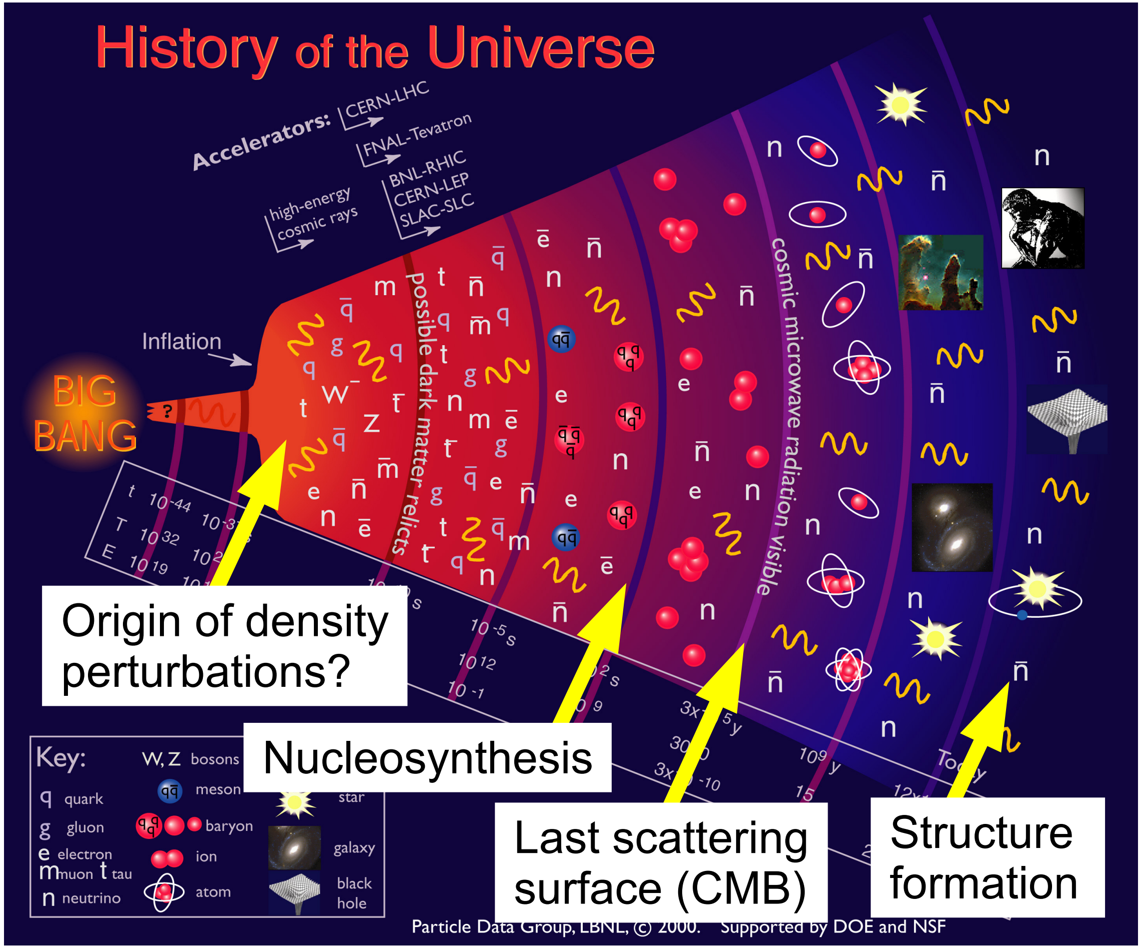
Neutrino mass from CMB and LSS

Yvonne Y. Y. Wong
RWTH Aachen

The future of neutrino mass measurement, Seattle, February 8--11, 2010



Relic neutrino background:

- **Temperature**:

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\mathrm{CMB},0} = 1.95\ \mathrm{K}$$

- **Number density** per flavour:

$$n_{\nu,0} = \frac{6}{4}\frac{\zeta(3)}{\pi^2} T_{\nu,0}^3 = 112\ \mathrm{cm}^{-3}$$

- **Energy density** per flavour:

If $m_\nu > 1$ meV

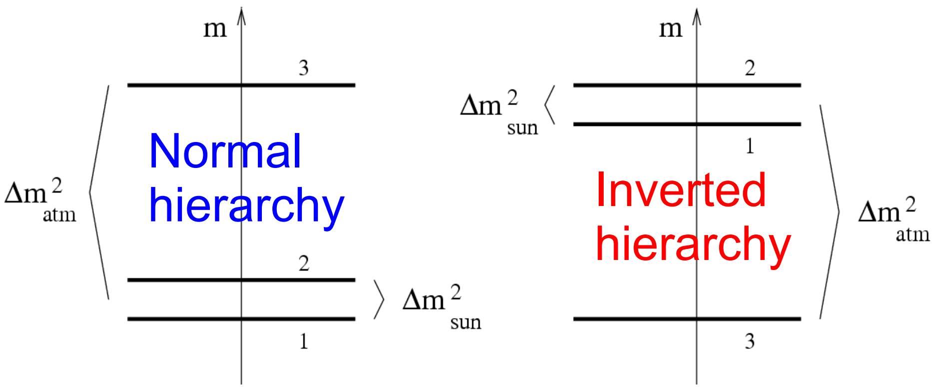
$$\Omega_\nu h^2 = \frac{m_\nu}{93\ \mathrm{eV}}$$

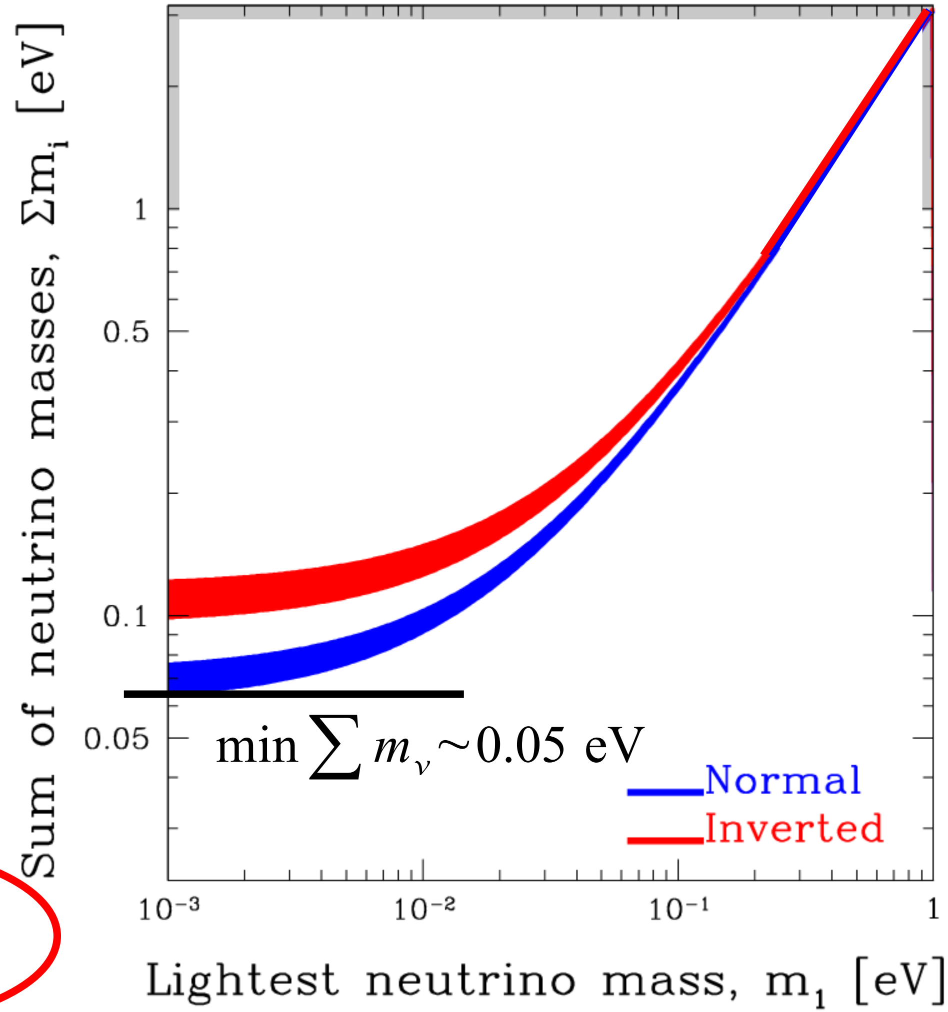
→ Neutrino dark matter

Neutrino dark matter...

- Neutrino oscillations:

$$\Delta m^2_{atm} \sim 10^{-3}\ \mathrm{eV}^2 \qquad \Delta m^2_{sun} \sim 10^{-5}\ \mathrm{eV}^2$$

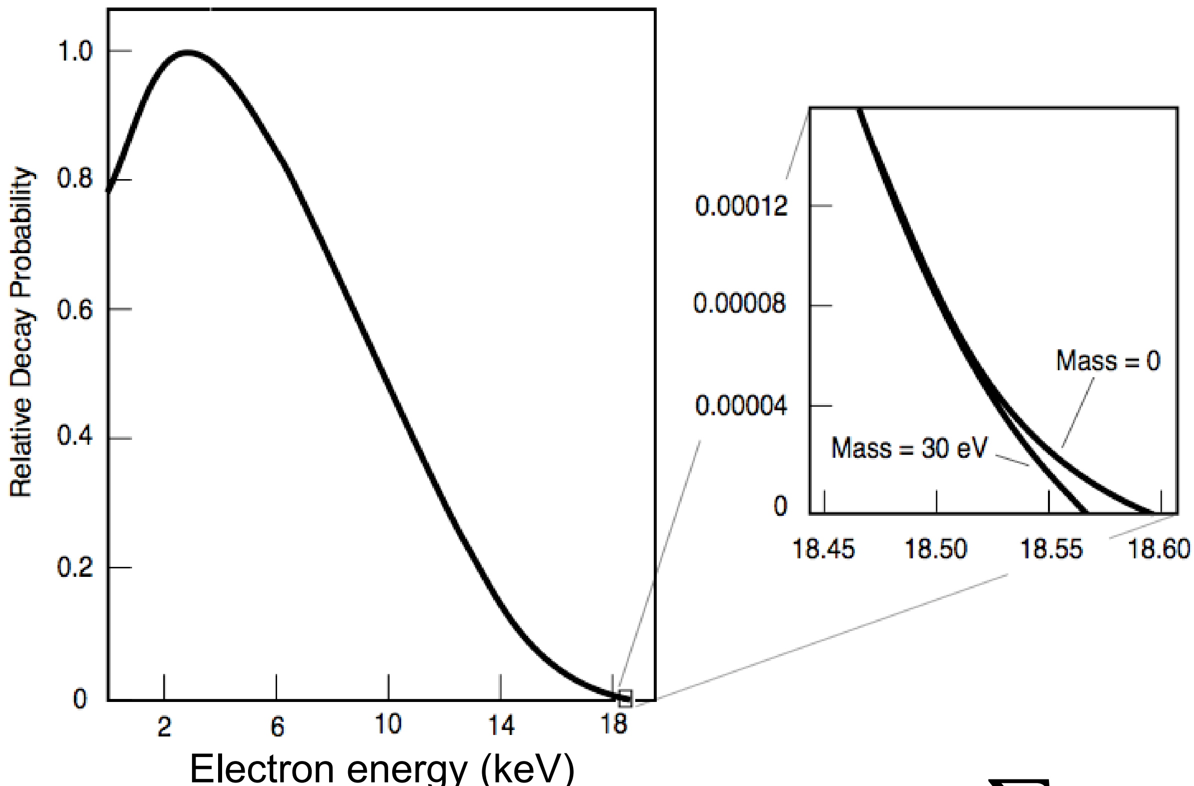




Mininum amount of neutrino dark matter

$$\min \sum m_\nu \sim 0.05\ \mathrm{eV} \rightarrow \min \Omega_\nu \sim 0.1\,\%$$

- Upper limit on neutrino masses from tritium β-decay:



Large mixing means

$|U_{ei}|^2 \sim O(0.1 \to 1)$

$$m_e \equiv \left(\sum_i |U_{ei}|^2 m_i^2 \right)^{1/2} < 2.2 \text{ eV}$$

$$\max \sum m_\nu \sim 7 \text{ eV} \to \max \Omega_\nu \sim 12\%$$

Light neutrinos cannot be the only dark matter component

Neutrino dark matter is hot...

- **Large velocity dispersion**:

$$\langle v_{\text{thermal}} \rangle \simeq 81(1+z)\left(\frac{\text{eV}}{m_\nu}\right) \text{ km s}^{-1}$$

  - A dwarf galaxy has a velocity dispersion of 10 km s^{-1} or less, a galaxy about 100 km s^{-1}.

  - Sub-eV neutrinos have too much thermal energy to be packed into galaxy-size self-gravitating systems.

    - Neutrinos **cannot** be the *dominant* Galactic dark matter.

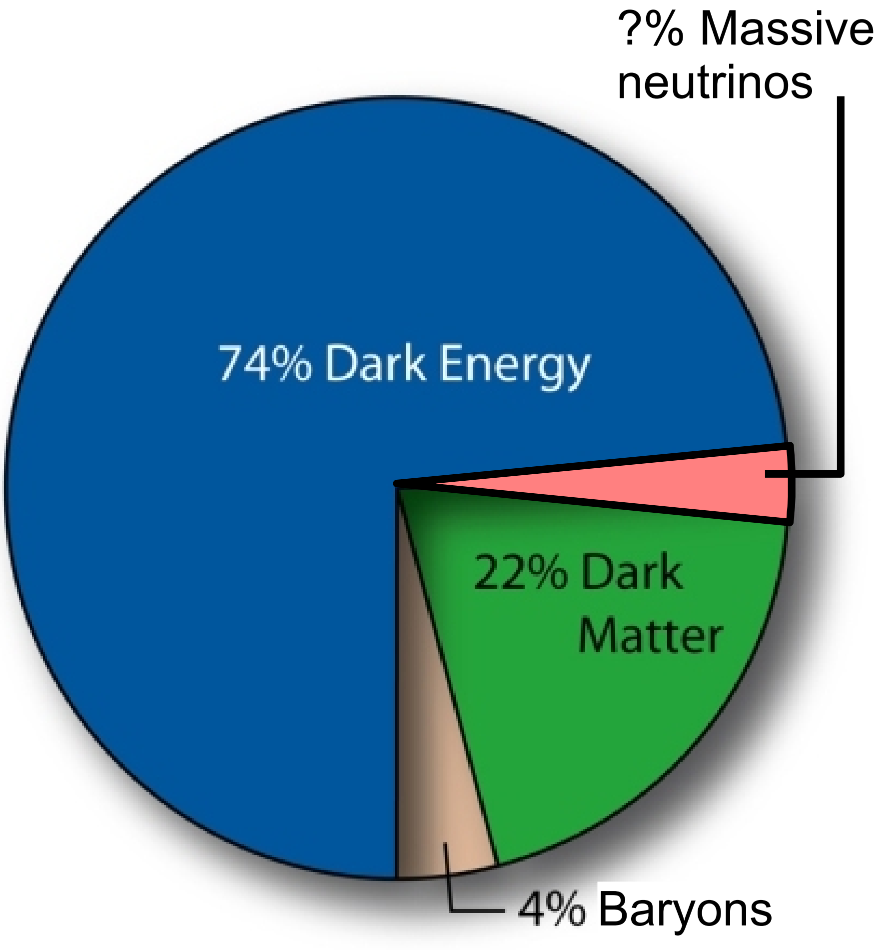
Why study neutrinos in cosmology...

- Hot dark matter leaves a distinctive imprint on the large-scale structure distribution.
  - We can learn about neutrino properties from cosmology.

- Cosmological probes are getting ever more precise:
  - Even a small neutrino mass can **bias** the inference of other cosmological parameters.

The concordance framework...

- We work within the ΛCDM framework extended with a subdominant component of massive neutrino dark matter.
    - Flat geometry.
    - Main dark matter is cold.
    - Initial conditions from single-field slow-roll inflation.



Possible alternatives...

- Broken scale invariance in the primordial density perturbation power spectrum.
- We live in a void.
- Interacting dark sectors.
- ...

Plan...

- What we can do **now**
- What we can do **in the future**
- The **nonlinear matter power spectrum**

1. What we can do now…

Two effects of massive neutrinos...

- On the **background**:
  - Shift in time of matter radiation equality.

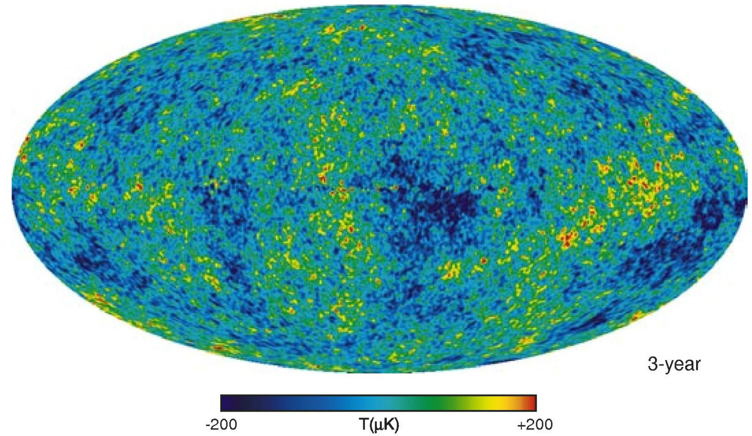
- On the **perturbations**:
  - Suppression of growth.

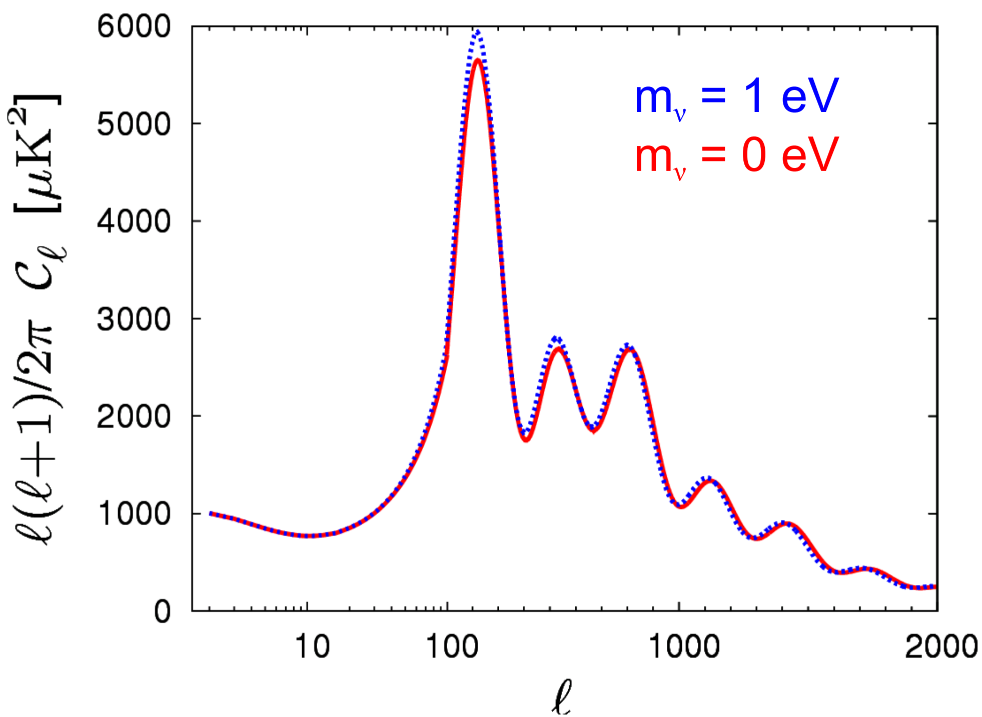
Background...

- Sub-eV neutrinos become nonrelativistic at **z<1000**:
  - Radiation at early times.
  - Matter at late times.

Comoving matter density today ≠ Comoving matter density before recombination

  - Shift in matter-radiation equality relative to model with zero neutrino mass.





Two effects of massive neutrinos...

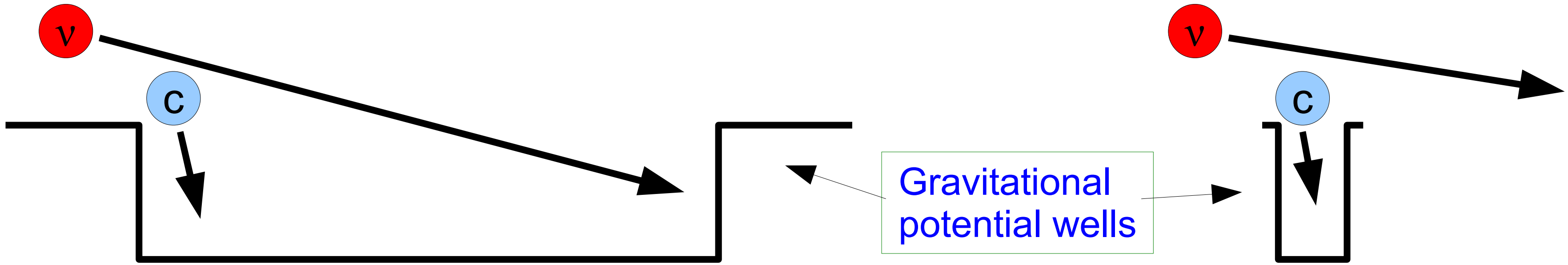
- On the **background**:
  - Shift in time of matter radiation equality.

- On the **perturbations**:
  - Suppression of growth.

Perturbations...

- At low redshifts, neutrinos become nonrelativistic:.
  - But still have large thermal speed: $c_\nu \simeq 81(1+z)\left(\frac{\text{eV}}{m_\nu}\right) \text{ km s}^{-1}$

    → hinder ν clustering on small scales.



- Free-streaming length scale & wavenumber:

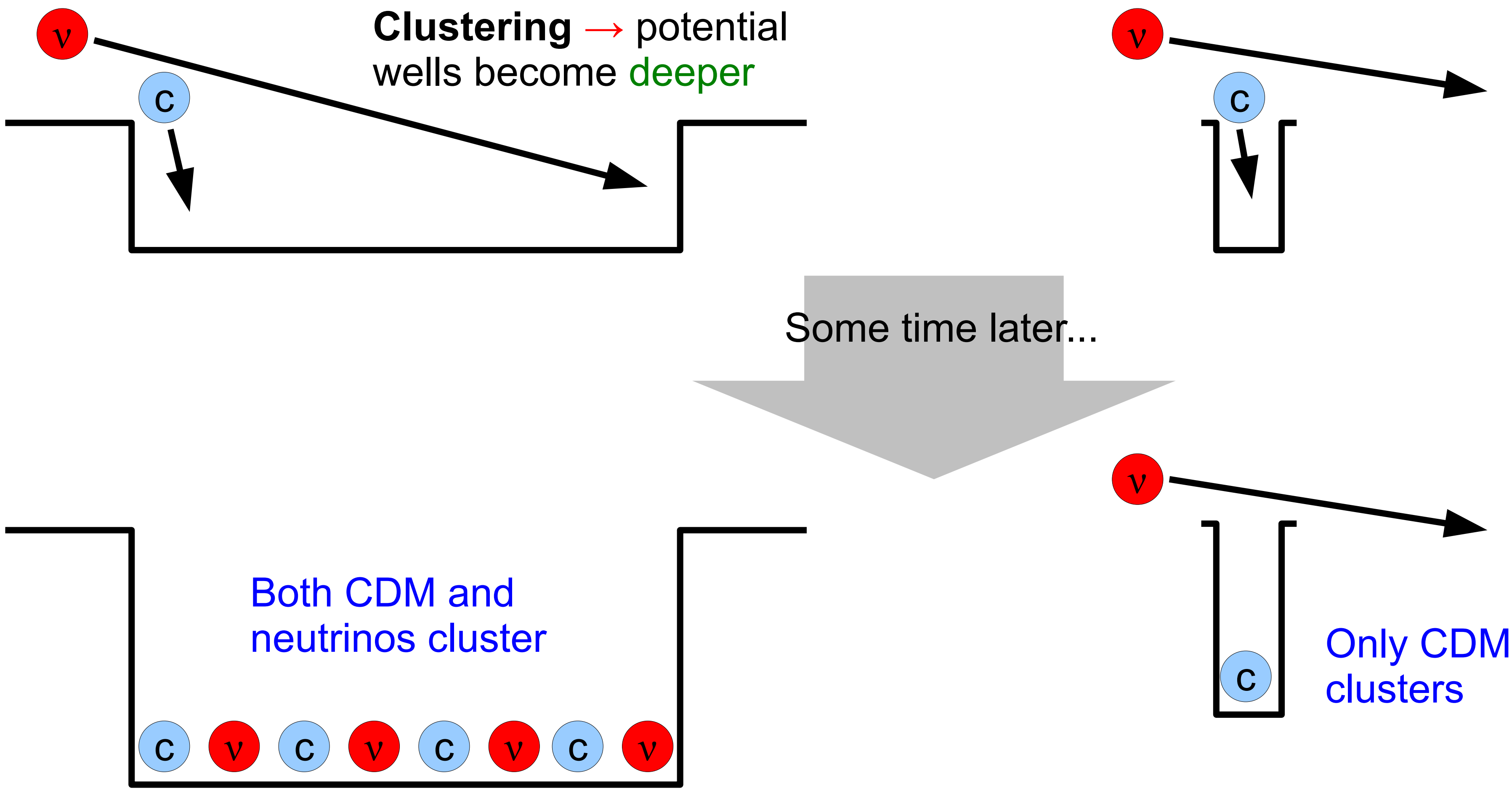
$$\lambda_{FS} \equiv \sqrt{\frac{8\pi^2 c_\nu^2}{3\Omega_m H^2}} \simeq 4.2 \sqrt{\frac{1+z}{\Omega_{m,0}}}\left(\frac{\text{eV}}{m_\nu}\right) \ h^{-1} \text{ Mpc}$$

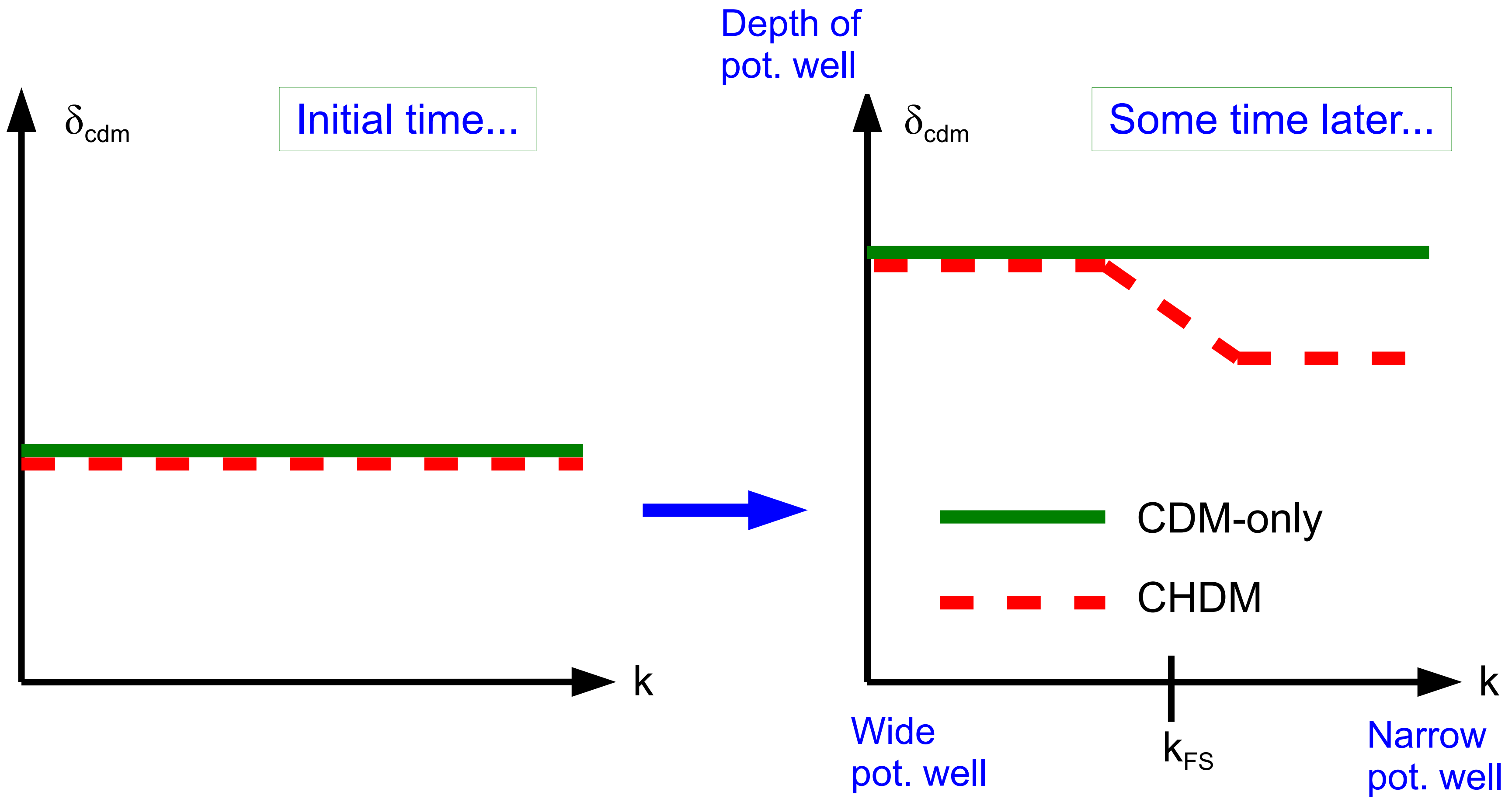
$$k_{FS} \equiv \frac{2\pi}{\lambda_{FS}}$$

$\lambda \gg \lambda_{FS}$, $k \ll k_{FS}$: Clustering

$\lambda \ll \lambda_{FS}$, $k \gg k_{FS}$: Non-clustering

- In turn, free-streaming (non-clustering) neutrinos slow down the growth of gravitational potential wells on **scales** $\lambda \ll \lambda_{FS}$ or **wavenumbers** $k \gg k_{FS}$.





- The presence of HDM slows down the growth of CDM perturbations at large wavenumbers k.
  - The density perturbation spectrum acquires a step-like feature.

Describing perturbations: CDM...

- Cold dark matter = collisionless, pressureless fluid:

Density perturbations

Continuity eqn $\dot{\delta}_c + \theta_c = 0$

Gravitational source

Euler eqn $\dot{\theta}_c + H\theta_c + \nabla^2 \Phi = 0$

Velocity divergence

Expansion

$f_\nu \equiv \frac{\Omega_\nu}{\Omega_m}$ Neutrino fraction

Poisson eqn $\nabla^2 \Phi = \frac{3}{2} H^2 \Omega_m [f_c \delta_c + f_\nu \delta_\nu]$

Describing perturbations: Neutrinos...

- Free-streaming neutrinos **cannot** be described by a perfect fluid.
  - Must solve (linearised) collisionless Boltzmann equation:

Nonrelativistic neutrinos

$$\frac{\partial(\delta f)}{\partial \tau} + \frac{\boldsymbol{p}}{m_\nu a} \cdot \nabla(\delta f) - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{p}} = 0$$

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0 + \delta f$$

Phase space density

Describing perturbations: Neutrinos...

- Free-streaming neutrinos **cannot** be described by a perfect fluid.
  - Must solve (linearised) collisionless Boltzmann equation:

Nonrelativistic neutrinos

$$\frac{\partial(\delta f)}{\partial \tau} + \frac{\boldsymbol{p}}{m_\nu a} \cdot \nabla(\delta f) - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{p}} = 0$$

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0 + \delta f$$

Phase space density

  - Momentum moments:

$$\delta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p}\, (\delta f)$$

Density perturbation

$$\theta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p}\, \frac{p_i}{a\, m_\nu} \partial_i (\delta f)$$

Velocity divergence

$$\sigma_{ij} \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p}\, \frac{p_i p_j}{a^2 m_\nu^2} (\delta f)$$

Pressure and anisotropic stress

$$\vdots$$

Describing perturbations: Neutrinos...

- Free-streaming neutrinos **cannot** be described by a perfect fluid.
  - Must solve (linearised) collisionless Boltzmann equation:

Nonrelativistic neutrinos

$$\frac{\partial(\delta f)}{\partial \tau} + \frac{\boldsymbol{p}}{m_\nu a} \cdot \nabla(\delta f) - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{p}} = 0$$

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0 + \delta f$$

Phase space density

- Momentum moments:

$$\delta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p} (\delta f)$$

Density perturbation

$$\theta_\nu \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p} \frac{p_i}{a\, m_\nu} \partial_i (\delta f)$$

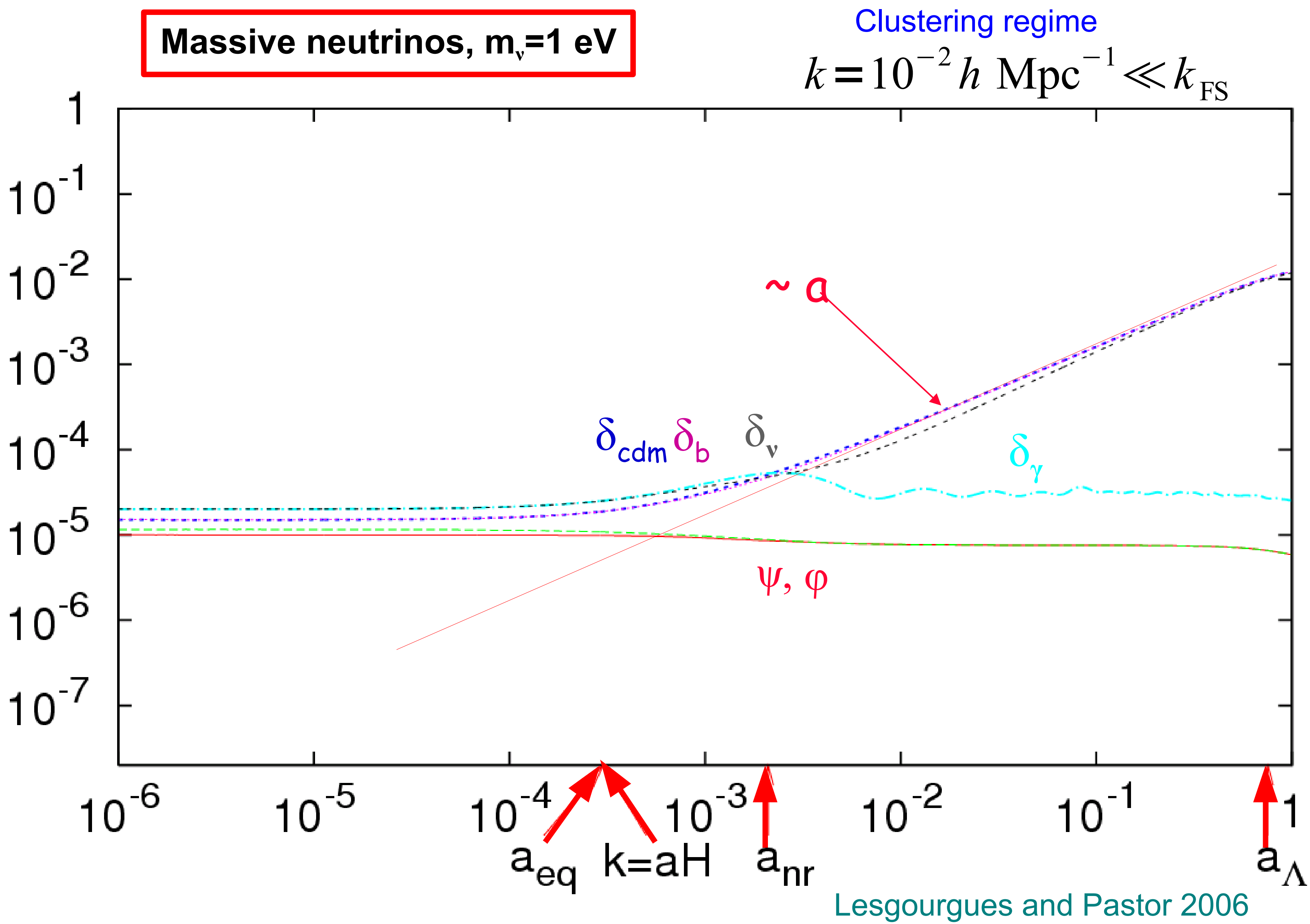
Velocity divergence

$$\sigma_{ij} \equiv \frac{1}{\bar{\rho}_\nu} \int d^3 \boldsymbol{p} \frac{p_i p_j}{a^2 m_\nu^2} (\delta f)$$

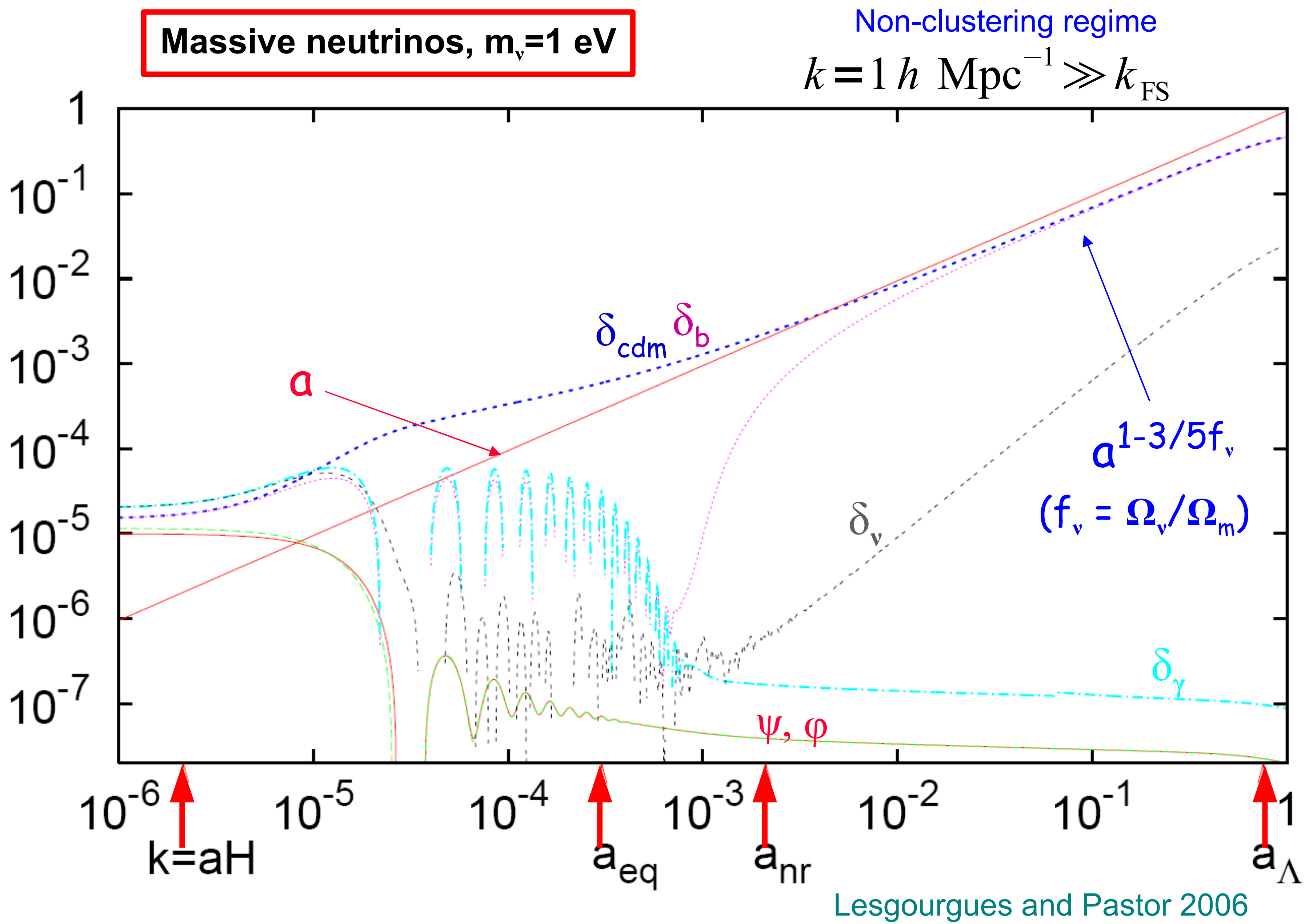
Pressure and anisotropic stress

$$\vdots$$

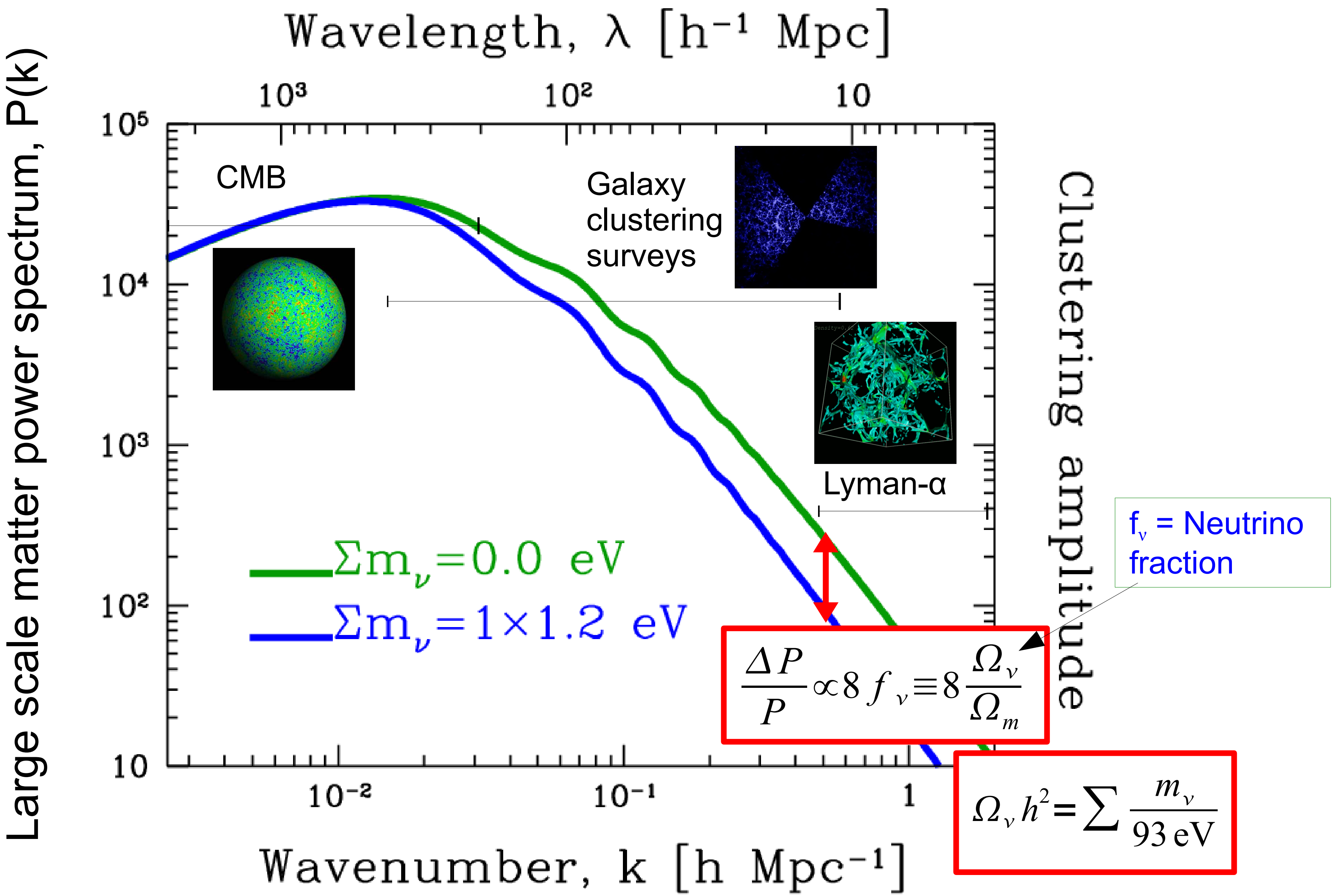
Give rise to free-streaming behaviour



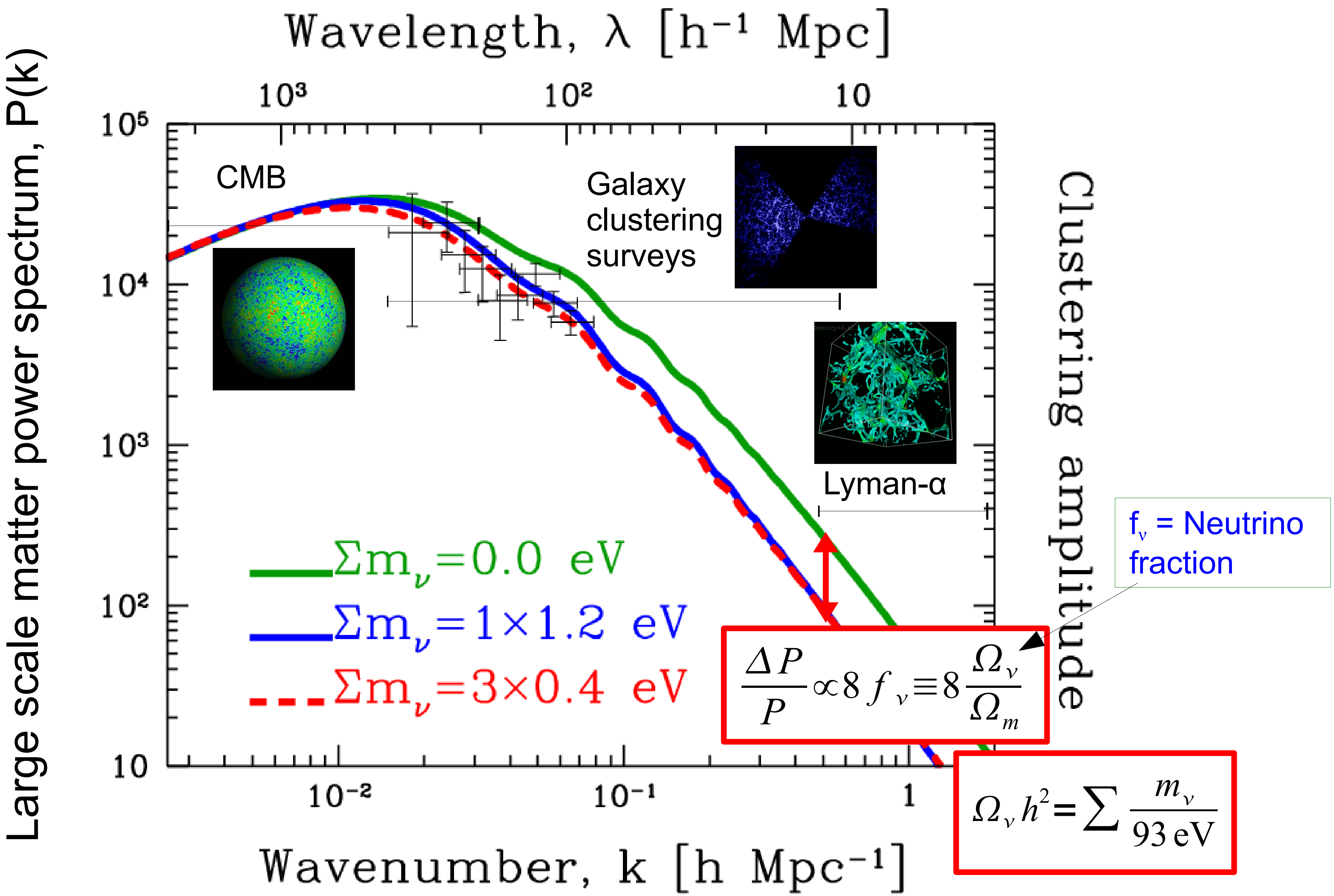
Massive neutrinos, m_ν=1 eV
Clustering regime
$k = 10^{-2}\,h\ \mathrm{Mpc}^{-1} \ll k_{FS}$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
~ a
δ_{cdm} δ_b
δ_ν
δ_γ
ψ, φ
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
a_{eq}
k=aH
a_{nr}
a_Λ
Lesgourgues and Pastor 2006



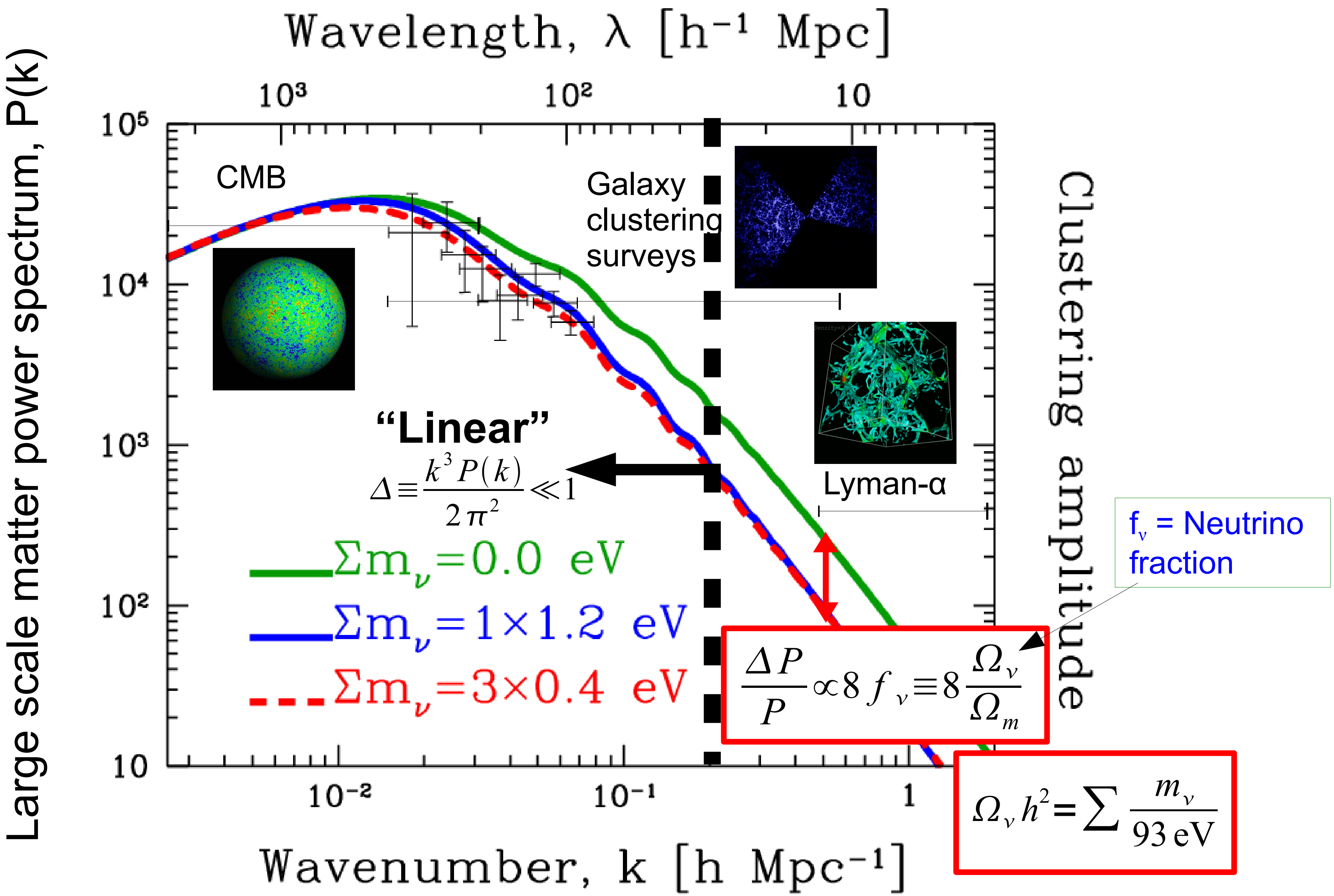
Massive neutrinos, m_ν=1 eV
Non-clustering regime
$k = 1\,h\ \mathrm{Mpc}^{-1} \gg k_{FS}$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
δ_{cdm} δ_b
a
$a^{1-3/5 f_\nu}$
($f_\nu = \Omega_\nu/\Omega_m$)
δ_ν
δ_γ
ψ, φ
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
k=aH
a_{eq}
a_{nr}
a_Λ
Lesgourgues and Pastor 2006



Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
10⁵
10⁴
10³
10²
10
10⁻²
10⁻¹
1
Large scale matter power spectrum, P(k)
Wavenumber, k [h Mpc⁻¹]
Clustering amplitude
CMB
Galaxy clustering surveys
Lyman-α
$\Sigma m_\nu = 0.0$ eV
$\Sigma m_\nu = 1\times 1.2$ eV
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$

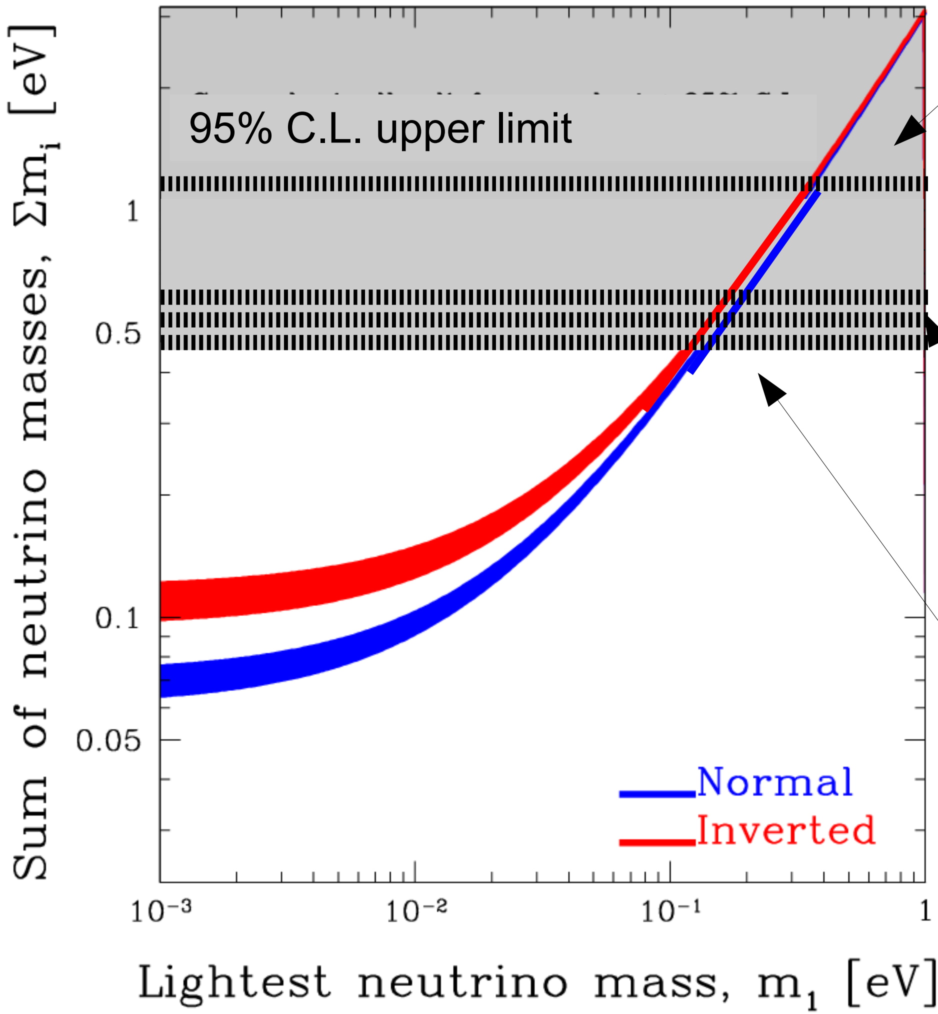


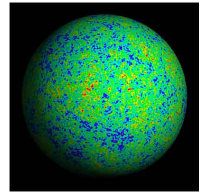
Wavelength, λ [h⁻¹ Mpc]
10³
10²
10
10⁵
10⁴
10³
10²
10
10⁻²
10⁻¹
1
Wavenumber, k [h Mpc⁻¹]
Large scale matter power spectrum, P(k)
Clustering amplitude
CMB
Galaxy clustering surveys
Lyman-α
Σmν=0.0 eV
Σmν=1×1.2 eV
Σmν=3×0.4 eV
fν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$



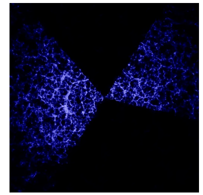
Wavelength, λ [h⁻¹ Mpc]
Large scale matter power spectrum, P(k)
CMB
Galaxy clustering surveys
Lyman-α
"Linear"
$\Delta \equiv \frac{k^3 P(k)}{2\pi^2} \ll 1$
$\Sigma m_\nu = 0.0$ eV
$\Sigma m_\nu = 1\times1.2$ eV
$\Sigma m_\nu = 3\times0.4$ eV
Clustering amplitude
f_ν = Neutrino fraction
$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$
$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$
Wavenumber, k [h Mpc⁻¹]

Present status...

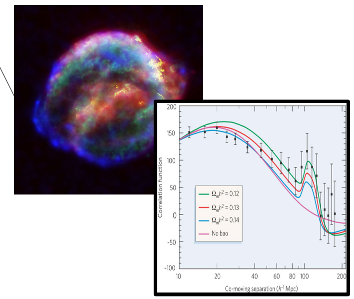




WMAP5 only
Dunkley et al. 2008

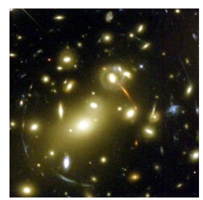


+ Galaxy clustering
Reid et al. 2009



+ Galaxy + SN + HST
Reid et al. 2009

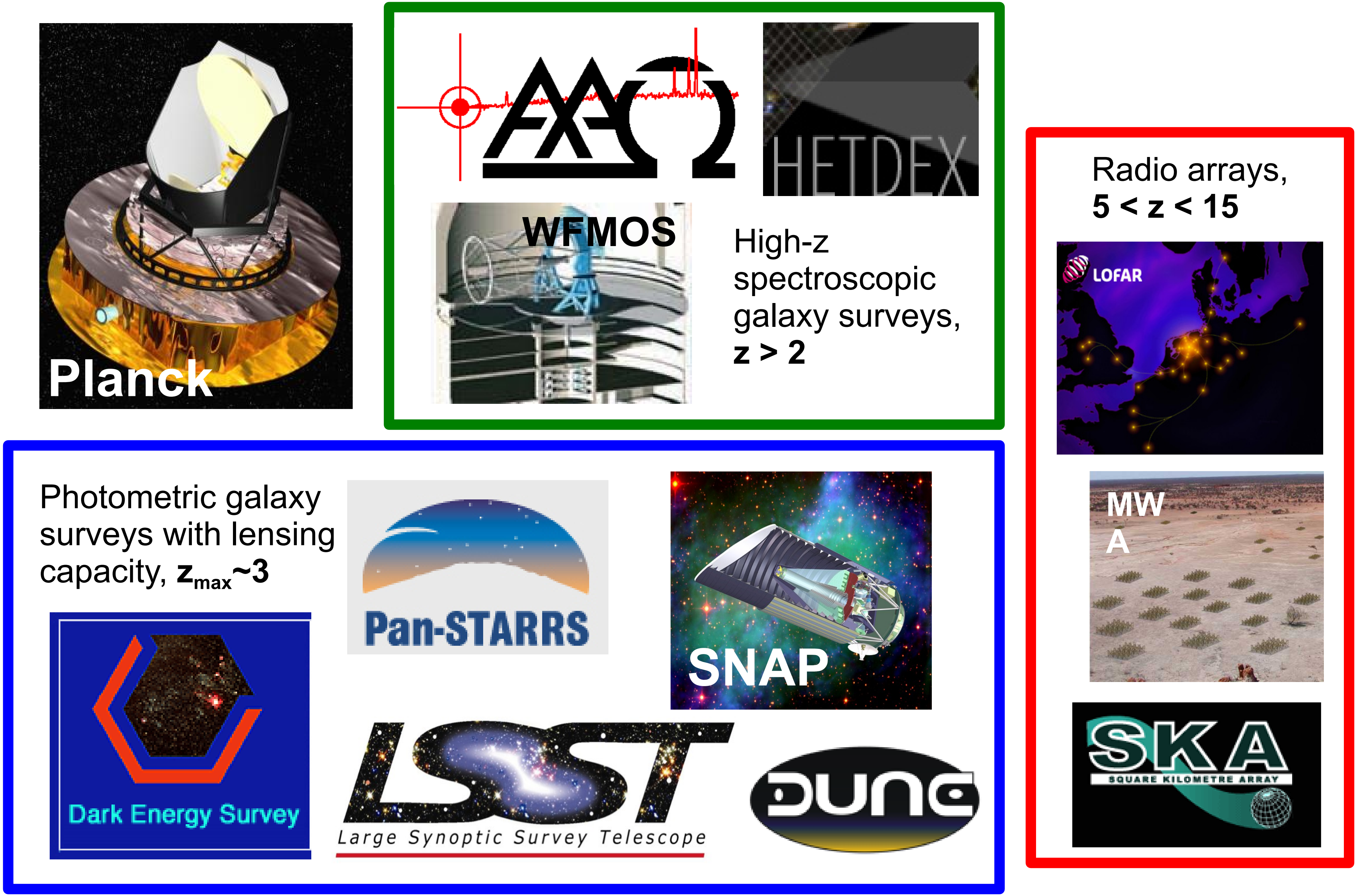
Break degenercies



+ Weak lensing
Tereno et al. 2008
Ichiki et al. 2008

... and many more.

2. What we can do in the future…


Planck
AAΩ
HETDEX
WFMOS
High-z spectroscopic galaxy surveys, z > 2
Radio arrays, 5 < z < 15
LOFAR
MW
A
SKA
SQUARE KILOMETRE ARRAY
Photometric galaxy surveys with lensing capacity, z_{max}~3
Pan-STARRS
SNAP
Dark Energy Survey
LSST
Large Synoptic Survey Telescope
DUNE

Possible new techniques...

- **Weak lensing**
  - of galaxies
  - of the CMB

Abazajian & Dodelson 2002
Song & Knox 2004
Hannestad, Tu & Y^3W 2006
Kitching et al. 2008

Lesgourgues et al. 2006
Perotto, Lesgourgues, Hannestad, Tu & Y^3W, 2006

- **21 cm emission**

Mao et al. 2008
Pritchard & Pierpaoli 2008
Metcalf 2009

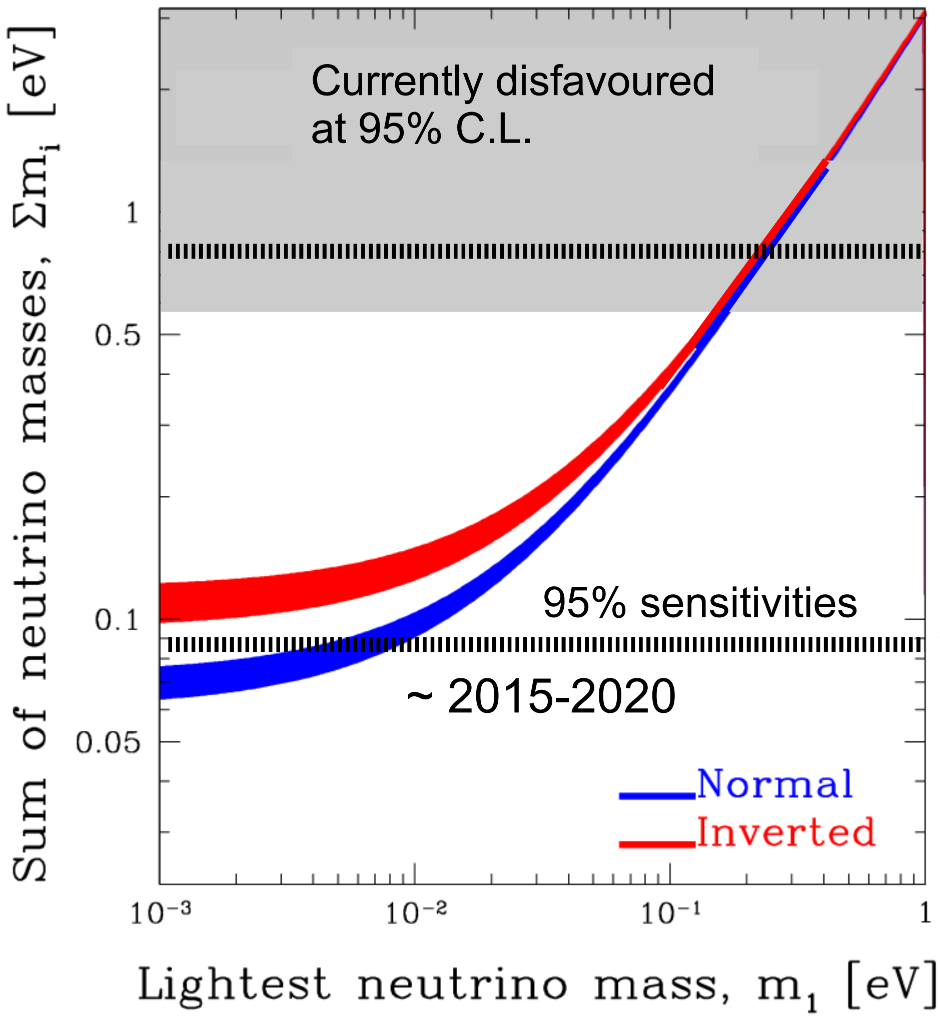
- **ISW effect**

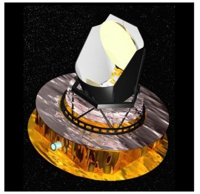
Ichikawa & Takahashi 2005
Lesgourgues, Valkenburg & Gaztañaga 2007

- **Cluster abundance**

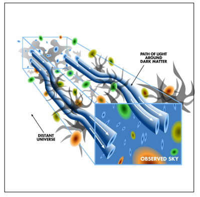
Wang et al. 2005

Projected sensitivities...





Planck (1 year)

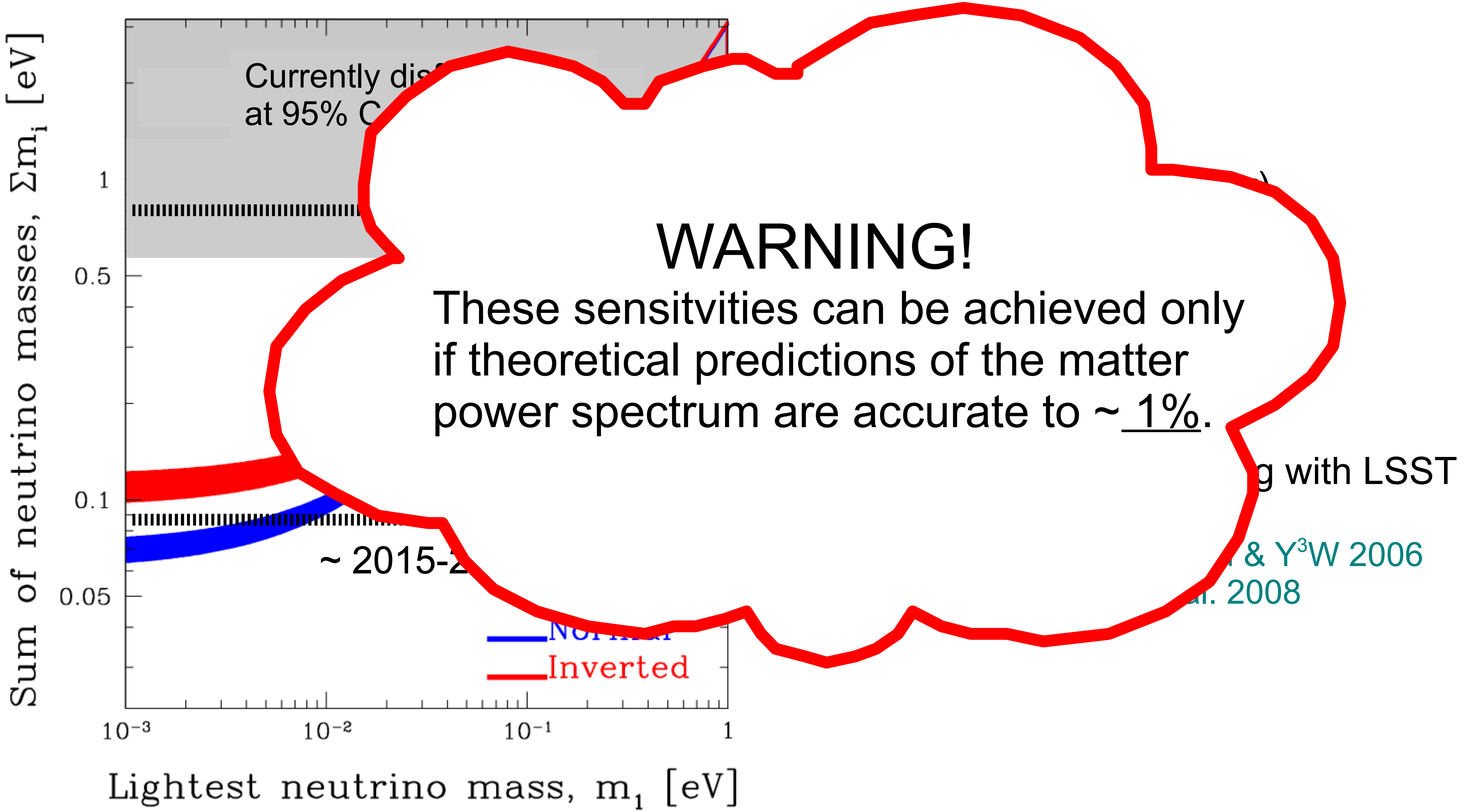


+ Weak lensing with LSST (tomography)

Hannestad, Tu & Y^3W 2006

Kitching et al. 2008

Projected sensitivities...

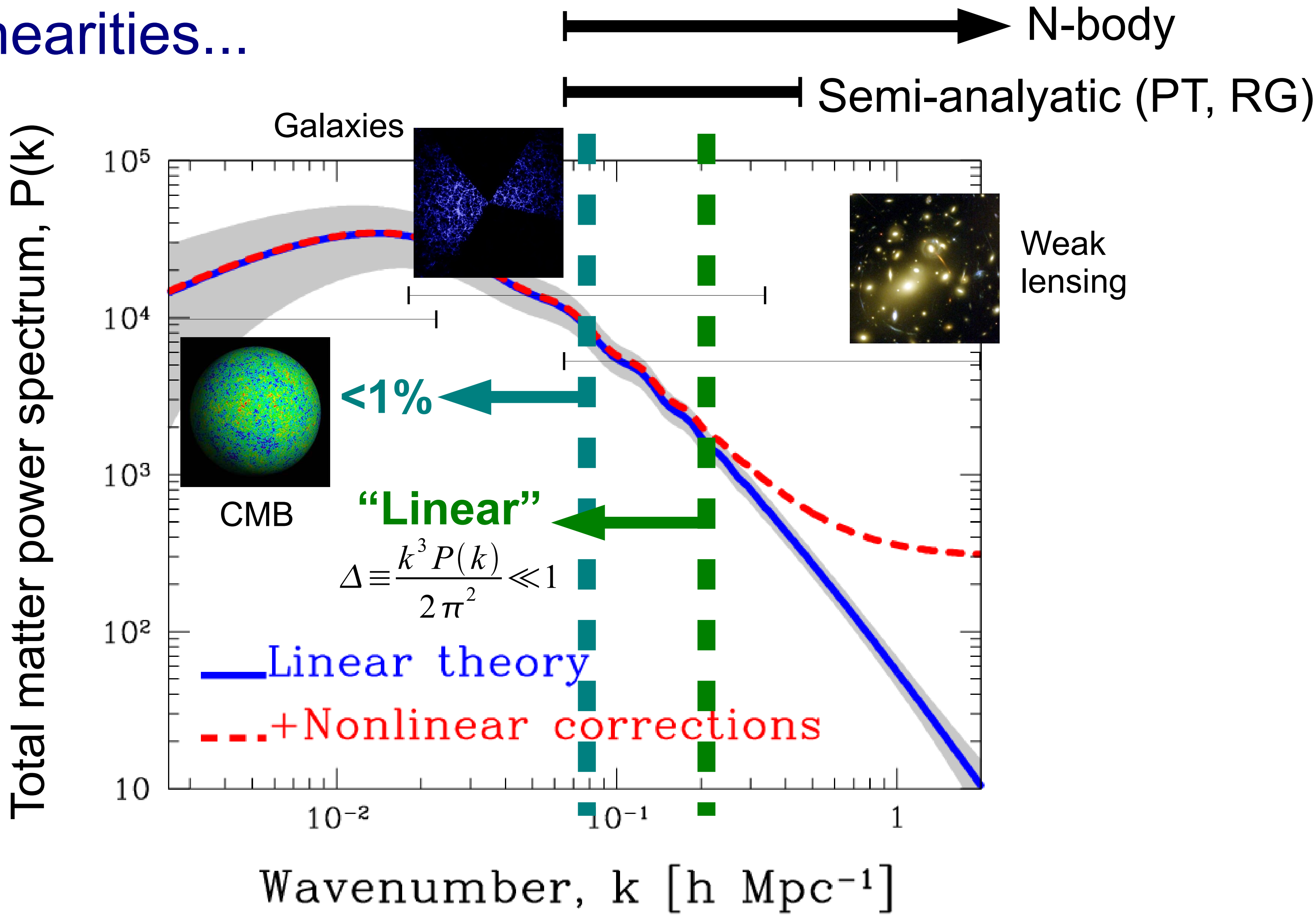


WARNING!

These sensitvities can be achieved only if theoretical predictions of the matter power spectrum are accurate to ~ 1%.

3. The nonlinear matter power spectrum...

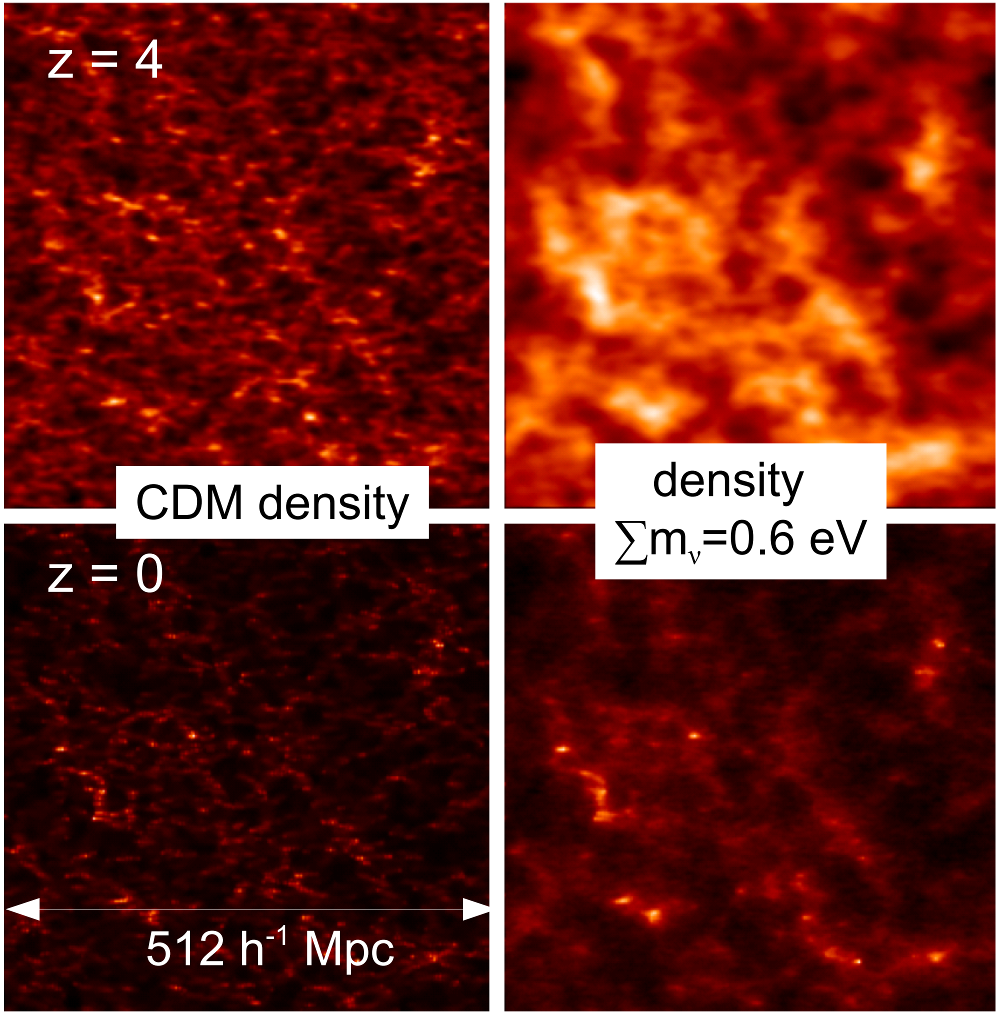
Nonlinearities...



N-body simulations with massive neutrinos...

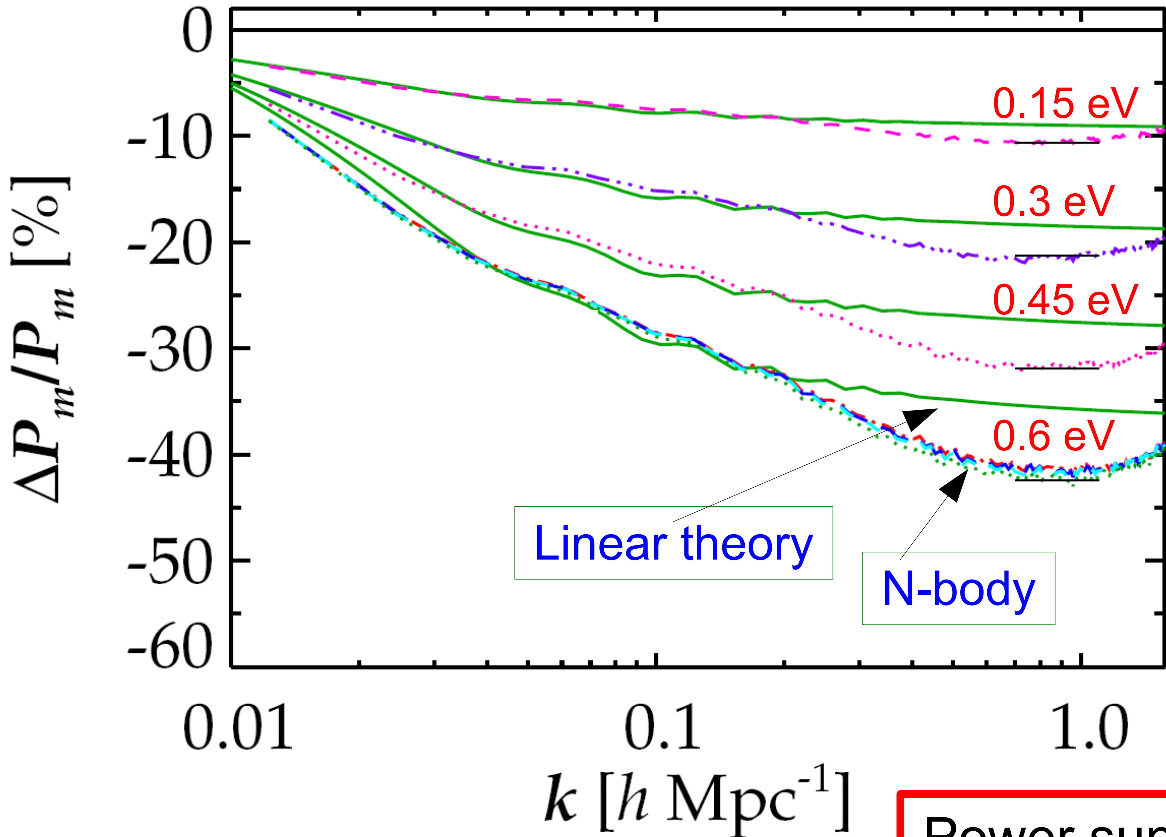
- Particle representation for **both** CDM and neutrinos.
  - Modified GADGET-2.
  - **Neutrino particles** drawn from Fermi-Dirac distribution.

Brandbyge, Hannestad, Haugbølle & Thomsen 2008
Brandbyge and Hannestad 2008, 2009



Change in the total matter power spectrum relative to the $f_\nu = 0$ case:

$$\frac{\Delta P_m}{P_m} \equiv \frac{P_{f_\nu \neq 0}(k) - P_{f_\nu = 0}(k)}{P_{f_\nu = 0}(k)}$$



Linear perturbation theory:

$$\frac{\Delta P_m}{P_m} \sim 8 \frac{\Omega_\nu}{\Omega_m}$$

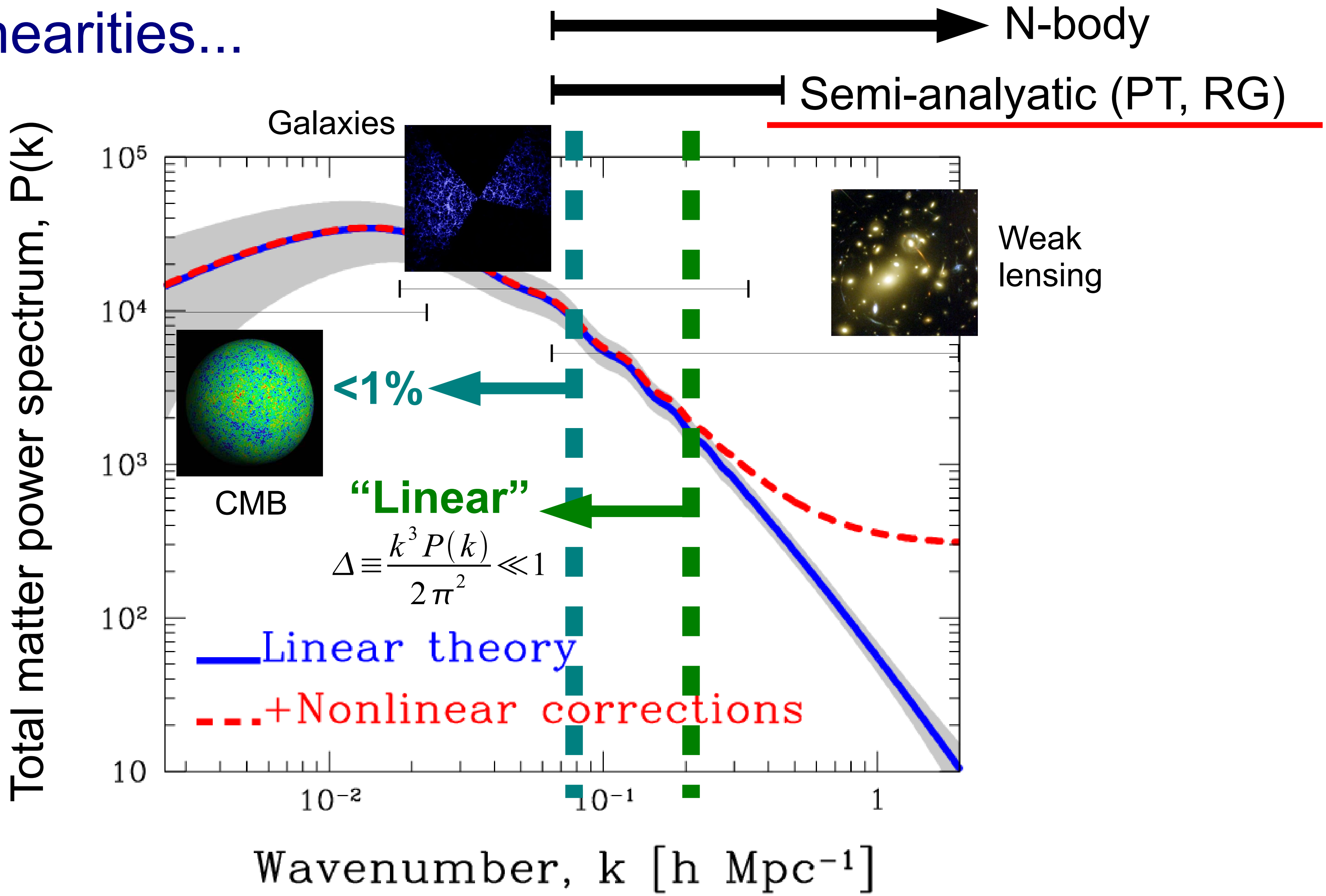
With nonlinear corrections:

$$\frac{\Delta P_m}{P_m} \sim 9.8 \frac{\Omega_\nu}{\Omega_m}$$

Brandbyge, Hannestad, Haugbølle & Thomsen 2008

Power suppression due to neutrino free-streaming is larger than predicted by linear perturbation theory.

Nonlinearities...



Perturbation theory and resummation/RG techniques...

- Going beyond linear perturbation theory?

✓ Nonlinear correction

CDM

$$\dot{\delta}_c + \theta_c = 0$$

Linearised continuity eqn

$$\dot{\theta}_c + H\theta_c + \nabla^2 \Phi = 0$$

Linearised Euler eqn

× **No** nonlinear correction

Neutrinos

$$\frac{\partial(\delta f)}{\partial \tau} + \frac{\boldsymbol{p}}{m_\nu a} \cdot \nabla(\delta f) - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{p}} = 0$$

Linearised Vlasov eqn

But see
Shoji & Komatsu 2009
Y^3W in prep

$$\nabla^2 \Phi = \frac{3}{2} H^2 \Omega_m [f_c \delta_c + f_\nu \delta_\nu]$$

Poisson eqn

Corrections to the CDM component...

- Fluid description (linear):

Continuity eqn

$$\dot{\delta}_c(\boldsymbol{k},\tau)+\theta_c(\boldsymbol{k},\tau)= 0$$

Euler eqn

$$\dot{\theta}_c(\boldsymbol{k},\tau)+H\,\theta_c(\boldsymbol{k},\tau)-k^2\Phi(\boldsymbol{k},\tau)= 0$$

Poisson eqn

$$k^2\Phi=-\frac{3}{2}H^2\Omega_m[f_c\delta_c+f_\nu\delta_\nu]$$

δ_c = CDM density perturbations
δ_ν = ν density perturbations
θ_c = Divergence of velocity field

Corrections to the CDM component...

- Fluid description (incl. **some** nonlinear terms):

Vertex

$$\gamma_{121}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\equiv\delta_D(\boldsymbol{k}-\boldsymbol{q}_{12})\frac{\boldsymbol{q}_{12}\cdot\boldsymbol{q}_1}{q_1^2}$$

Continuity eqn

$$\dot{\delta}_c(\boldsymbol{k},\tau)+\theta_c(\boldsymbol{k},\tau)= \underline{-\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2\gamma_{121}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\theta_c(\boldsymbol{q}_1,\tau)\delta_c(\boldsymbol{q}_2,\tau)}$$

Euler eqn

Mode coupling

$$\dot{\theta}_c(\boldsymbol{k},\tau)+H\,\theta_c(\boldsymbol{k},\tau)-k^2\Phi(\boldsymbol{k},\tau)= \underline{-\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2\gamma_{222}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\theta_c(\boldsymbol{q}_1,\tau)\theta_c(\boldsymbol{q}_2,\tau)}$$

Poisson eqn

$$k^2\Phi=-\frac{3}{2}H^2\Omega_m[f_c\delta_c+f_\nu\delta_\nu]$$

Vertex

$$\gamma_{222}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\equiv\delta_D(\boldsymbol{k}-\boldsymbol{q}_{12})\frac{q_{12}^2(\boldsymbol{q}_1\cdot\boldsymbol{q}_2)}{2\,q_1^2 q_2^2}$$

δ_c = CDM density perturbations
δ_ν = ν density perturbations
θ_c = Divergence of velocity field

Starting point of **most** semi-analytic calculations in the literature.

Standard perturbation theory...

- Solve by perturbative expansion:

$$\varphi(\boldsymbol{k},\tau) \equiv \begin{pmatrix} \delta_c(\boldsymbol{k},\tau) \\ -\theta_c(\boldsymbol{k},\tau)/H \end{pmatrix} \qquad \varphi(\boldsymbol{k},\tau) = \sum_{m=1}^{\infty} \varphi^{(n)}(\boldsymbol{k},\tau)$$

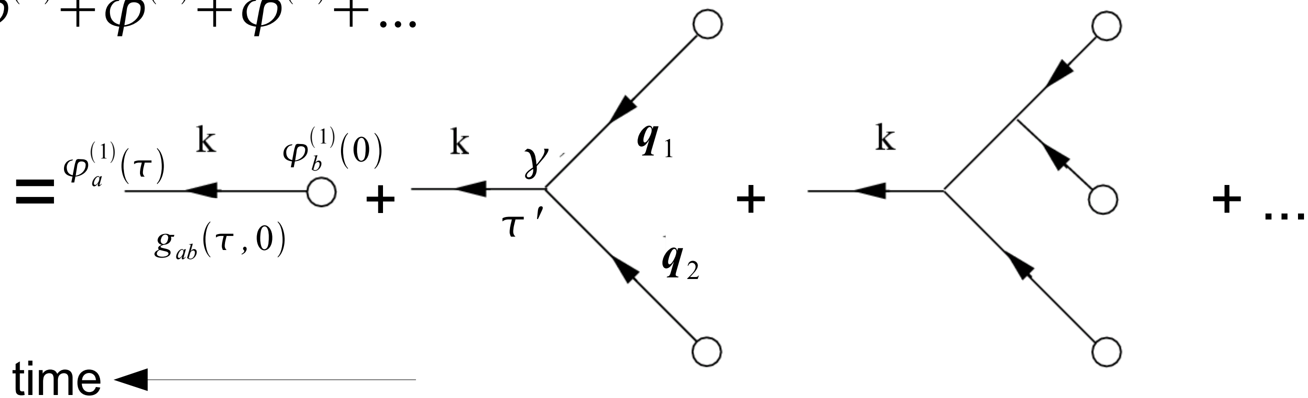
- nth order solution:

$$\varphi_a^{(n)}(\boldsymbol{k},\tau) = g_{ab}(\tau,0)\,\varphi_b^{(n)}(\boldsymbol{k},0) + \int d^3\boldsymbol{q}_1 \int d^3\boldsymbol{q}_2 \int_0^{\tau} d\tau'\, g_{ab}(\tau,\tau')\,\gamma_{bcd}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2) \sum_{m=1}^{n-1} \varphi_c^{(n-m)}(\boldsymbol{q}_1,\tau')\,\varphi_d^{(m)}(\boldsymbol{q}_2,\tau')$$

Crocce & Scoccimarro 2006
Matarrese & Pietroni 2007

$$\varphi(\boldsymbol{k}) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \ldots$$

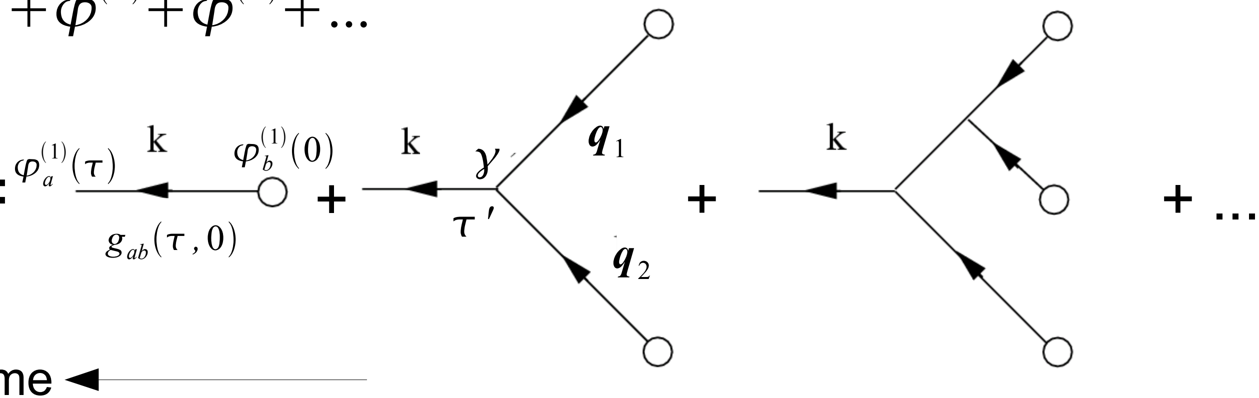
Density/
Velocity =



Crocce & Scoccimarro 2006
Matarrese & Pietroni 2007

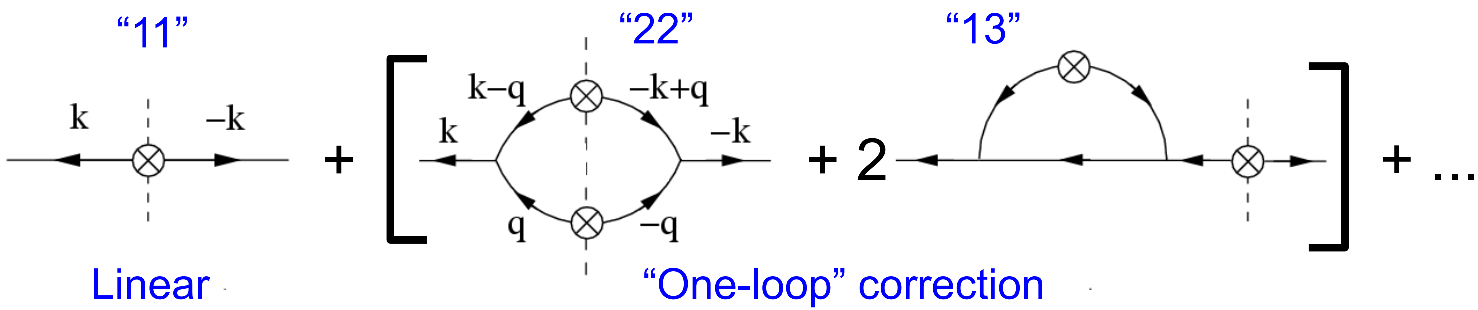
$$\varphi(\boldsymbol{k}) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \dots$$

Density/ Velocity =



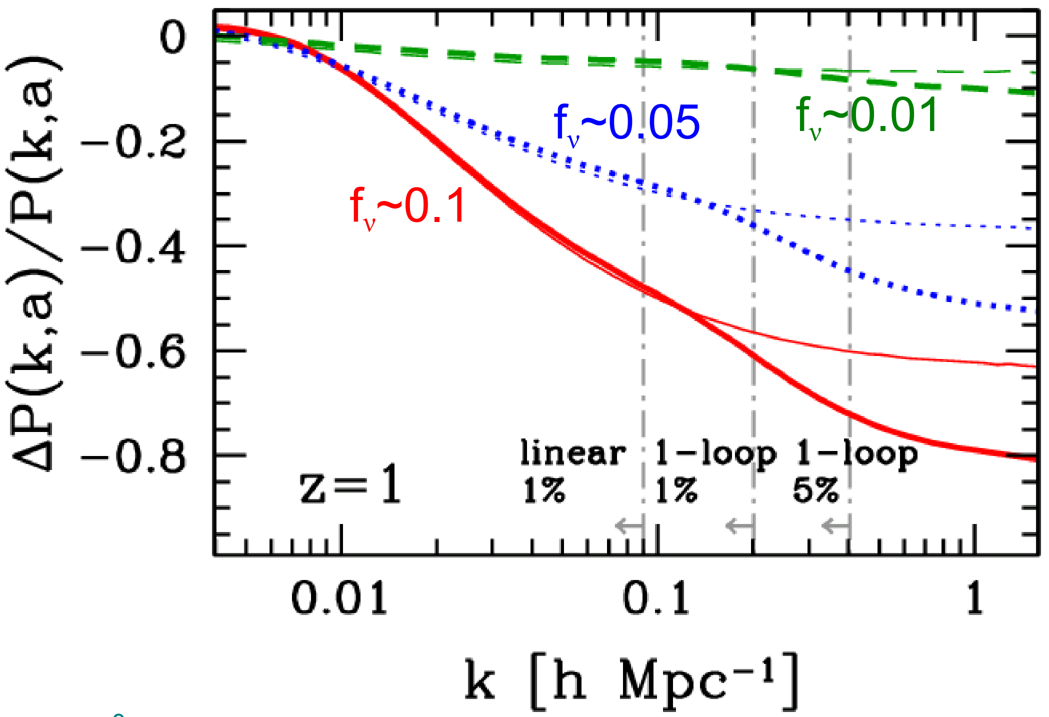
$$P(k)\,\delta_D(\boldsymbol{k}+\boldsymbol{k}') \equiv \langle \varphi(\boldsymbol{k})\,\varphi(\boldsymbol{k}')\rangle = \langle \varphi^{(1)}\varphi^{(1)}\rangle + [\langle \varphi^{(2)}\varphi^{(2)}\rangle + 2\langle \varphi^{(1)}\varphi^{(3)}\rangle] + \dots$$

Power spectrum =



Free-streaming suppression: One-loop corrected...

Thin lines = linear
Thick lines = 1-loop corrected



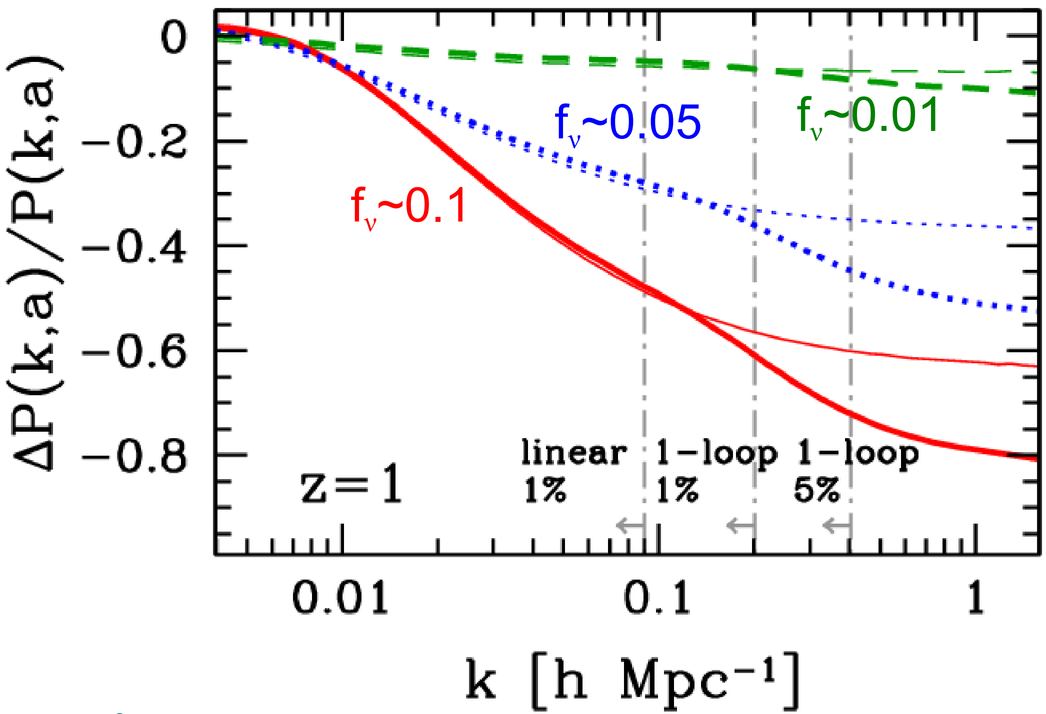
Change in power spectrum relative to the $f_\nu = 0$ case:

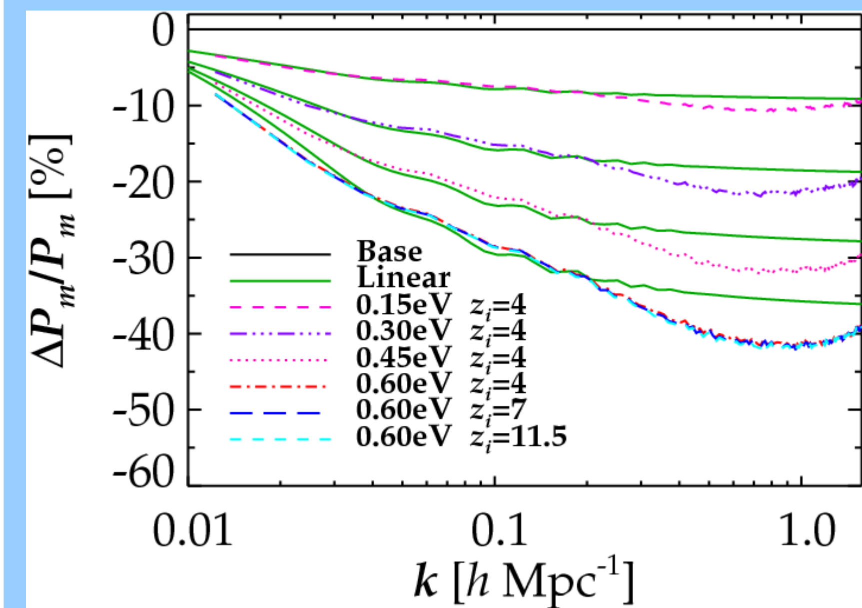
$$\frac{\Delta P}{P} \equiv \frac{P_{f_\nu \neq 0}(k) - P_{f_\nu = 0}(k)}{P_{f_\nu = 0}(k)}$$

Y[3]W 2008
also Saito et al. 2008, 2009

Free-streaming suppression: One-loop corrected...

Thin lines = linear
Thick lines = 1-loop corrected





N-body simulations, Brandbyge et al. 2008

Y[3]W 2008
also Saito et al. 2008, 2009

Resummation and renormalisation group techniques...

- Many schemes have been proposed that go beyond standard perturbation theory:

  Crocce & Scoccimarro 2006, 2008

  Taruya & Hiramatsu 2007

  McDonald 2007

  Matarresse & Pietroni 2007, 2008

  Matsubara 2008

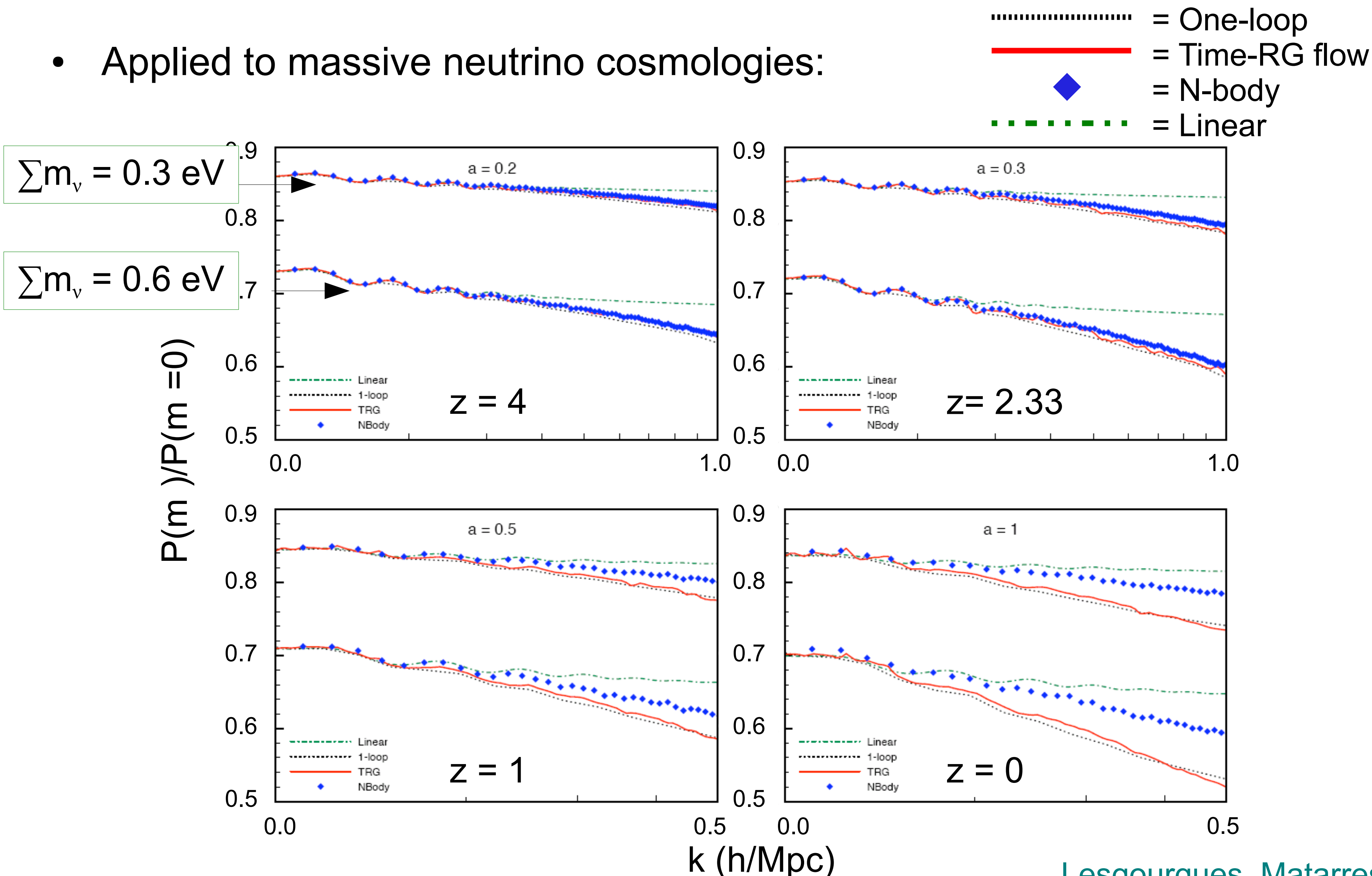
  Valageas 2007

  Pietroni 2008

  Hiramatsu & Taruya 2009

  etc..

- Applied to massive neutrino cosmologies:



Lesgourgues, Matarrese, Pietroni & Riotto 2009

Summary...

- Using the large-scale structure distribution to probe neutrino physics is still fun.
  - We can do **even better** in the future with forthcoming probes/new techniques.

- But we must make sure our theoretical predictions are reliable (1% accurate) at the (nonlinear) scales of interest.
  - N-body simulations are the definitive way to go.
  - Semi-analytic PT & RG techniques are also of some (limited) use.