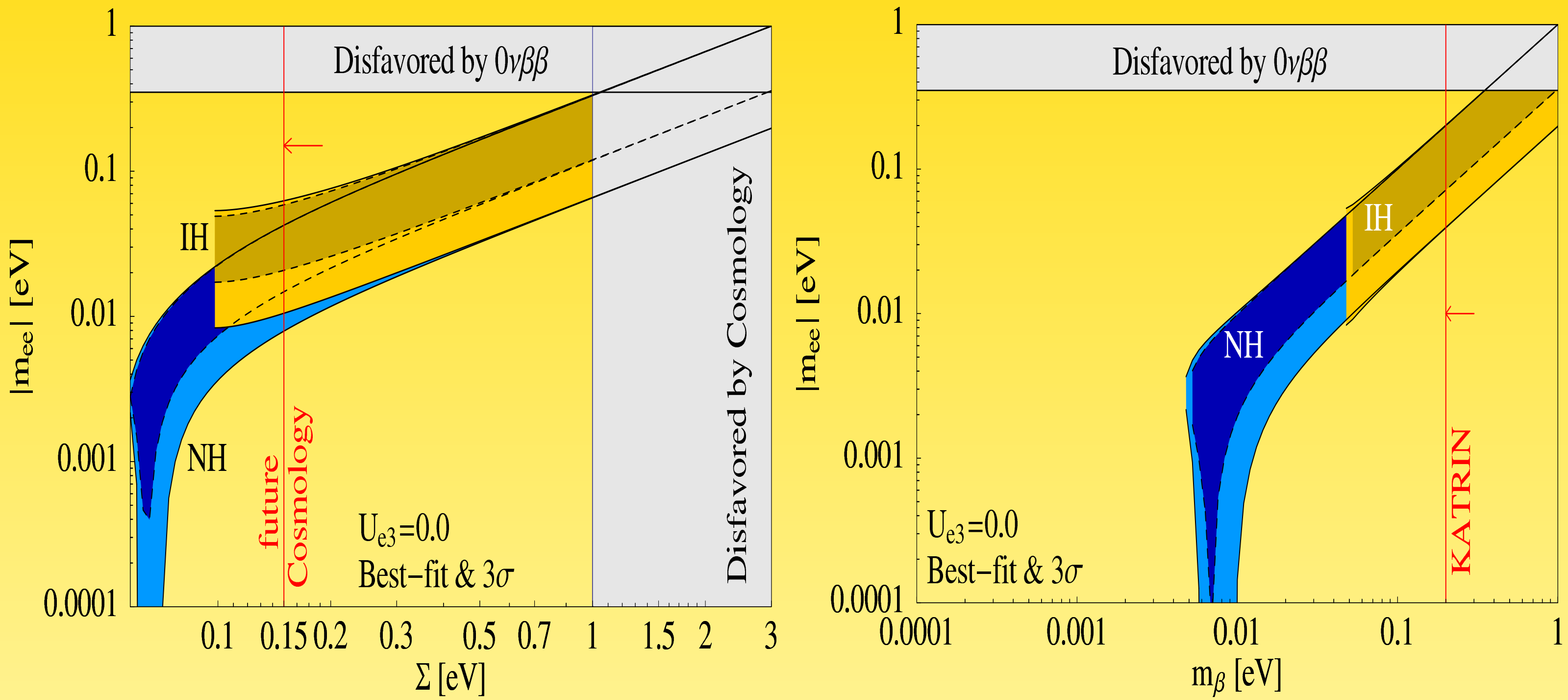




Note: importance of U_{e3}

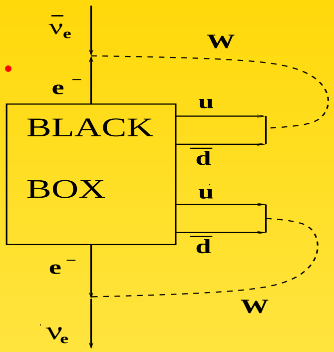


Lindner, Merle, W.R., Phys. Rev. D **73**, 053005 (2006)



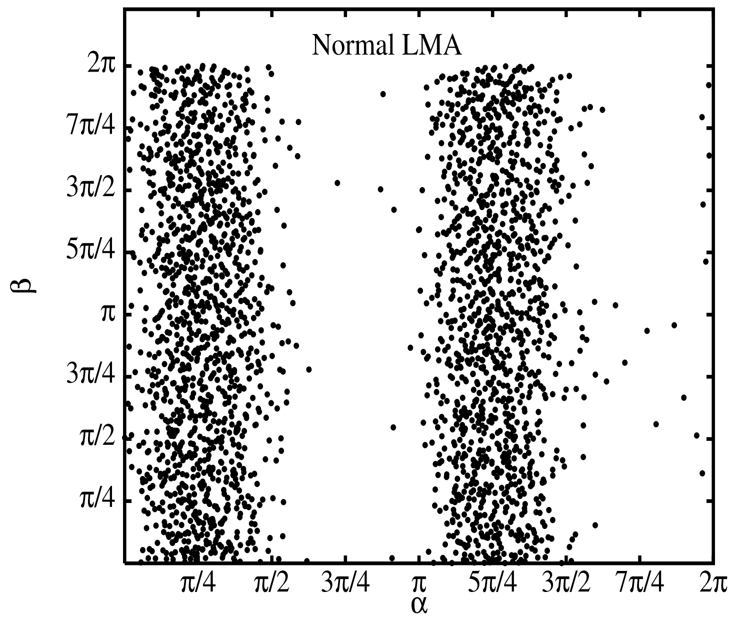
Disfavored by 0νββ
Disfavored by Cosmology
IH
NH
future Cosmology
U_{e3}=0.0
Best−fit & 3σ
$|m_{ee}|$ [eV]
Σ [eV]
1
0.1
0.01
0.001
0.0001
0.1 0.15 0.2 0.3 0.5 0.7 1 1.5 2 3
Disfavored by 0νββ
IH
NH
KATRIN
U_{e3}=0.0
Best−fit & 3σ
$|m_{ee}|$ [eV]
m_β [eV]
0.0001 0.001 0.01 0.1 1

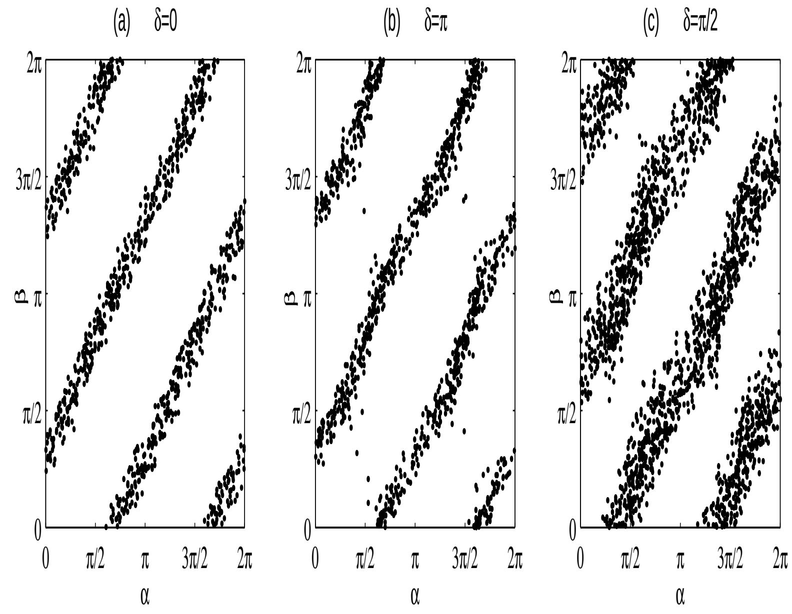
If we observe $0\nu\beta\beta$...



- Neutrinos are Majorana (Schechter-Valle)
- we still need to identify the mechanism of $0\nu\beta\beta$: SUSY, RH currents, heavy Majorana neutrinos,...
  solution:...
- reduce/check NME uncertainty: same solution
- $|m_{ee}|$ can rule out models
- we have tested a prediction of the see-saw mechanism(s)!
- GUTs predict LNV!
- we will believe much more firmly in leptogenesis!

Leptogenesis





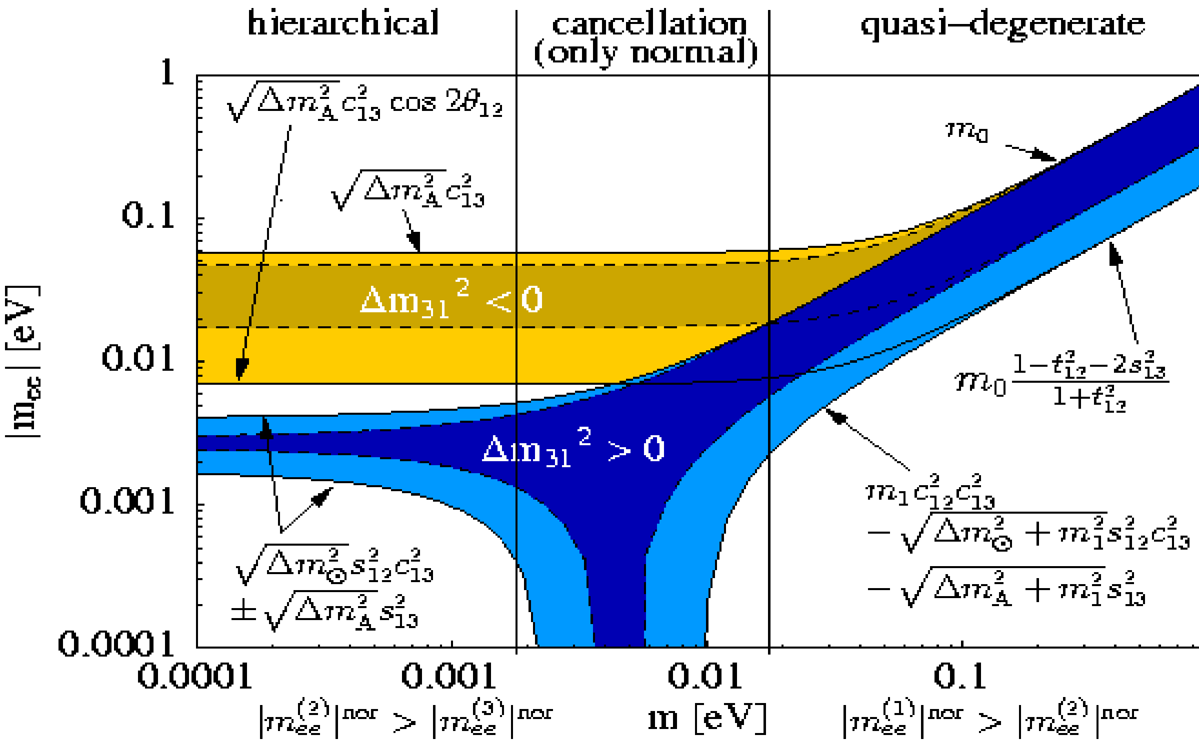
Paschos, Joshipura, W.R.,

JHEP **01**, 029 (2001)

Branco *et al.*,

Nucl. Phys. B **640**, 202 (2002)

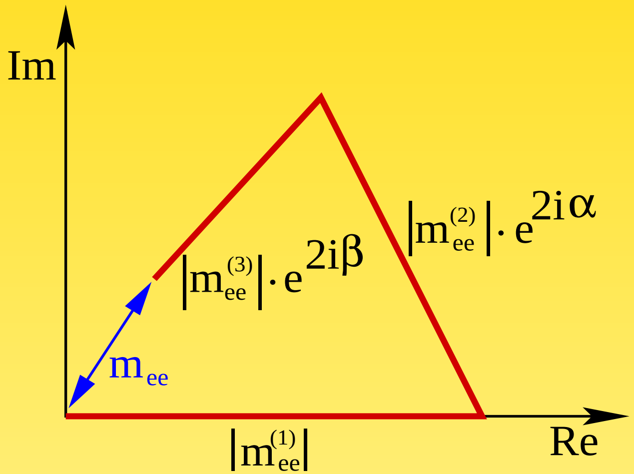
What if $|m_{ee}| = 0$?



The "chimney"

if we don't observe $0\nu\beta\beta$: vanishing $|m_{ee}|$

- a triangle can be formed! $\left| |m_{ee}^{(1)}| + |m_{ee}^{(2)}| \, e^{2i\alpha} + |m_{ee}^{(3)}| \, e^{2i\beta} \right| = 0$

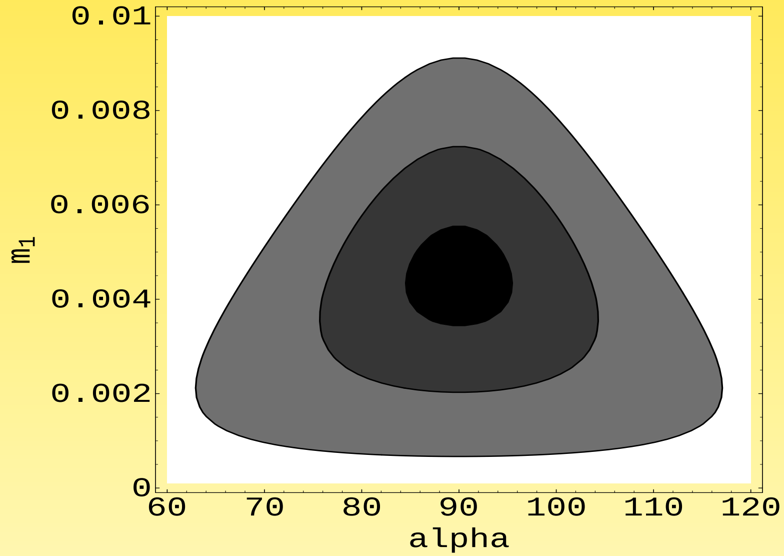


- texture zero in charged lepton basis! (only $m_{e\mu}$ or $m_{e\tau}$ can in addition be zero)

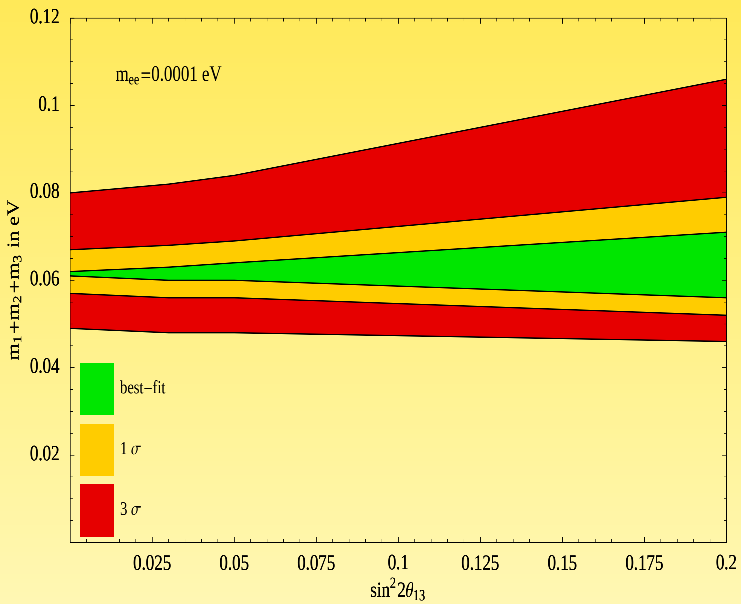
$$\cos 2\alpha = \frac{|m_{ee}^{(1)}|^2 + |m_{ee}^{(2)}|^2 - |m_{ee}^{(3)}|^2}{2|m_{ee}^{(1)}| \, |m_{ee}^{(2)}|} =$$

$$\frac{m_1^2 \, \left(c_{13}^4 \left(s_{12}^4 + c_{12}^4\right) - s_{13}^4\right) + \Delta m_\odot^2 \, s_{12}^4 \, c_{13}^4 - \Delta m_A^2 \, s_{13}^4}{2m_1 \sqrt{m_1^2 + \Delta m_\odot^2} \, s_{12}^2 \, c_{12}^2 \, c_{13}^4}$$

- only possible for normal ordering
- if $\theta_{13} = 0$: $m_1 = \sin^2\theta_{12} \sqrt{\frac{\Delta m^2_\odot}{\cos 2\theta_{12}}} \simeq 4.5\ (2.8 \div 8.4)\ \text{meV}$
- if $m_1 = 0$: $\sin^2 2\theta_{13} \simeq 4\ \sin^2\theta_{12} \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}} \simeq 0.24\ (0.14 \div 0.40)$



Dev, Kumar, hep-ph/0607048



Lindner, Merle, W.R., PRD **73**, 053005 (2006)

Does it stay zero?

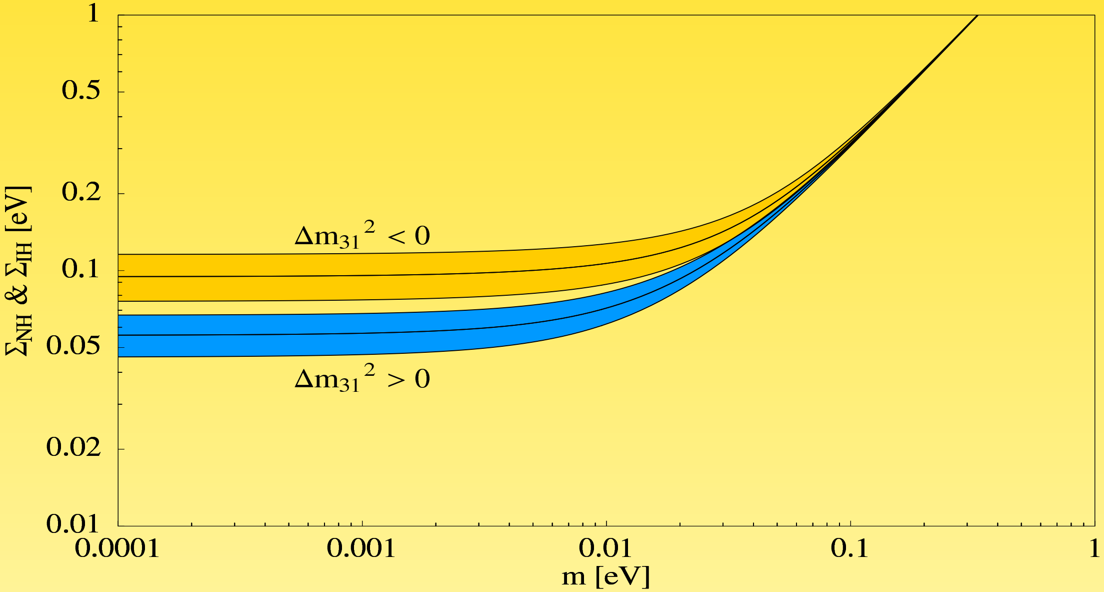
- RG effects!
- actually:

$$\mathcal{M} \propto \frac{U_{ei}^2\, m_i}{q^2 - m_i^2} \simeq \frac{|m_{ee}|}{q^2} + \mathcal{O}(m_i^3/q^4)$$

Other (non-oscillation) probes

1) Cosmology: $\Sigma = \sum m_i$

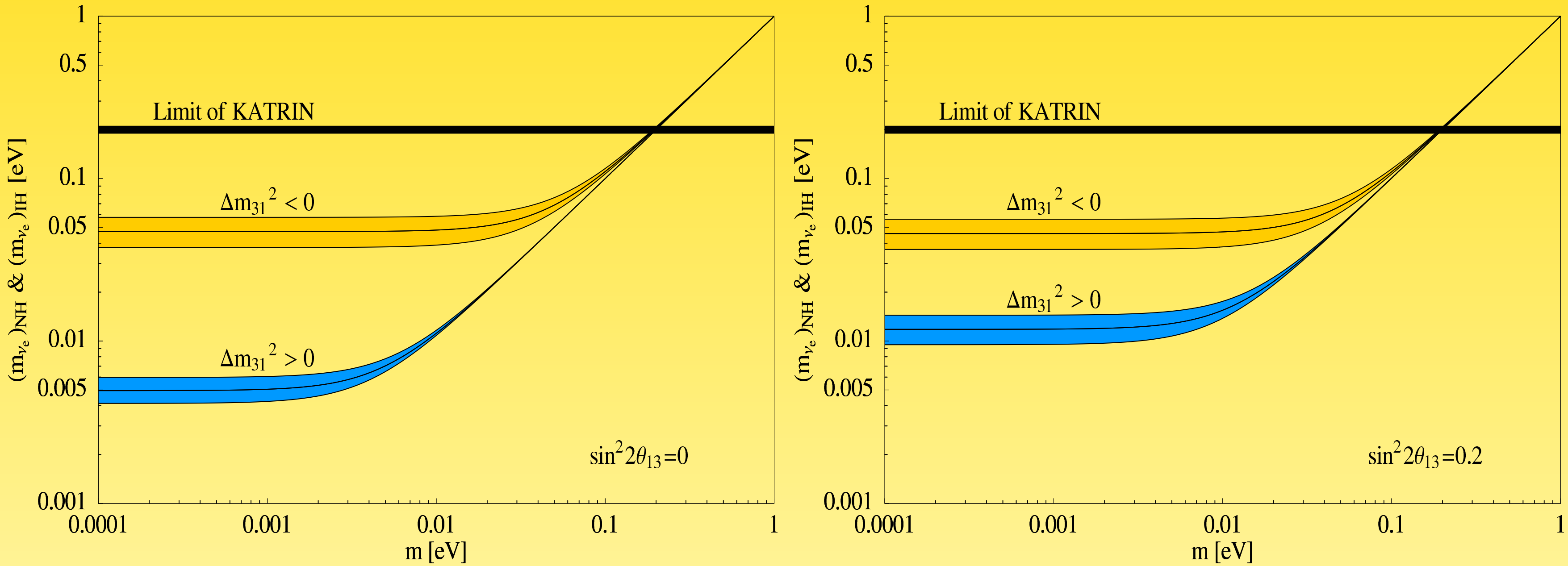
$$\Sigma^{\rm NH} \simeq \sqrt{\Delta m^2_{\rm A}} < \Sigma^{\rm IH} \simeq 2\sqrt{\Delta m^2_{\rm A}}$$



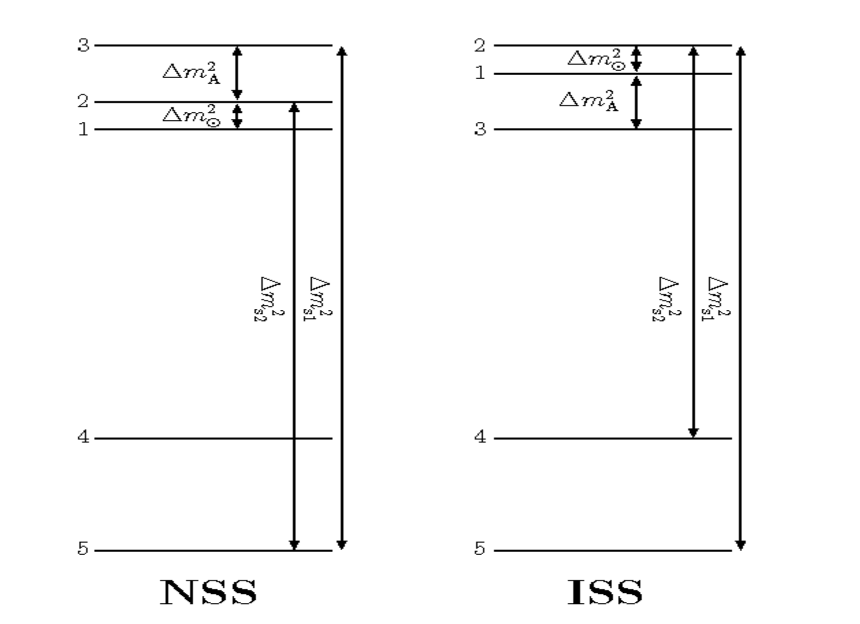
- independent on mixing angles
- requires $\sigma(\Sigma) \lesssim 0.05$ eV

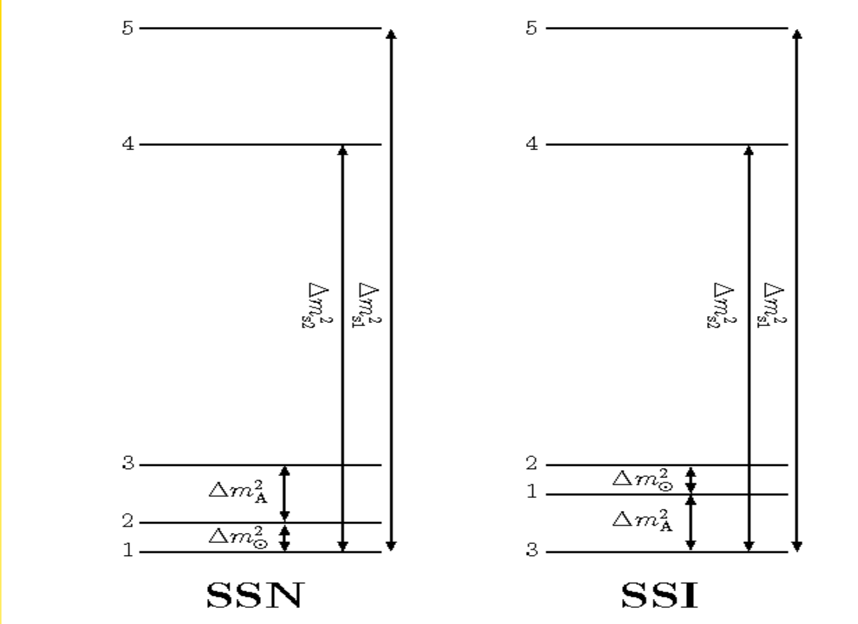
2) β-decay: $m_\beta = \sqrt{\sum |U_{ei}|^2 \, m_i^2}$

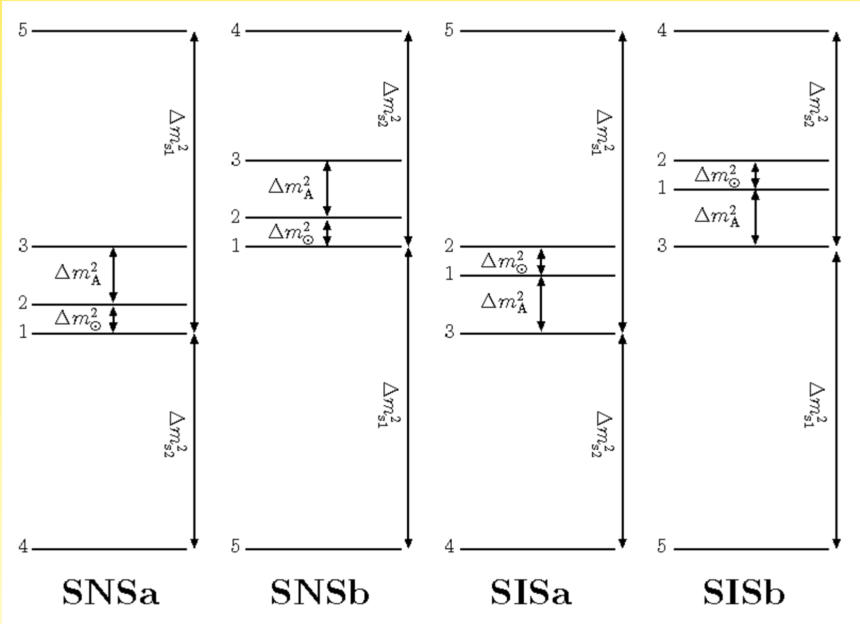
$$m_\beta^{\rm NH} \simeq \sqrt{s_{12}^2 \, c_{13}^2 \, \Delta m_\odot^2 + s_{13}^2 \, \Delta m_{\rm A}^2} \ll m_\beta^{\rm IH} \simeq \sqrt{c_{13}^2 \, \Delta m_{\rm A}^2}$$



- almost independent on mixing angles
- difference of normal and inverted shows up well below KATRIN limit
- different for sterile (LSND/MiniBooNE!!) neutrinos


NSS
ISS


SSN
SSI


SNSa
SNSb
SISa
SISb

KATRIN and $0\nu\beta\beta$

$0\nu\beta\beta$	$\Delta m^2_{\rm A}$	KATRIN	Conclusion
yes	> 0	yes	QD, Majorana
yes	> 0	no	QD, Majorana or
			NH, Majorana + heavy particles
yes	< 0	no	IH, Majorana
yes	< 0	yes	QD, Majorana
no	> 0	no	NH, Dirac or Majorana
no	< 0	no	Dirac
no	< 0	yes	Dirac
no	> 0	yes	Dirac

APS study, Mohapatra *et al.*, hep-ph/0510213

	Σ	m_β	$\lvert m_{ee}\rvert$
NH	$\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_\odot + \lvert U_{e3}\rvert^2 \Delta m^2_{\rm A}}$	$\left\lvert\sqrt{\Delta m^2_\odot} + \lvert U_{e3}\rvert^2 \sqrt{\Delta m^2_{\rm A}}\, e^{i(\alpha-\beta)}\right\rvert$
IH	$2\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_{\rm A}}\sqrt{1-\sin^2 2\theta_\odot \sin^2\alpha}$
QD	$3\, m_0$	m_0	$m_0\sqrt{1-\sin^2 2\theta_\odot \sin^2\alpha}$

corrections due to splitting:

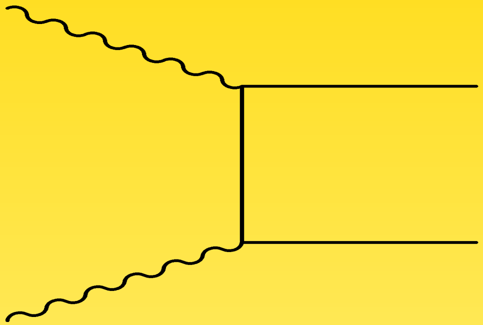
$$\frac{1}{3}\,\Sigma - m_\beta \simeq \frac{1}{3}\,\Sigma - \lvert m_{ee}\rvert^{\rm max} \simeq \frac{m_0}{6}\left((3\cos 2\theta_{13} - 2)\,\eta_{\rm A} + \eta_{\rm A}^2 + (1 + 3\,\cos 2\theta_{12})\,\eta_\odot\right)$$

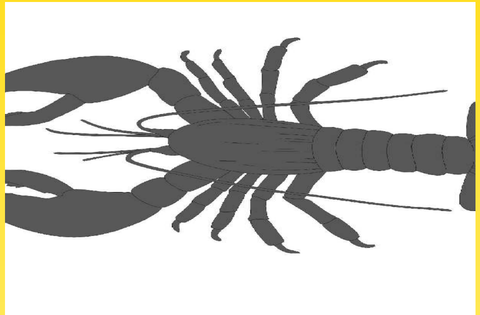
with $\eta_{\rm A} = \Delta m^2_{\rm A}/(2\, m_0^2) \simeq 0.013$ and $\eta_\odot = \Delta m^2_\odot/(2\, m_0^2) \simeq 0.0004$

(assuming unitarity of PMNS matrix)

Other (non-oscillation) probes

3) Other elements of m_ν: "the lobster"





Recall: $(A,Z) \to (A,Z+2) + 2\,e^-$ is proportional to

$$m_{ee} = \sum_i U_{ei}^2\, m_i$$

is ee element of mass matrix

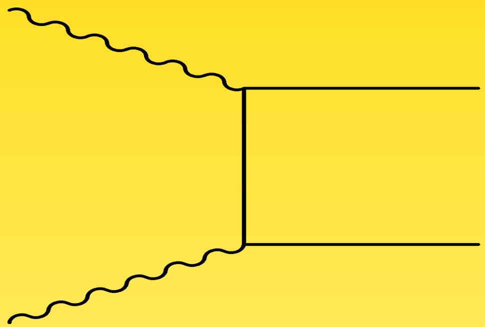
Therefore, $K^+ \to \pi^-\, e^+\, \mu^+$ is proportional to

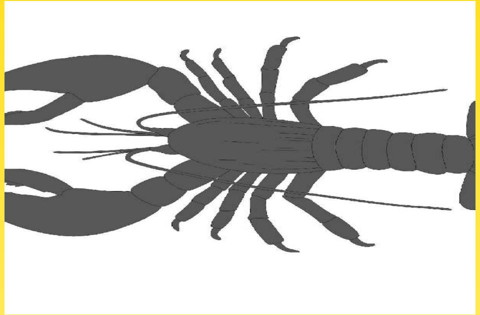
$$\sum_i U_{ei}\, U_{\mu i}\, m_i$$

$e\mu$ element of mass matrix

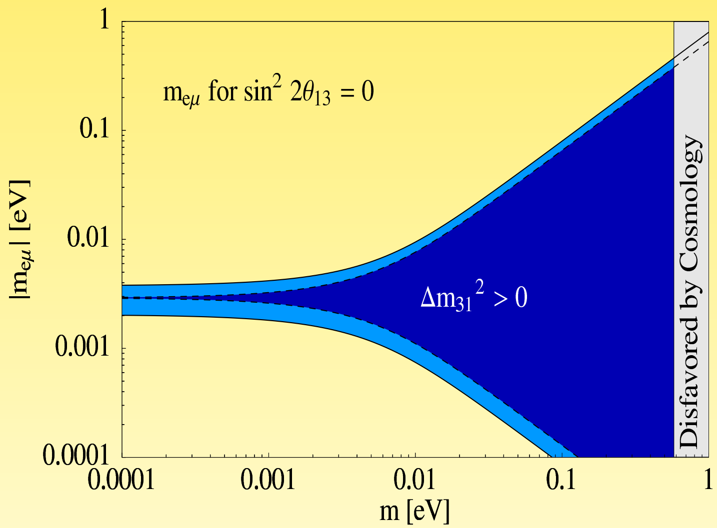
Other (non-oscillation) probes

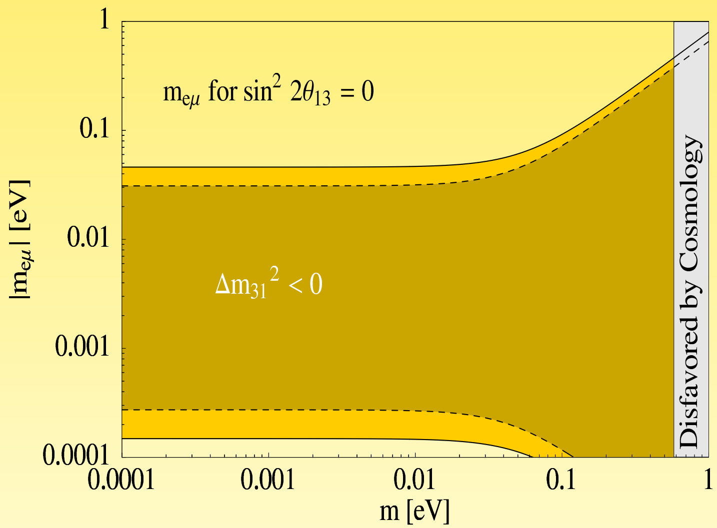
3) Other elements of m_ν: "the lobster"



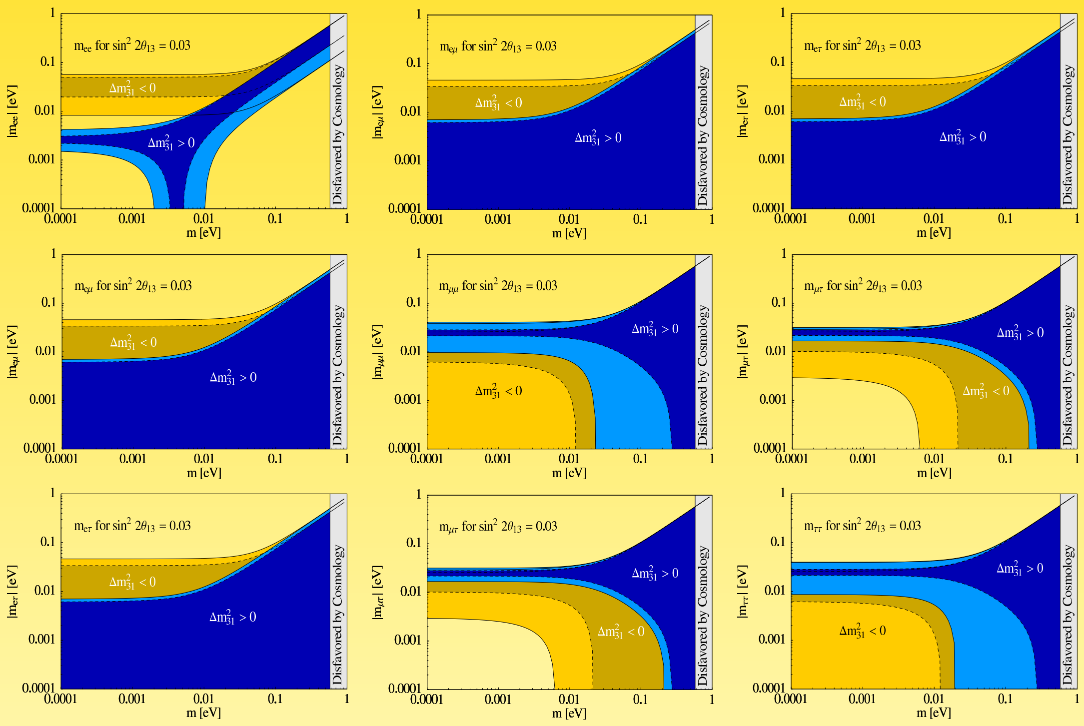


$$\mathrm{BR}(K^+ \to \pi^- e^+ \mu^+) \propto |m_{e\mu}|^2 = \left| \sum U_{ei} U_{\mu i} m_i \right|^2 \sim 10^{-30} \left(\frac{|m_{e\mu}|}{\mathrm{eV}} \right)^2$$

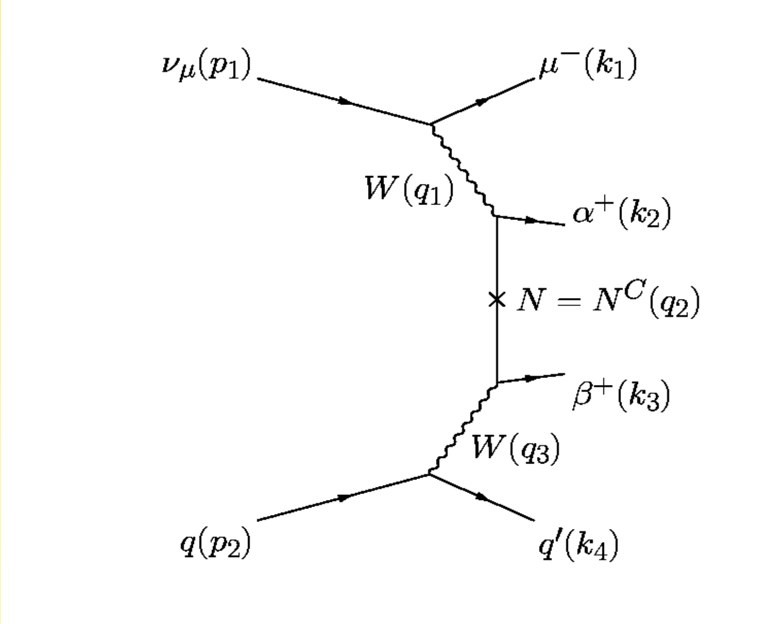


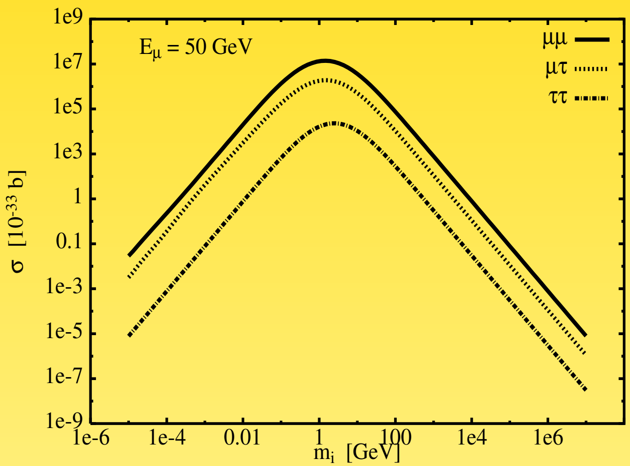


$$(m_\nu)_{\alpha\beta} =$$



3) Other elements of m_ν: "the lobster"



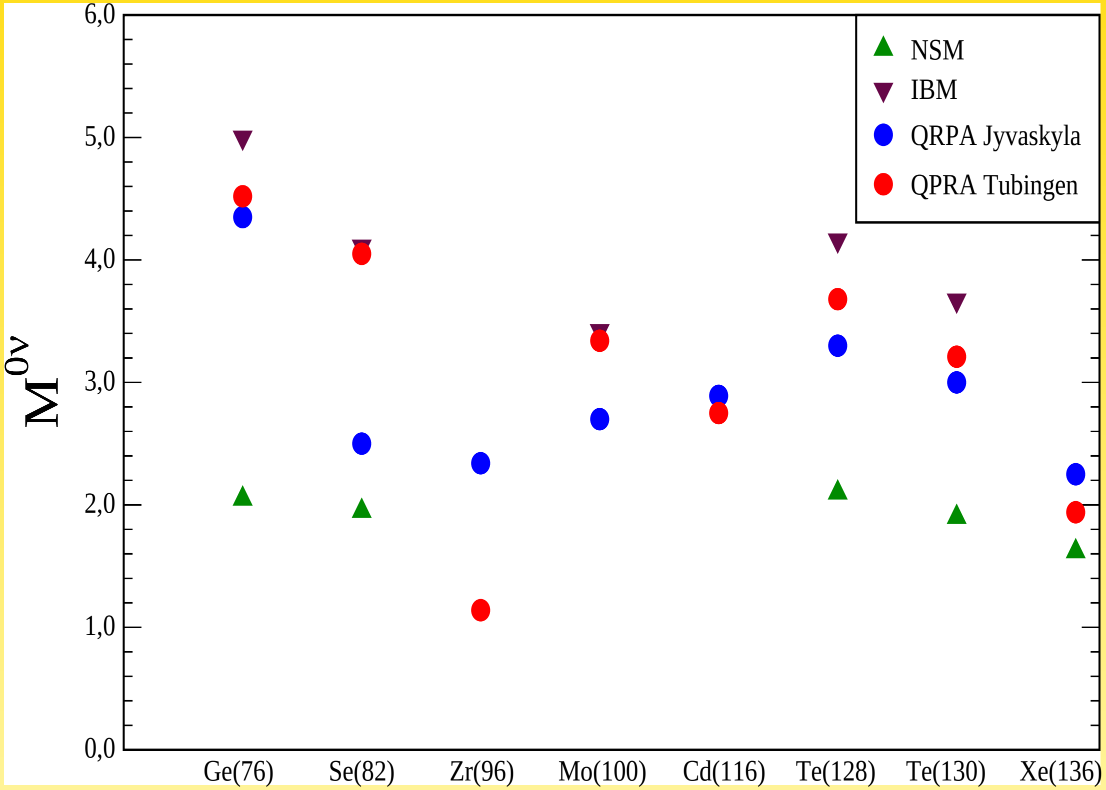


$$\nu_\mu\, N \longrightarrow \mu^-\, \alpha^+ \beta^+ X$$

(νN scattering, ν-fac, HERA,...)

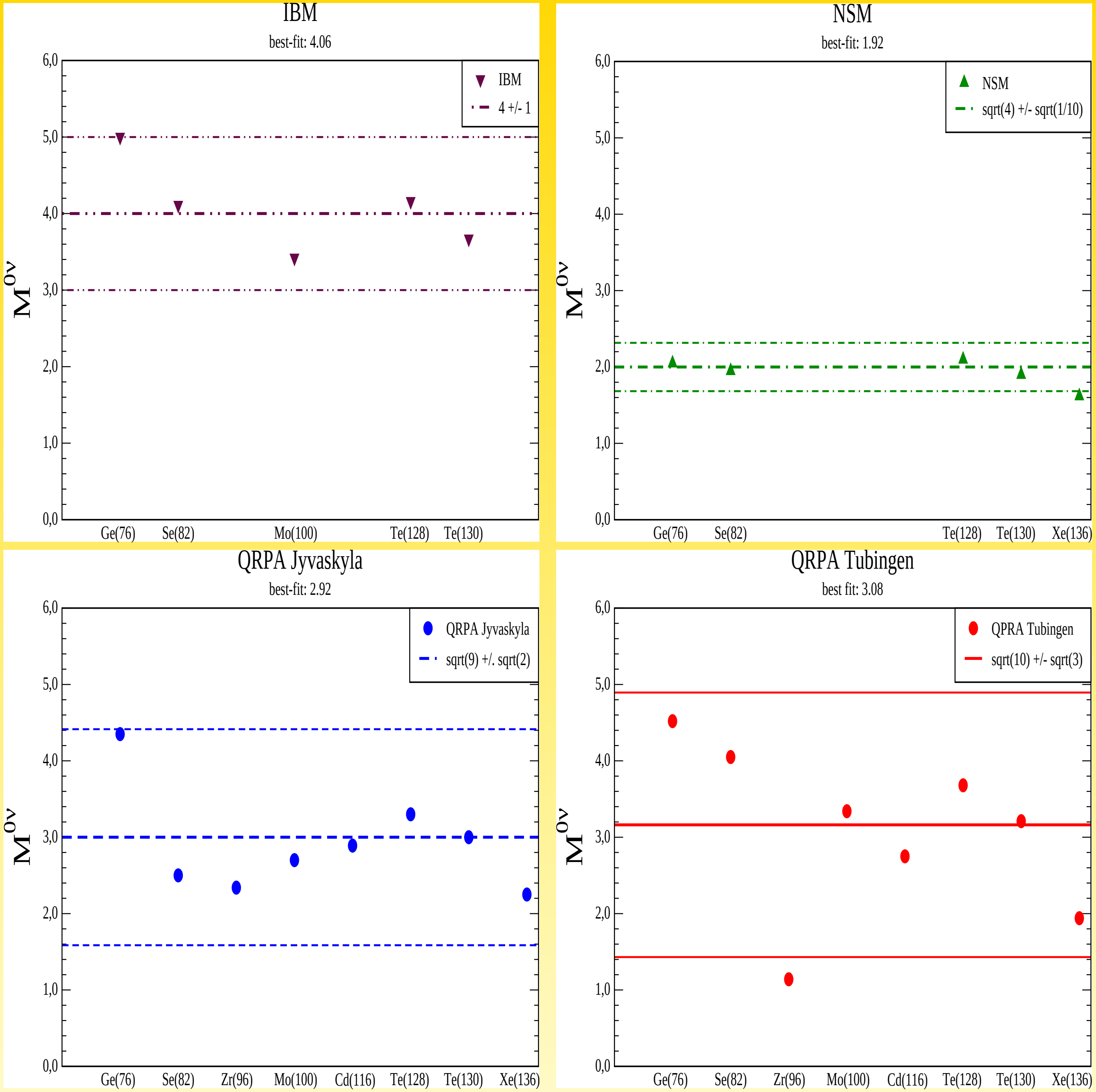
$$\text{BR},\ \Gamma,\ \sigma \propto \frac{m_i^2}{(q^2 - m_i^2)^2} \simeq \begin{cases} m_i^2 & q^2 \gg m_i^2 \\ \dfrac{1}{m_i^2} & q^2 \ll m_i^2 \end{cases}$$

Nuclear Matrix Elements



Rodin, arXiv:0910.5866 [nucl-th]

without NSM: 50 % for $\Gamma(0\nu\beta\beta)$


IBM
best-fit: 4.06
IBM
4 +/- 1
$M^{0\nu}$
Ge(76) Se(82) Mo(100) Te(128) Te(130)
NSM
best-fit: 1.92
NSM
sqrt(4) +/- sqrt(1/10)
Ge(76) Se(82) Te(128) Te(130) Xe(136)
QRPA Jyvaskyla
best-fit: 2.92
QRPA Jyvaskyla
sqrt(9) +/. sqrt(2)
QRPA Tubingen
best fit: 3.08
QPRA Tubingen
sqrt(10) +/- sqrt(3)
Ge(76) Se(82) Zr(96) Mo(100) Cd(116) Te(128) Te(130) Xe(136)

Towards Statistics with NMEs and $0\nu\beta\beta$

Majorana phases: consider IH spectrum

$$\sin^2\alpha = \left(1 - \frac{|m_{ee}|}{\sqrt{|\Delta m^2_{\rm A}|}\,(1 - |U_{e3}|^2)}\right)^2 \frac{1}{\sin^2 2\theta_{12}}$$

- uncertainties on $|m_{ee}|$ from NME smaller than 2
- $\sigma(|m_{ee}|) \lesssim 15\%$
- $\sigma(\Delta m^2_{\rm A}) \lesssim 10\%$ (IH) or $\sigma(m_0) \lesssim 10\%$ (QD)
- $\sin^2\theta_{12} \gtrsim 0.29$
- $2\alpha \in [\pi/4, 3\pi/4]$ or $[5\pi/4, 7\pi/4]$

Pascoli, Petcov, W.R., Phys. Lett. B **549**, 177 (2002)

No to "no-go" from Barger *et al.*, Phys. Lett. B **540**, 247 (2002)

What's more to $0\nu\beta\beta$?

Pascoli, Petcov, W.R., Phys. Lett. B **549**, 177 (2002)

used simple error multiplication on

$$\sin^2\alpha = \left(1 - \frac{|m_{ee}|}{\sqrt{|\Delta m^2_{\rm A}|}\,(1-|U_{e3}|^2)}\right)^2 \frac{1}{\sin^2 2\theta_{12}}$$

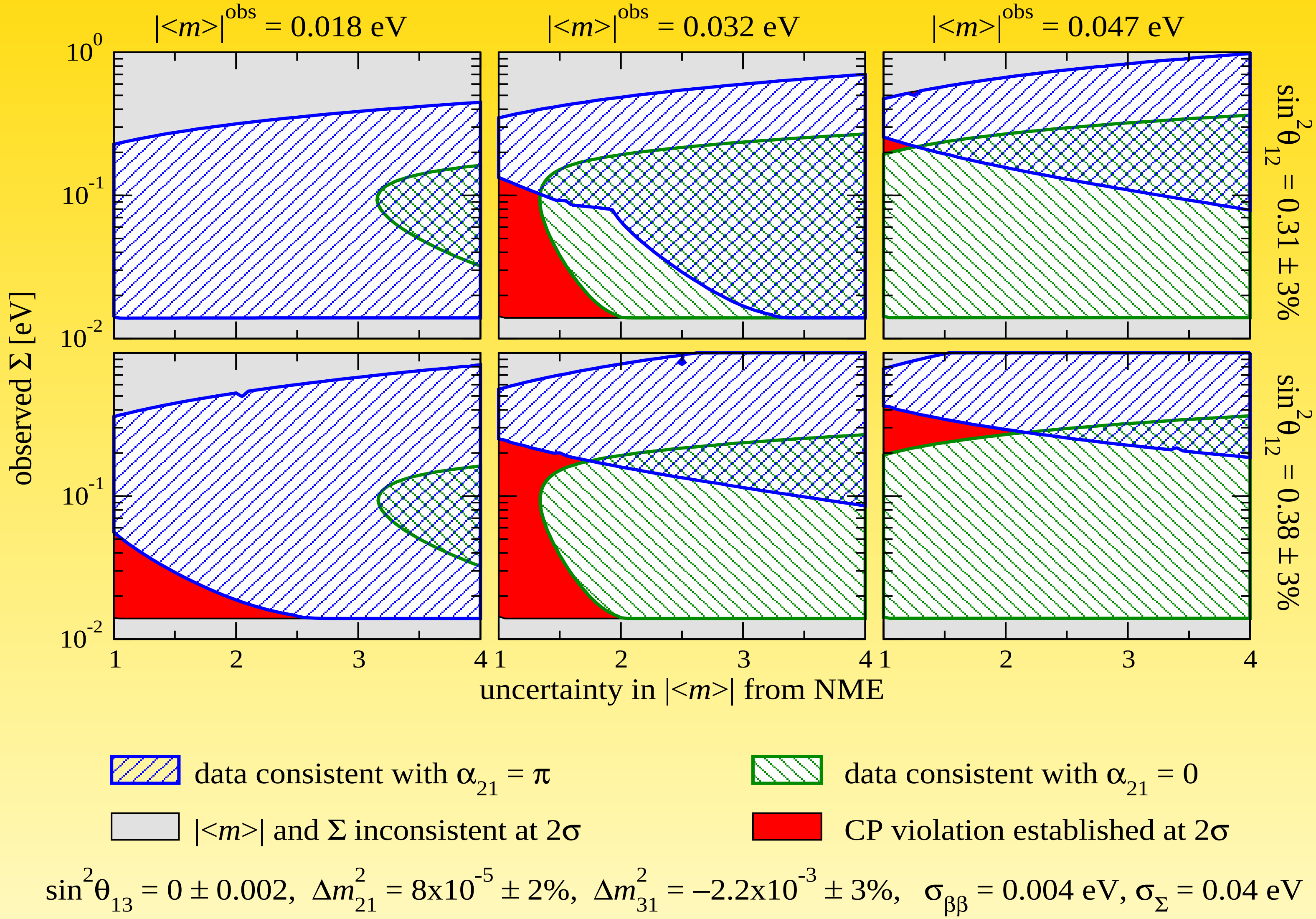
with $|m_{ee}| = \zeta\left(|m_{ee}|_{\rm min} \pm \sigma(|m_{ee}|)\right)$ and

$$\sigma(|m_{ee}|) = |m_{ee}|\,\sqrt{\mathrm{stat}^2 + \mathrm{sys}^2}$$

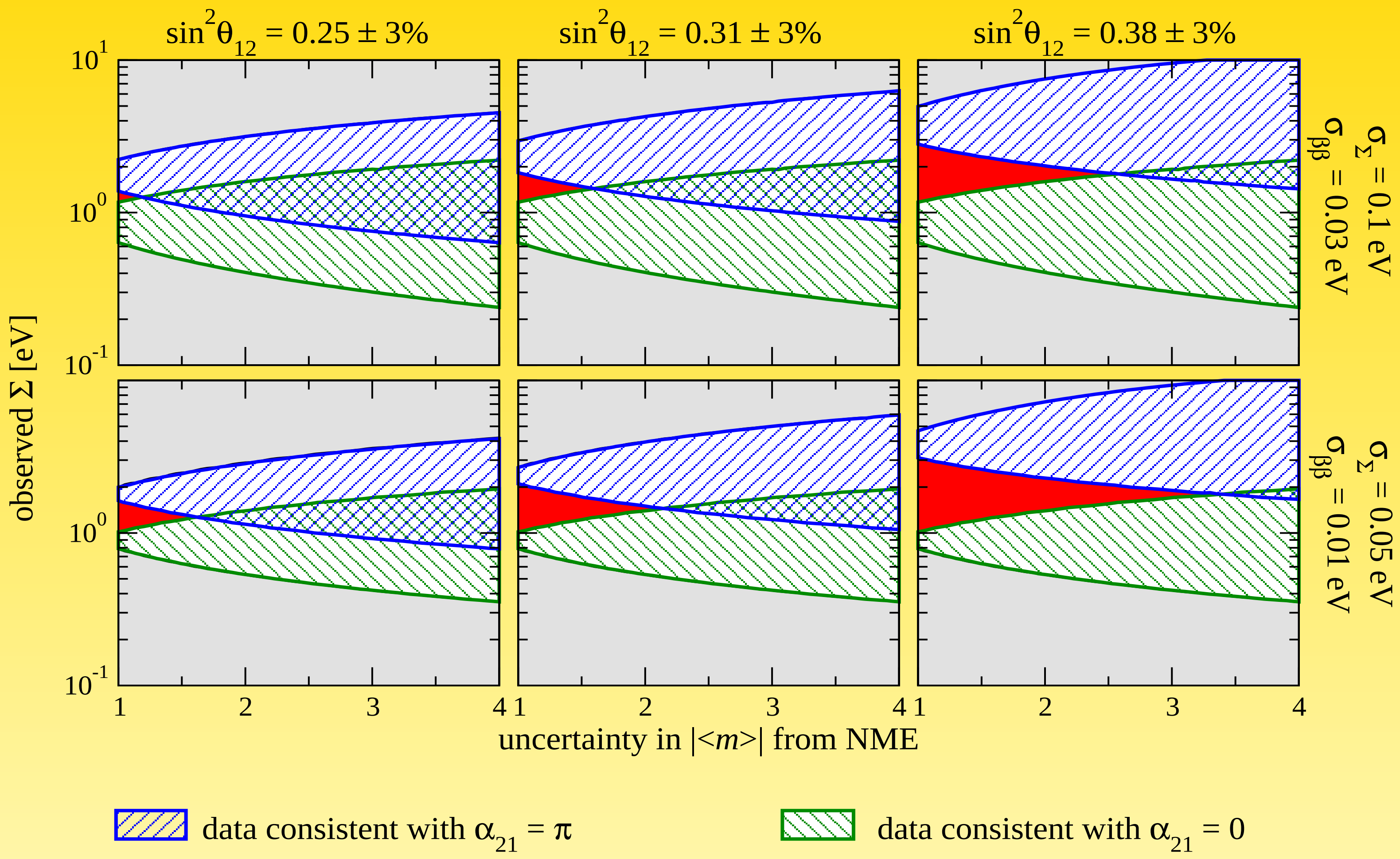
- statistical error: 0.028 eV/$|m_{ee}|$
- systematical error: 0.05

which gives in total 15 % at $|m_{ee}| = 0.2$ eV

Pascoli, Petcov, Schwetz, Nucl. Phys. B **734**, 24 (2006)



Pascoli, Petcov, Schwetz, Nucl. Phys. B **734**, 24 (2006)



$\sin^2\theta_{13} = 0 \pm 0.002$, $\Delta m^2_{21} = 8\text{x}10^{-5} \pm 2\%$, $\Delta m^2_{31} = 2.2\text{x}10^{-3} \pm 3\%$ observed $|\langle m\rangle| = 0.3$ eV

What's more to $0\nu\beta\beta$?

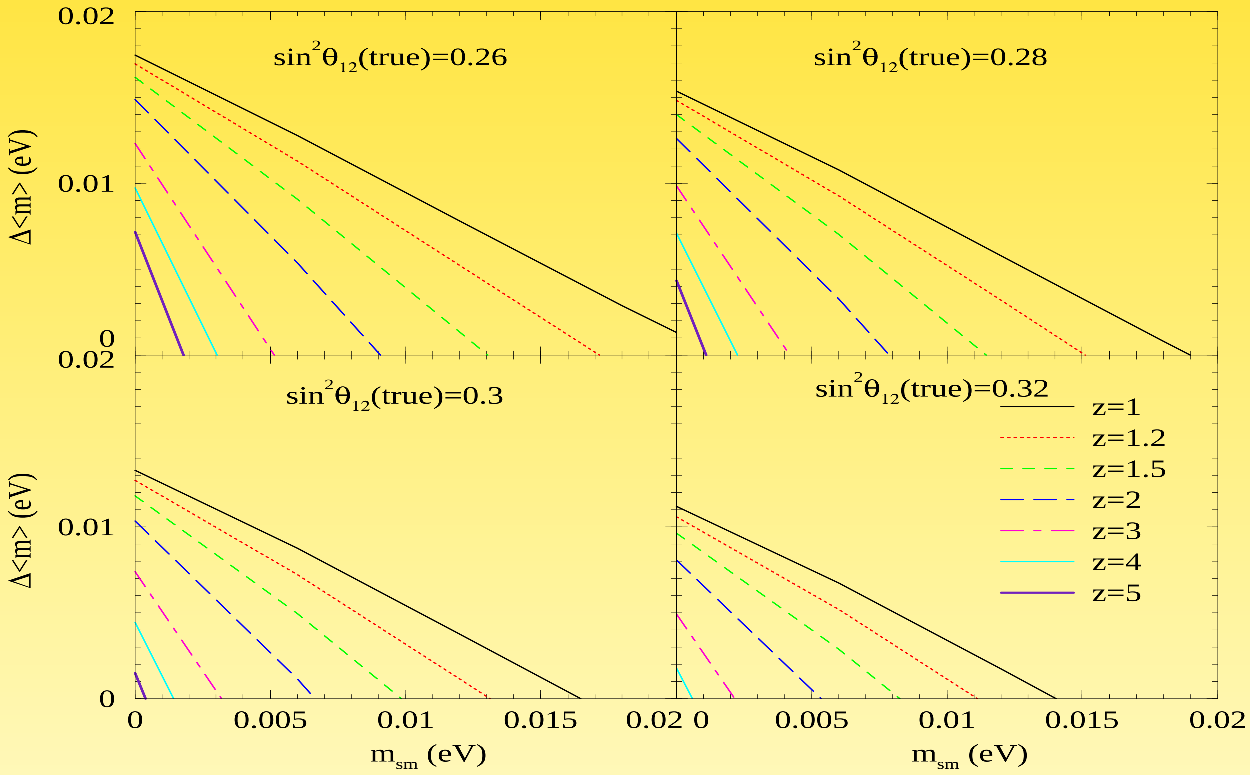
Mass scale: consider QD spectrum

$$m_0 \le \frac{1+\tan^2\theta_{12}}{1-\tan^2\theta_{12}-2\,|U_{e3}|^2}\,|m_{ee}|_{\rm lim} \simeq \frac{1}{\cos 2\theta_{12}}\,|m_{ee}|_{\rm lim} \simeq 3\,|m_{ee}|_{\rm lim}$$

Spectrum

$$\zeta \, |m_{ee}|^{\rm NH}_{\rm max} \stackrel{!}{<} |m_{ee}|^{\rm IH}_{\rm min}$$

gives



Statistical Analysis

- how well can we reconstruct possible scenarios?
- focus on "near future", i.e. no normal hierarchy
- what if cosmology gives wrong/inconsistent result?

We consider 3 scenarios with "true values":

Scenario	m_3 [eV]	$\|m_{ee}\|$ [eV]	m_β [eV]	Σ [eV]
$\mathcal{QD}$	0.3	$0.11 - 0.30$	0.30	0.91
$\mathcal{INT}$	0.1	$0.04 - 0.11$	(0.11)	0.32
$\mathcal{IH}$	0.003	$0.02 - 0.05$	(0.05)	(0.10)

W. Maneschg, A. Merle, W.R., Europhys. Lett. **85**, 51002 (2009) [arXiv:0812.0479 [hep-ph]]

Statistical Analysis

- $|m_{ee}|$
  - "experimental error"

$$\sigma(|m_{ee}|_{\rm exp}) = \frac{|m_{ee}|_{\rm exp}}{2} \, \frac{\sigma(\Gamma_{\rm obs})}{\Gamma_{\rm obs}}$$

    GERDA: $\sigma(\Gamma_{\rm obs})/\Gamma_{\rm obs} \simeq 23.3\%$

    (Phase I: 6 ± 1.4 events if Klapdor is right)

  - "theoretical error" NMEs

$$\sigma(|m_{ee}|) = (1 + \zeta) \left(|m_{ee}| + \sigma(|m_{ee}|_{\rm exp}) \right) - |m_{ee}|$$

    ($\leftrightarrow$ subtract $\simeq 1$ from uncertainty in PPS, sorry. . .)

- $\sigma(m_\beta^2) = 0.025$ eV^2
- $\sigma(\Sigma) = 0.05$ eV
- future 3σ ranges of oscillation parameters: current 1σ ranges (not important)

Statistical Analysis

covariance matrix:

$$S_{ab} \equiv \delta_{ab}\, \sigma^2(a) + \sum_i \frac{\partial T_a}{\partial x_i} \frac{\partial T_b}{\partial x_i}\, \sigma_i^2 \,,$$

where $T_1 = |m_{ee}|$, $T_2 = \Sigma$ and $T_3 = m_\beta^2$

$v_a = T_a - (T_a)_{\rm exp}$, where $(T_a)_{\rm exp}$ is experimental value of T_a

$$\chi^2 = v^T\, S^{-1}\, v$$

minimize χ^2 with respect to Majorana phases ($\chi^2_{\rm res}$) and then

$\Delta\chi^2 = \chi^2_{\rm res} - \chi^2_{\rm res,min} = 1, 4, 9$ is $1, 2, 3\sigma$ range for m_3 (only free parameter)

($\Delta\chi^2 = 0$ for true region)

- Pascoli, Petcov, Schwetz have defined

$$\chi^2(\boldsymbol{x}^{0\nu}_{\beta\beta}, F) = \min_{\xi\in[1/\sqrt{F},\sqrt{F}]} \frac{\left[\xi\, |m_{ee}|(\boldsymbol{x}) - |m_{ee}|^{\rm obs}\right]^2}{\sigma^2_{\beta\beta} + \xi^2\sigma^2_{\rm th}} \text{ with } \xi = \frac{|\mathcal{M}|}{|\mathcal{M}_0|}$$

where $\mathcal{M}_0$ is NME to obtain $|m_{ee}|^{\rm obs}$ and $\mathcal{M}$ is true NME

no weight on any NME $\leftrightarrow$ flat priors for theoretical errors in CKM fits

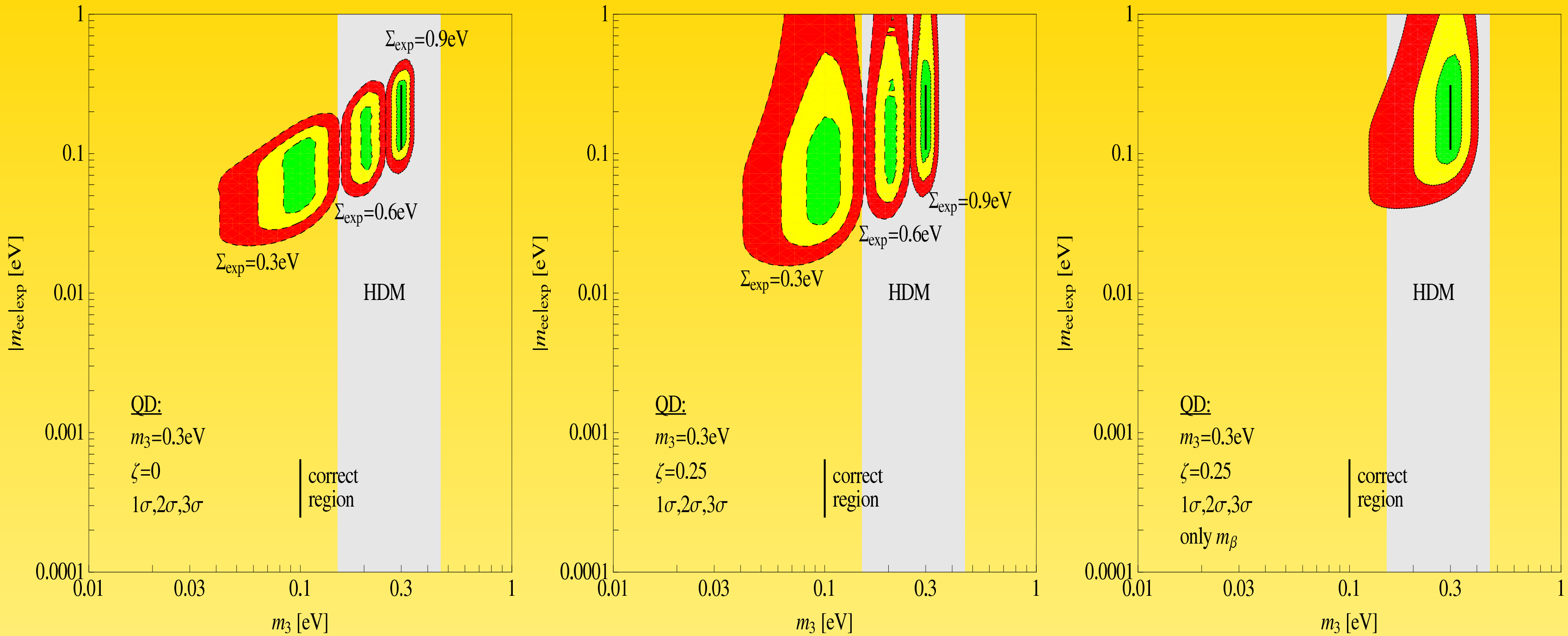
- Fogli *et al.* linearize $|m_{ee}|^2 = m_e^2\,\Gamma/|C_{mm}|^2$

$$2\log_{10}\left(\frac{m_{\beta\beta}}{\rm eV}\right) = 2\log_{10}\left(\frac{m_e}{\rm eV}\right) - \log_{10}\left(\frac{|C_{mm}|^2}{\rm yr^{-1}}\right) - \log_{10}\left(\frac{T^{0\nu}_{1/2}}{\rm yr}\right)$$

because non-linear expression and large uncertainties

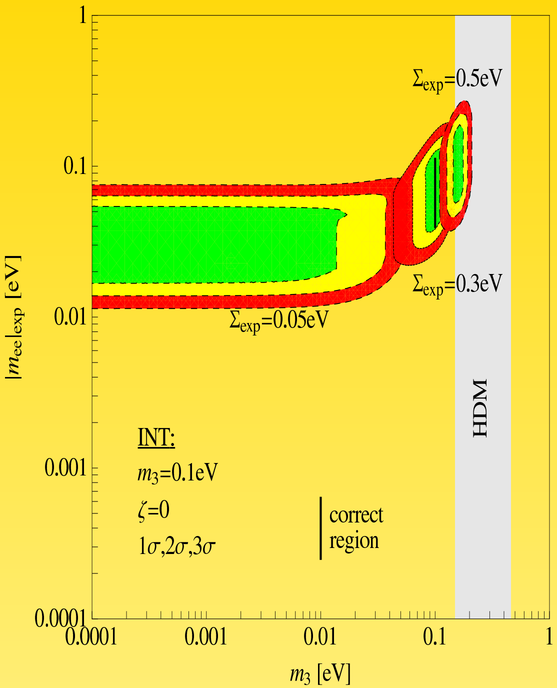
linear error propagation for log$|\mathcal{M}|^2$ rather than $|\mathcal{M}|^2$ ("tractable uncertainties")

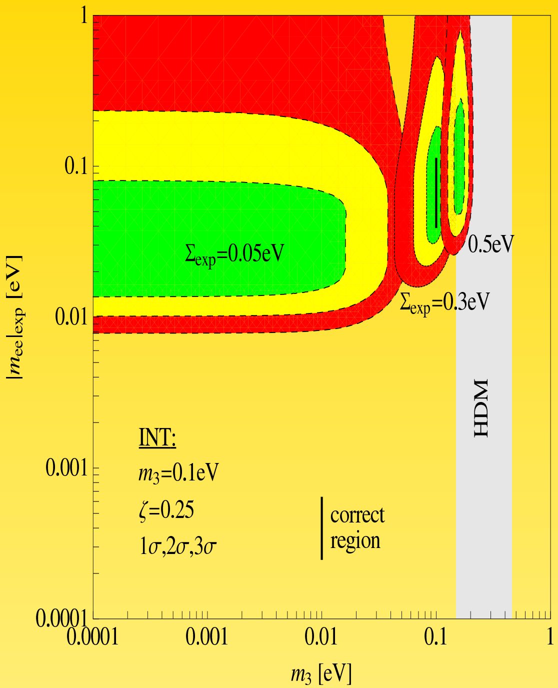
take minimal and maximal $|\mathcal{M}|^2$ and from (max $\pm$ min) get 1σ range

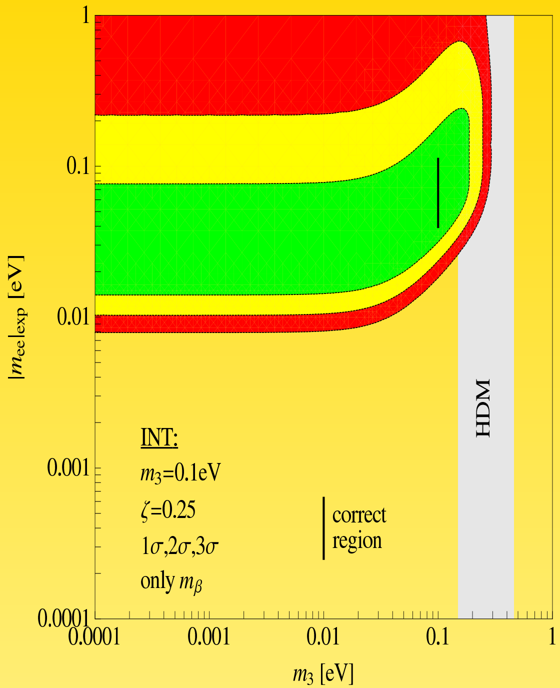


$\mathcal{QD}$ with $|m_{ee}|_{\rm exp} = 0.20$ eV

- if $\zeta = 0$: $\sigma(m_3) \simeq 15\%$ at 3σ
- if $\zeta = 0.25$: $\sigma(m_3) \simeq 25\%$
- if Σ wrong: reconstruction of m_3 wrong by one order of magnitude
- leave Σ out: $\sigma(m_3) \simeq 50\%$

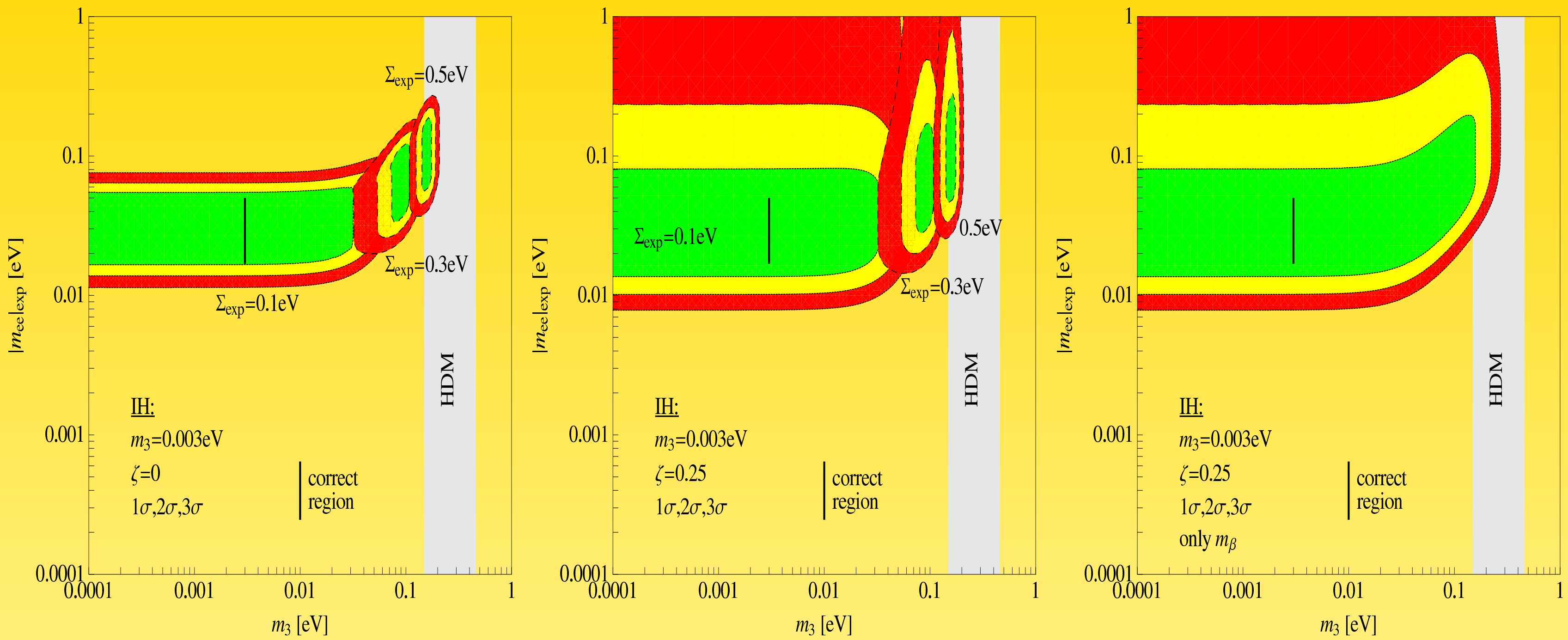






$\mathcal{INT}$ with $|m_{ee}|_{\rm exp} = 0.08$ eV

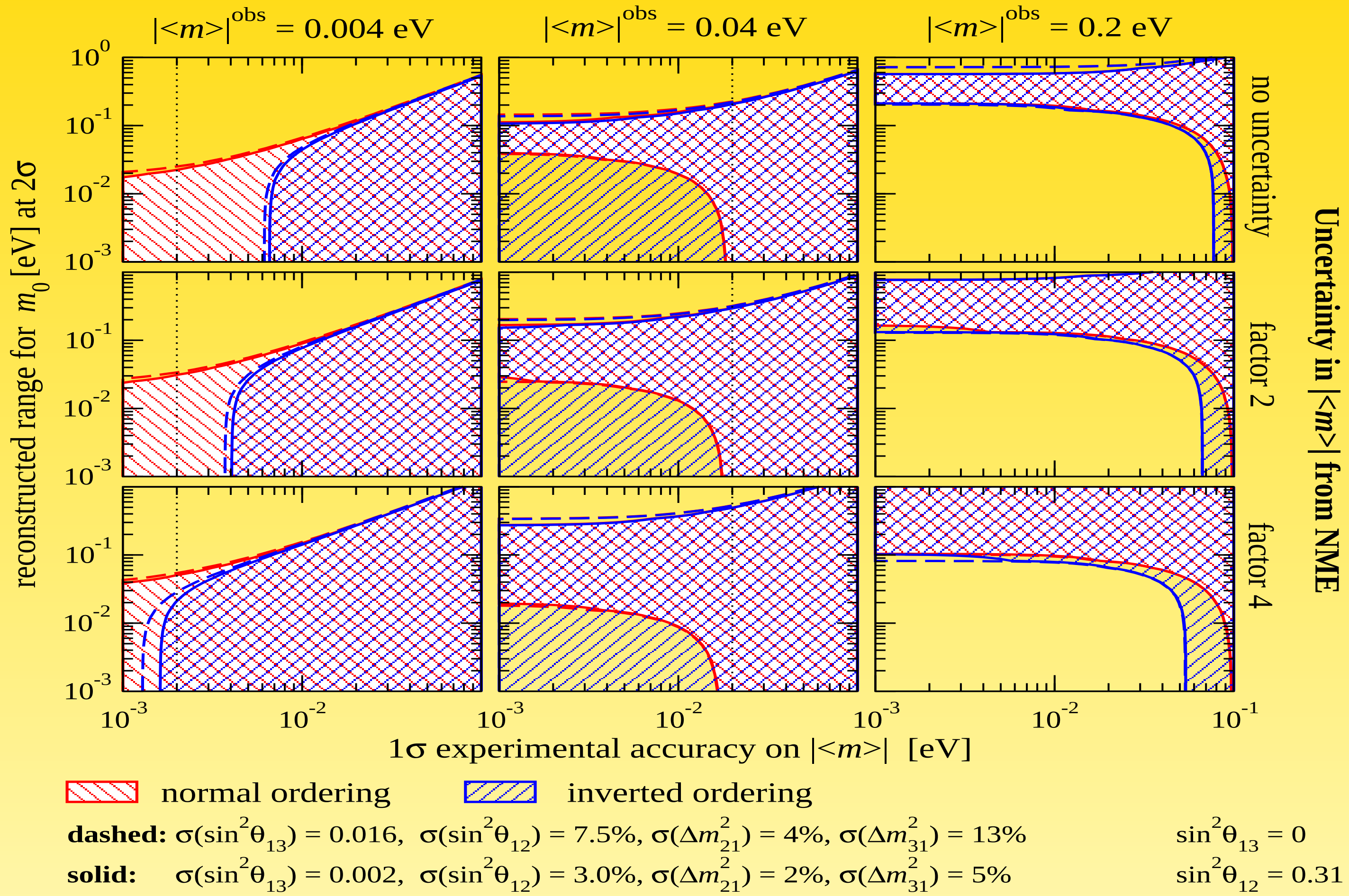
- if $\zeta = 0$: $\sigma(m_3) \simeq 50\%$ at 3σ
- if $\zeta = 0.25$: $\sigma(m_3) \simeq 60\%$
- if Σ wrong: no lower limit
- leave Σ out: no lower limit



$\mathcal{IH}$ with $|m_{ee}|_{\rm exp} = 0.04$ eV

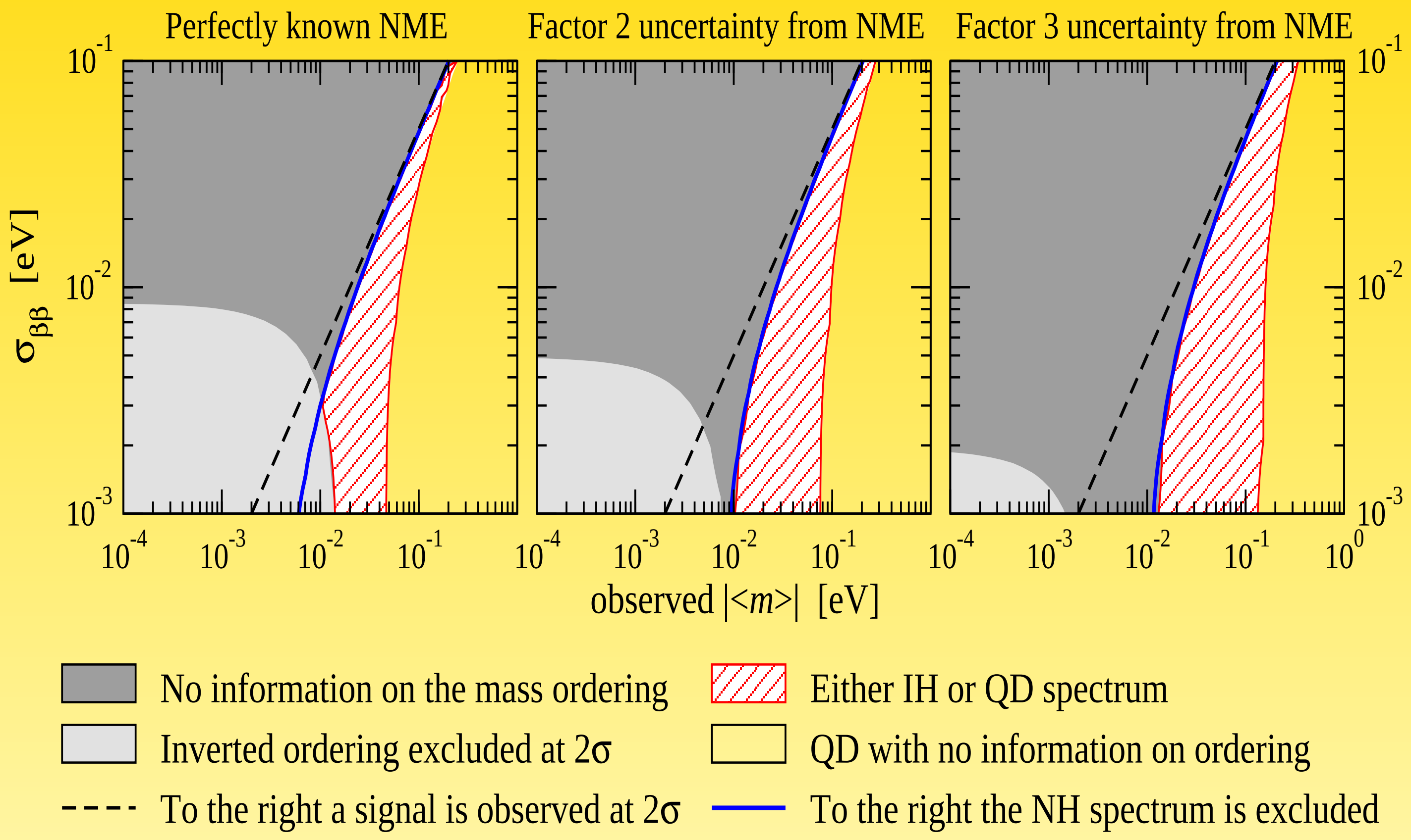
- $m_3 \leq 0.07$ at 3σ
- if Σ wrong: reconstruction of m_3 wrong by two orders of magnitude
- leave Σ out: not much difference to true case

Pascoli, Petcov, Schwetz, Nucl. Phys. B **734**, 24 (2006)



$\mathcal{QD}$ corresponds to $\sigma(|m_{ee}|) \simeq 0.023$ eV $\Rightarrow$ $\sigma(m_3) \simeq 50\%$ at 2σ for $\zeta = 0$

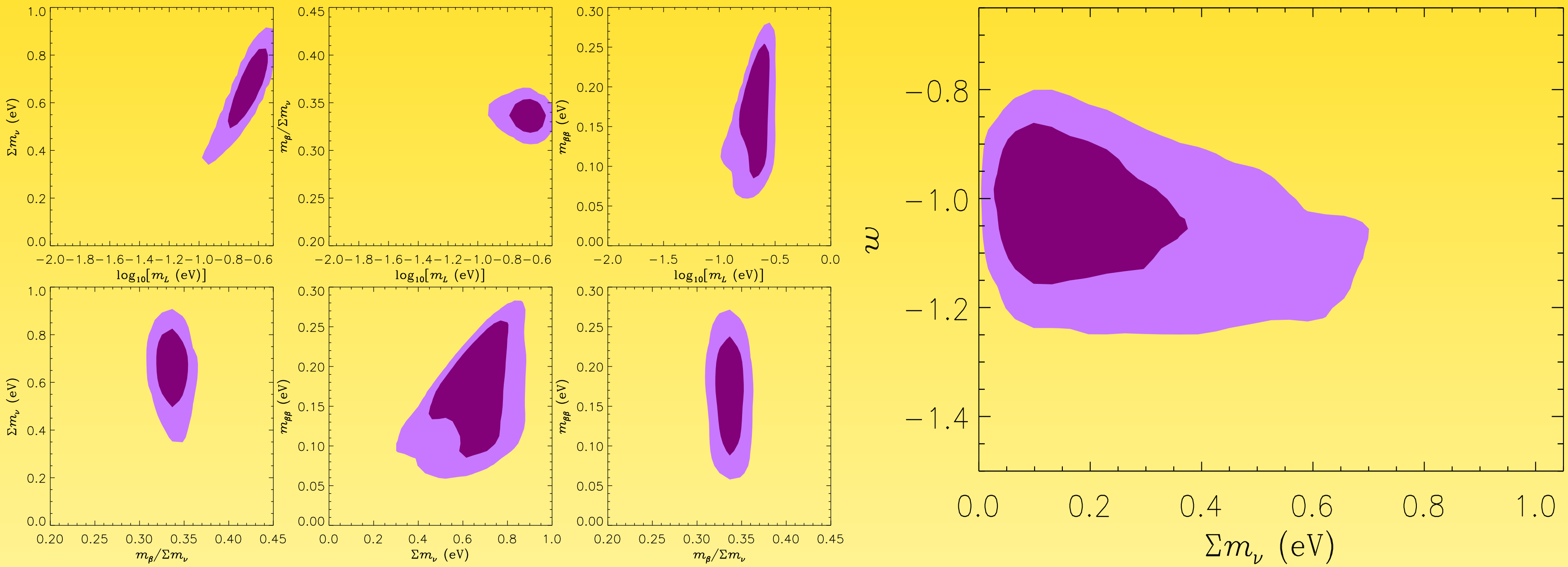
Pascoli, Petcov, Schwetz, Nucl. Phys. B **734**, 24 (2006)



$\sin^2\theta_{13} = 0.03 \pm 0.006$, $\sin^2\theta_{12} = 0.31 \pm 3\%$, $\Delta m^2_{21} = 8\text{x}10^{-5} \pm 2\%$, $|\Delta m^2_{31}| = 2.2\text{x}10^{-3} \pm 3\%$

`Hannestad, arXiv:0710.1952 [hep-ph]`

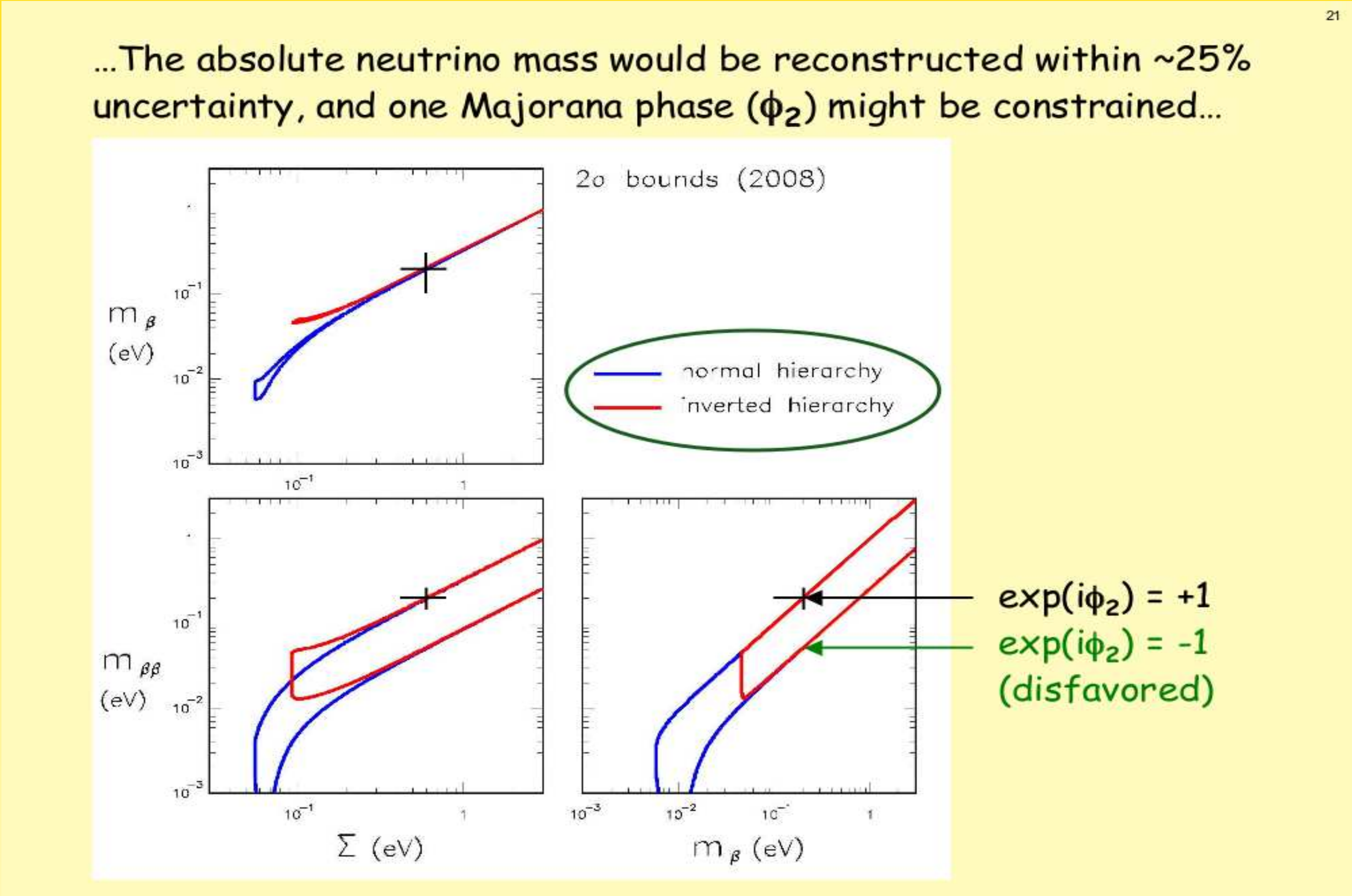
example $m_\beta = 0.28$ eV and $|m_{ee}| = 0.18$ eV with $\sigma(|m_{ee}|^2) = 0.01$ eV^2



gives $m_{\rm sm} = 0.2\,(0.035\ldots0.789)$ eV, or without cosmology
$m_{\rm sm} = 0.28\,(0.057\ldots0.836)$ eV from (MC)^2

assume $m_1 = m_2 = m_3 = 0.2$ eV and measurements with 1σ errors

$$m_\beta = 0.2\,(1 \pm 0.5)\,\mathrm{eV}\ , \quad |m_{ee}| \simeq 0.2\,(1 \pm 0.3)\,\mathrm{eV}\ , \quad \Sigma \simeq 0.6\,(1 \pm 0.3)\,\mathrm{eV}$$



Lisi, talk at Erice 09

Summary

- reconstruction of m_0 possible, best case 20 %
- QD neutrinos would be great
- the smaller m_0, the less observables, the less we can say
- inconsistencies could lead to wrong results