Neutrino mass and cosmic simulations

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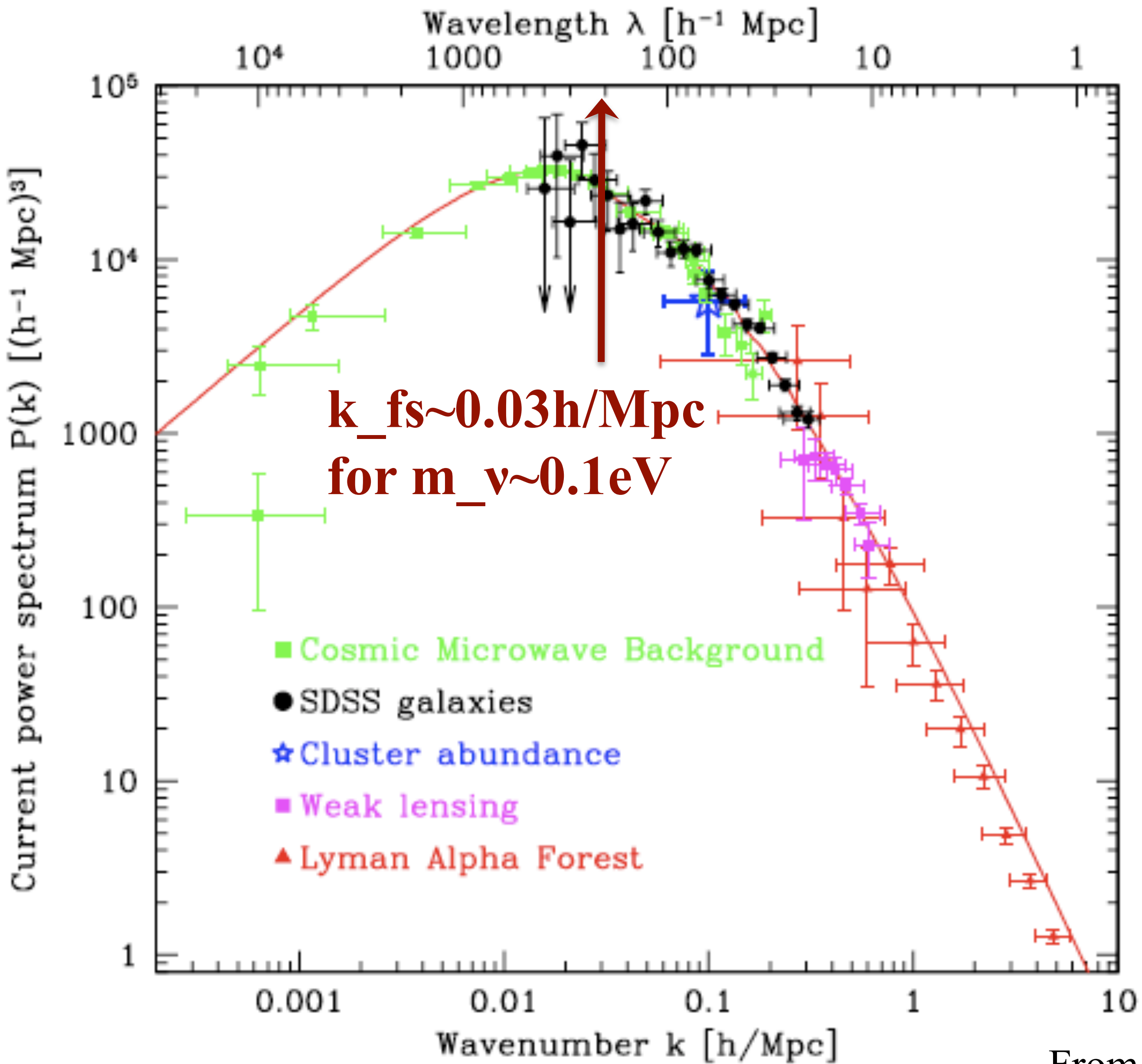
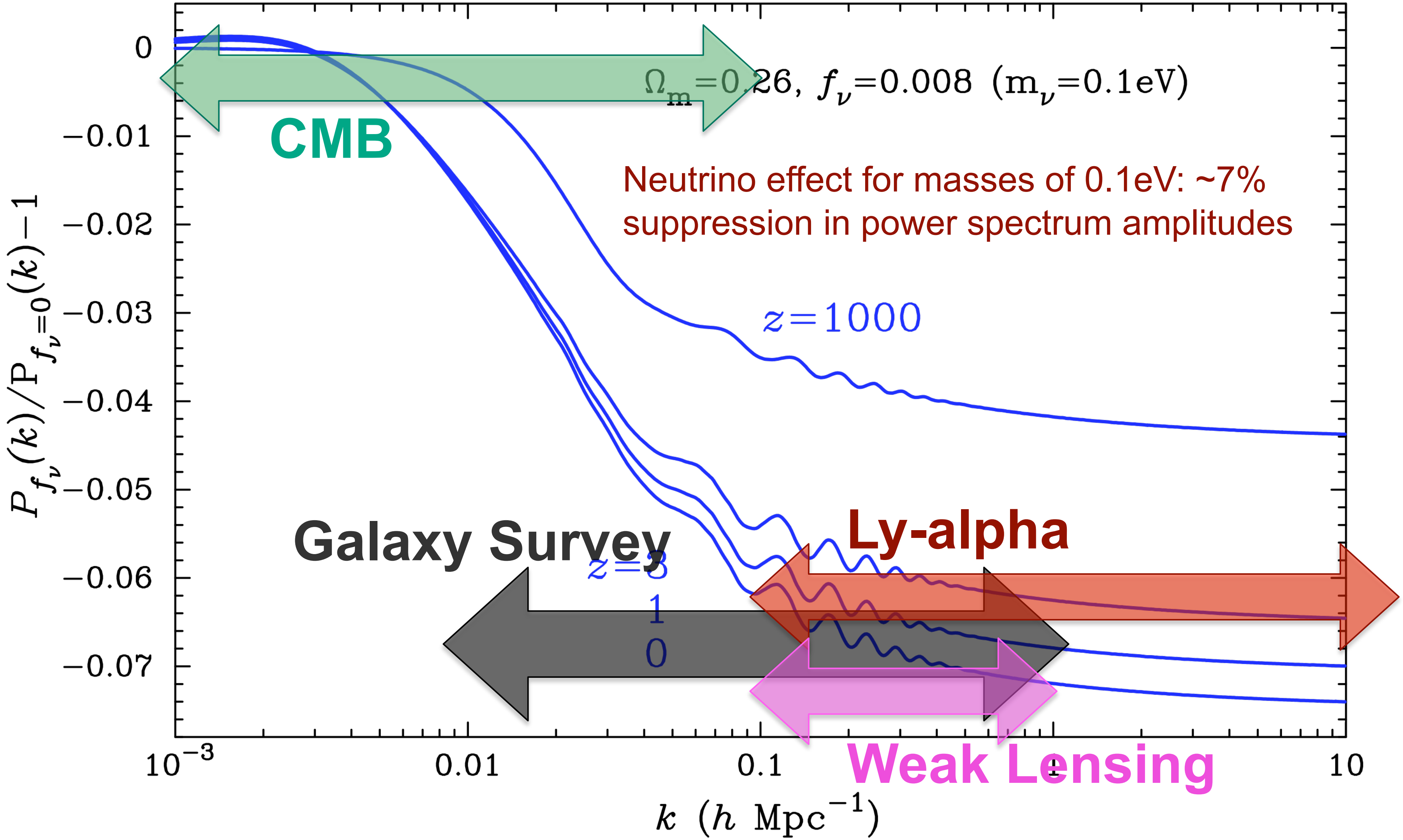
Large-scale structure probes

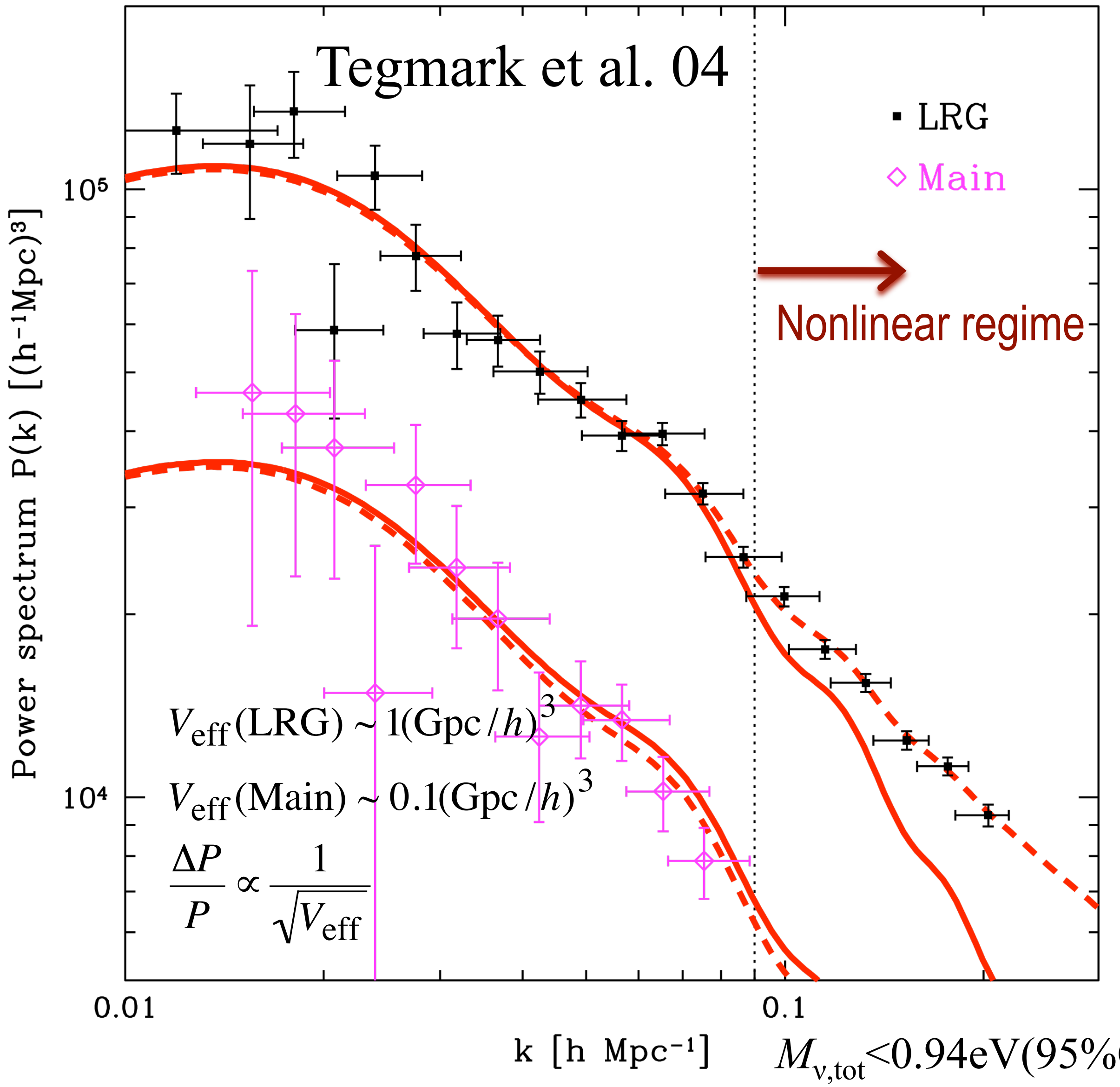


From Tegmark+04

Sensitivity window of each probe



Nonlinearities in structure formation

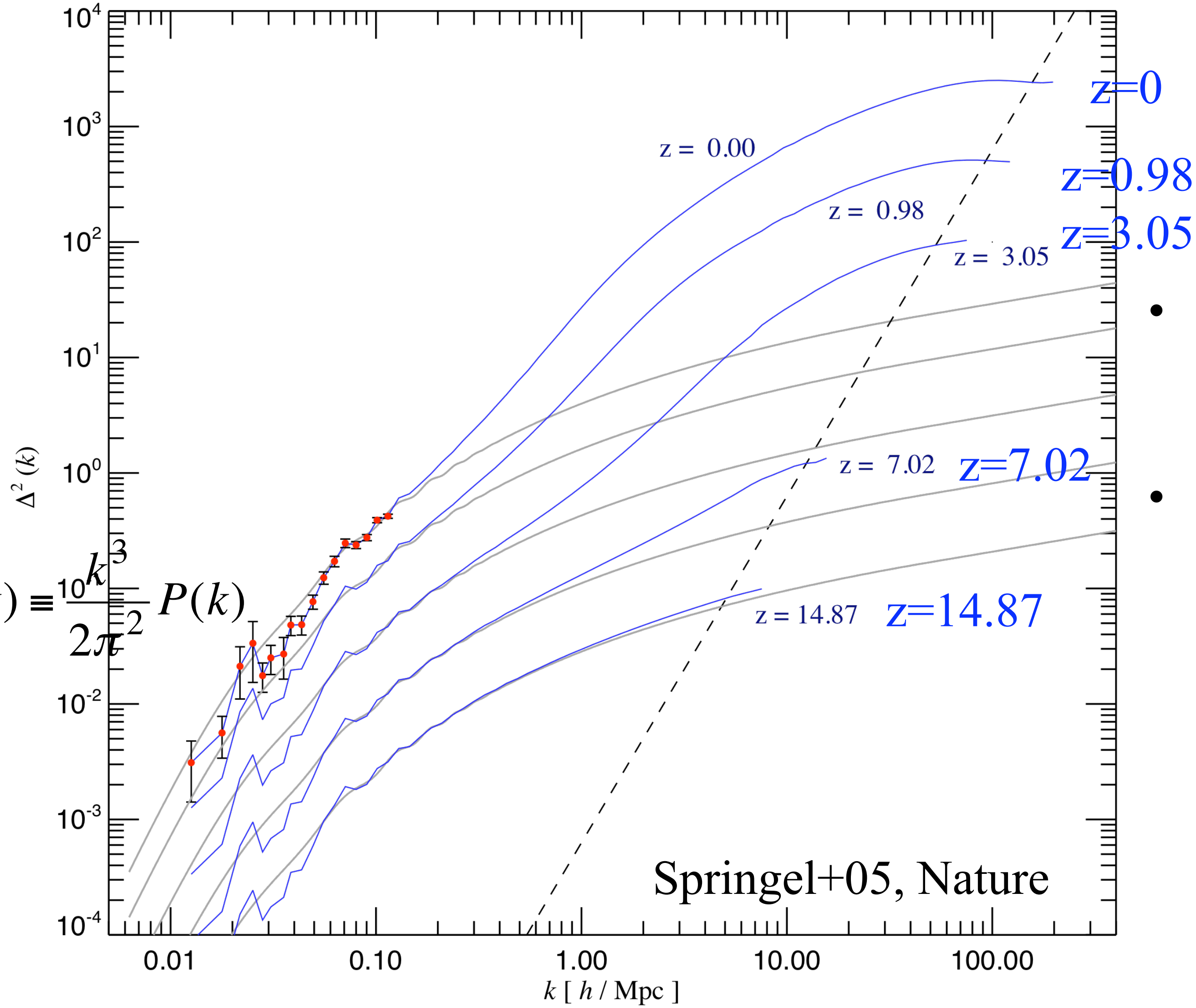


$M_{\nu,\rm tot}$<0.94eV(95%C.L.)

$$P_g(k) = b^2 P_m^L(k) \frac{1 + Q_{\rm nl} k^2}{1 + 1.4k}$$

- The linear theory ceases to be accurate even on these large length scales, k>0.1/Mpc (~50Mpc: δ~O(0.1))
- An empirical model is employed: nuisance parameters to model galaxy biases (also see Reid et al where the halo model was used)

Remark: smaller nonlinearities at higher redshifts



$$\Delta^2(k) \equiv \frac{k^3}{2\pi^2} P(k)$$

- Nonlinear clustering effect is smaller at higher redshifts
- Cosmological probes targeting high-redshift structures (e.g. 21cm) may allow a cleaner interpretation of the measurement

Cosmic simulations for CDM cosmologies

- At large scales and over relevant relevant redshifts, gravity dominates, so need to solve the Vlasov-Poisson equations in the non-relativistic limit
  - Note equations are in 1+6D, and intrinsically nonlinear
  - Use N-body method: sample the phase-space PDF with N-body particles and evolve the system of interacting particles
  - The systematic errors are well understood?

Cosmological Vlasov-Poisson equations

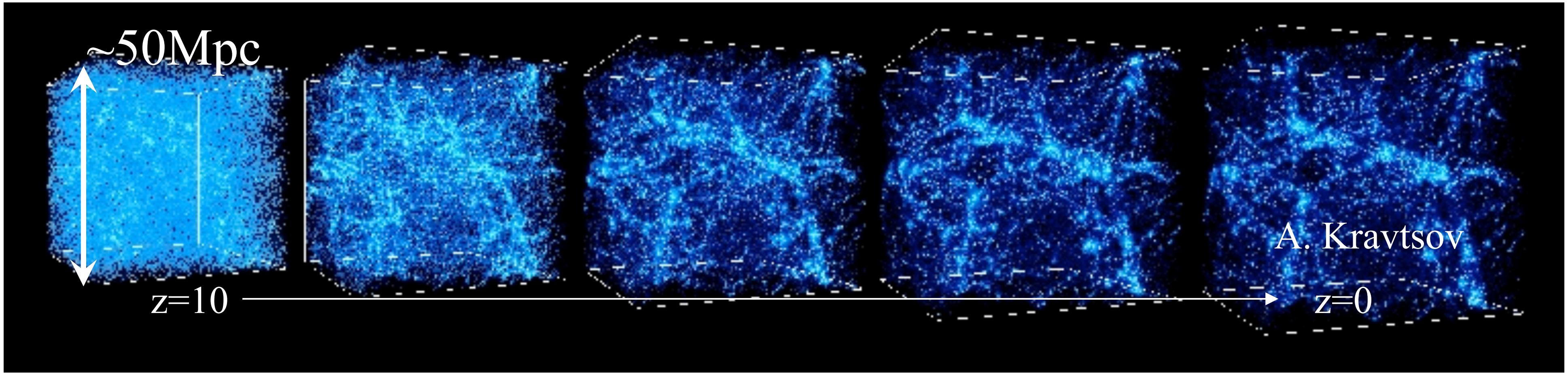
$$\frac{\partial f_i}{\partial t} + \dot{\boldsymbol{x}}\frac{\partial f_i}{\partial \boldsymbol{x}} - \nabla\Phi\frac{\partial f_i}{\partial \boldsymbol{p}} = 0, \quad p = a^2\dot{\boldsymbol{x}}$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}_m \delta_m$$

$$\delta_m(\boldsymbol{x},t) = \rho_m(\boldsymbol{x},t)/\bar{\rho}_m - 1$$

$$\rho_m(\boldsymbol{x},t) = a^{-3}\sum_i m_i \int d^3\boldsymbol{p} f_i(\boldsymbol{x},\boldsymbol{p},t)$$

- At small scales, need to add gas physics and astrophysical feedback
  - Simulations get harder
  - These physics are poorly understood
  - Need to calibrate the simulations with observational constraints. This is doable?
- *Goal: The nonlinear power spectrum at a 1%-level accuracy up to k~1/Mpc*



Gravity-only cosmology code

(from slide of S. Habib at SF09 workshop)

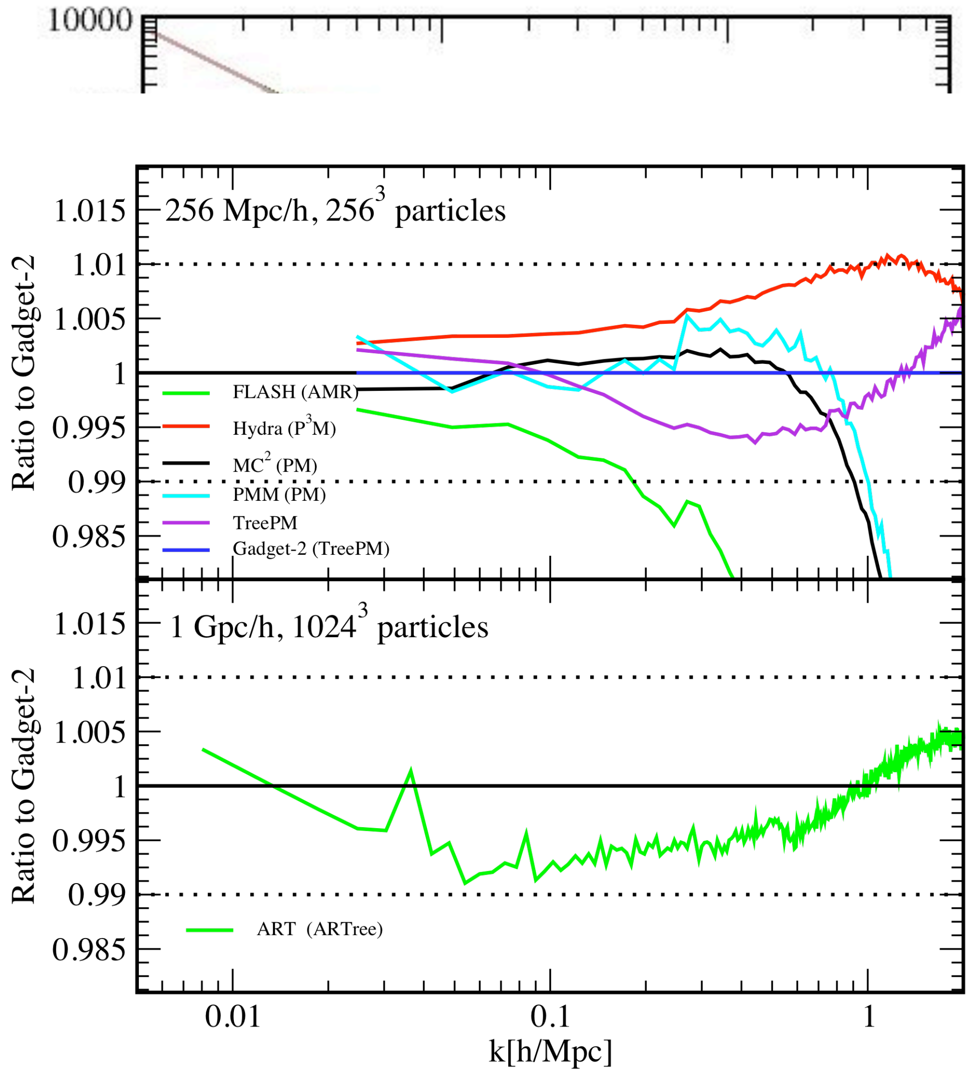
- Initial conditions
  - Simulation begins with the initial conditions well constrained by CMB: (i) Pick the initial power spectrum of CDM perturbations; (ii) Generate a single realization of the density field in k-space, use the Poisson equation to solve for the velocity potential; (iii) use the Zel'dovich (or some other approximation) to move each particles off a grid (quite start, i.e. at a sufficiently high initial redshift)
- Time-stepping
  - Use symplectic/leap-frog integration; first "stream" particles, then compute inter-particle forces, update velocities, do next "stream" and redo
- Force computation
  - Direct particle-particle force evaluation is computationally expensive given large cosmological volume; use approximate tricks (tree, PM, AMR, …) to reduce order of algorithm to NlogN
- Tests
  - Many sources of systematic errors: develop suite of tests for robustness of simulation results, check everything multiple times
- Analysis
  - Compute P(k), etc. Make halo catalogs, merger trees, Mock observational data

The current state-of-art of CDM simulations

The Coyote Universe Project: aimed at calibrating nonlinear power spectrum at 1%-level precision for CDM cosmologies

(Heitmann et al. 07, 08, 09)

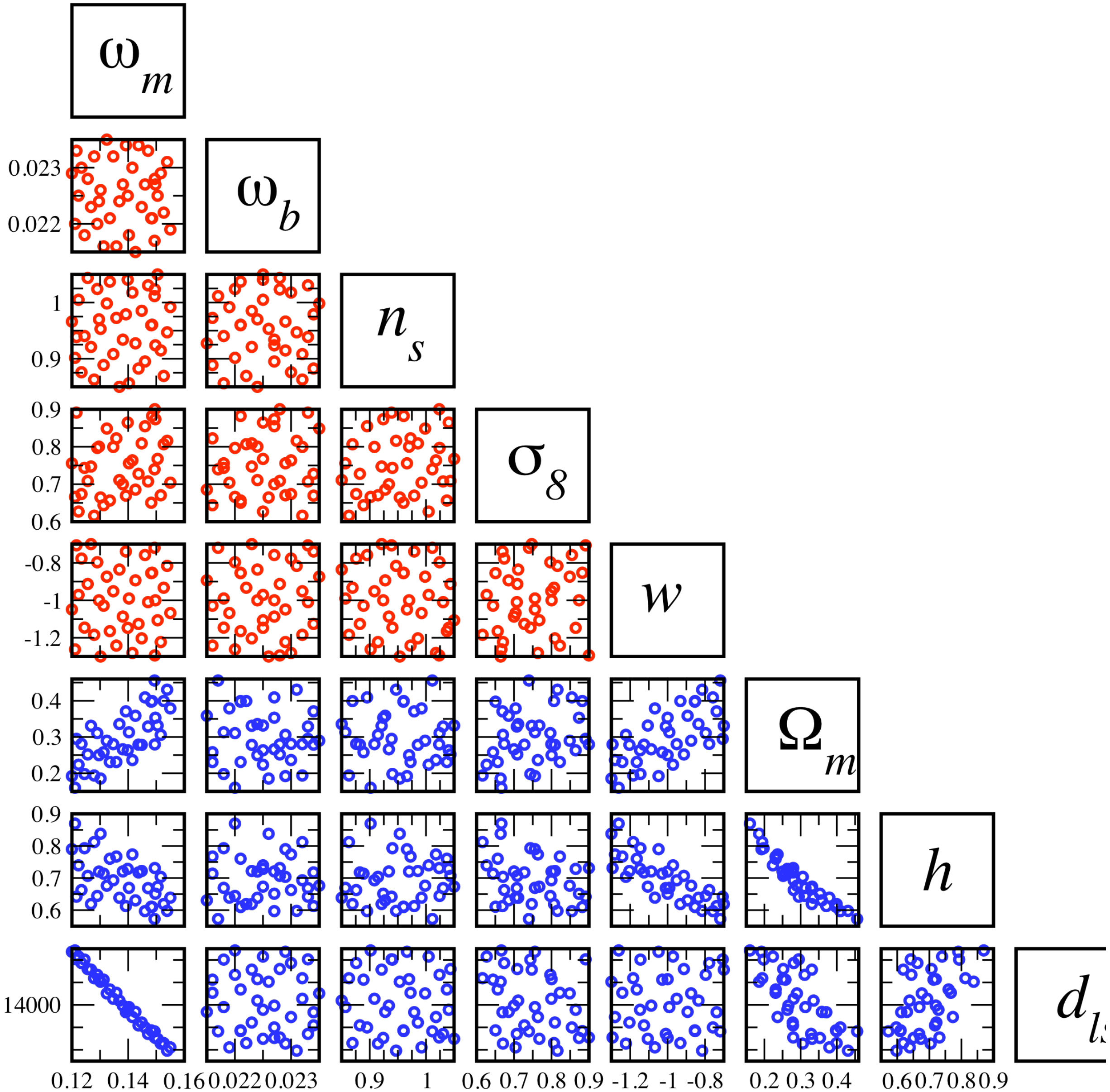
- Code comparison
- Testing sources of systematic errors
  - Initial conditions (initial condition generation, initial redshift)
  - Resolution tests (box size, particle numbers, mass resolution, time stepping)
- Secure parameters: *1Gpc box, 1024^3 particles, z_i~200, TreePM*
- A 1%-level accuracy at k<1/Mpc achieved!
- Fitting formula (Halofit) underestimates by ~5%



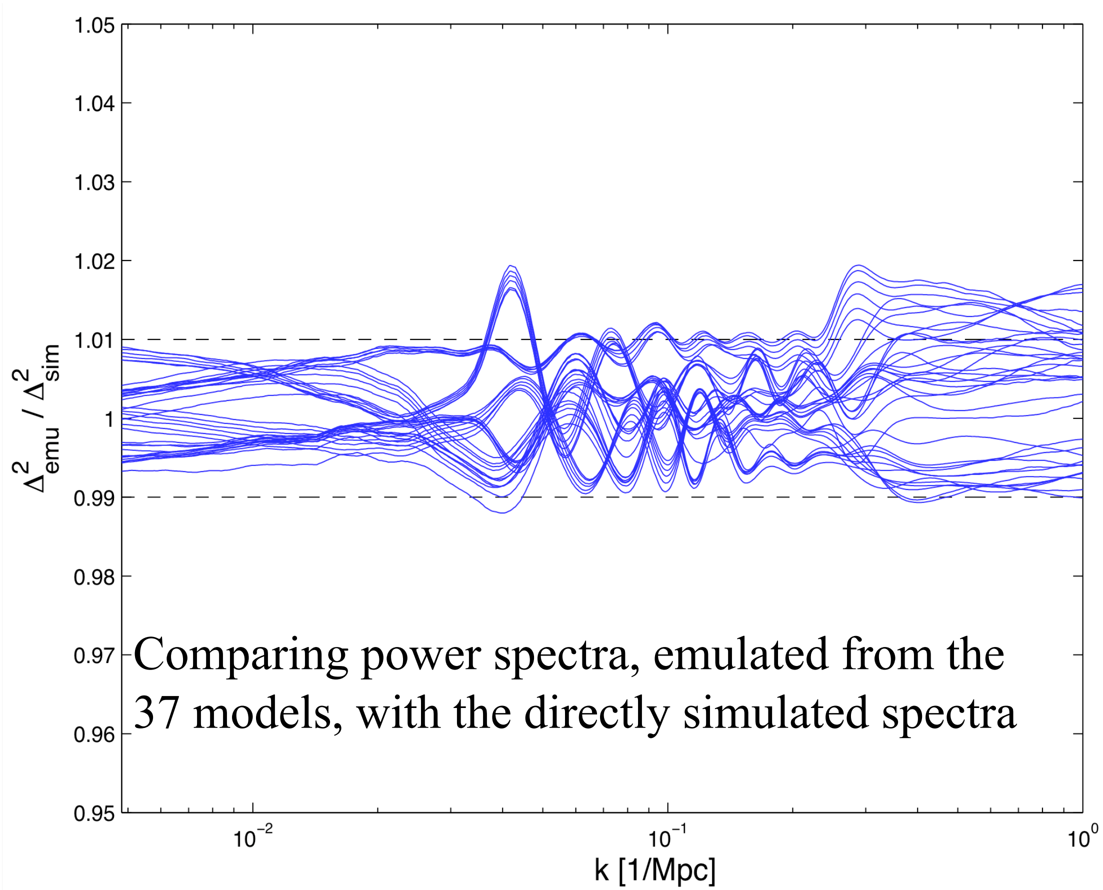
Emulator of nonlinear power spectrum

Heitmann et al.

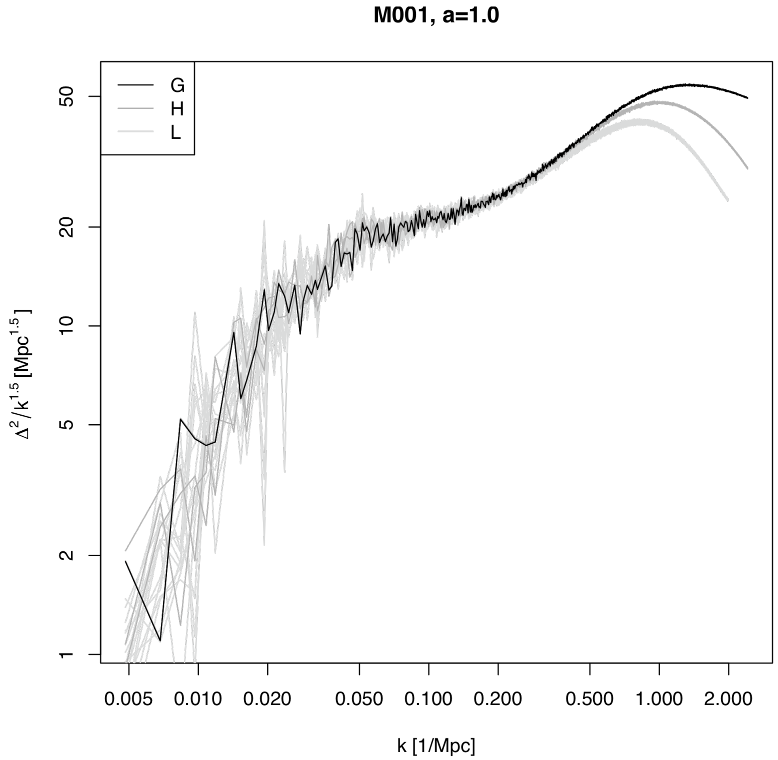
- Selected 37 cosmological models in 5D parameter space (ω_m, ω_b, n_s, σ_8, w), over ranges of parameters allowed by WMAP
- Use the sampling method of the model space based on the Latin hypercubic + orthogonal array design
- Ran about 1000 realizations for the 37 models
  - For each model: 20 realizations (16 low-resolutions, 4 medium resolutions) + 1 high-resolution run
- Linux cluster: 2580 AMD Opeterons (2.6GHz)



Emulator of nonlinear P(k) (contd.)



Comparing power spectra, emulated from the 37 models, with the directly simulated spectra



- Emulate the nonlinear power spectrum at arbitrary point in 5D parameter space by interpolating the 37 model results
- Used the PCA decomposition of the nonlinear P(k), and the Gaussian Process Modeling for the interpolation
- Achieve ~1% accuracy

Analytical approach: Perturbation theory

Jaskiewicz (81), Vishniac (83), Goroff+(86), Suto & Sasaki (91)…

- Assumption: pressure-less and irrotational fluid for CDM + baryon perturbations
- On sub-horizon scales, the self-gravitating dynamics in an expanding universe is described as

$$\frac{\partial\delta}{\partial t} + \frac{1}{a}\nabla\cdot\left[(1+\delta)\vec{v}\right] = 0$$

$$\frac{\partial\vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{a}\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\bar{\rho}_m a^2\delta$$

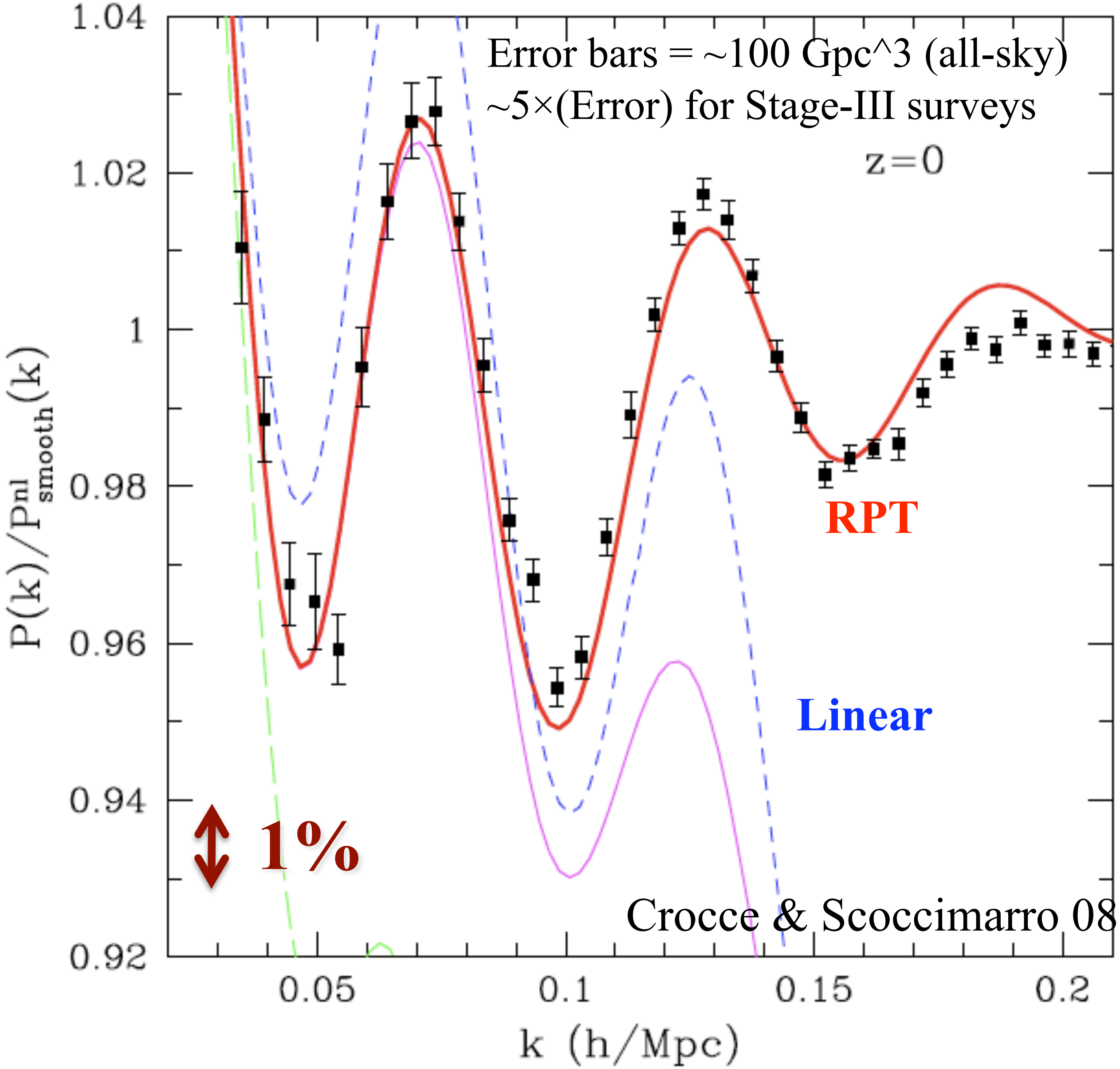
- Perturbative expansion

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots, \quad \vec{v} = \vec{v}^{(1)} + \vec{v}^{(2)} + \vec{v}^{(3)} + \cdots$$

- Compute the nonlinear P(k) including the higher-order corrections

$$P(k) \Leftarrow \left\langle \left(\delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots\right)^2 \right\rangle$$

Analytical approach (contd.)



- Several refined PT modeling developed
- Impressive agreement with the simulations to 1%-level accuracy up to k~0.1/Mpc
- Break down at higher k
- Complementary to N-body simulations

Also see
Jeong & Komatsu 06
Taruya & Hiramatsu 08
Matsubara 08

C+H DM model (CDM + Neutrinos)

- The real universe should have two dark matter components (CDM + neutrinos)

$$\Omega_{\mathrm{m0}} = \Omega_{\mathrm{cdm0}} + \Omega_{\mathrm{baryon0}} + \Omega_{\nu 0}$$

$$f_\nu \equiv \frac{\Omega_{\nu 0}}{\Omega_{\mathrm{m0}}} = \frac{m_{\nu,\mathrm{tot}}}{94.1\mathrm{eV}\Omega_m h^2} > 0.005$$

- The phase space density of relic neutrinos obey the (perturbed) Fermi-Dirac distribution

$$f_\nu(p) = \frac{1}{\exp[p/T]+1}$$

- Neutrinos have large thermal motion (> velocity disp. of galaxy clusters)

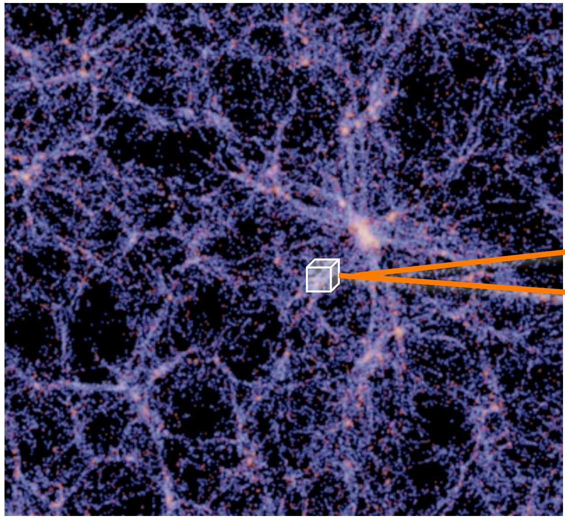
$$\sigma_\nu(z) = \sqrt{\left\langle \frac{p^2}{2m_\nu} \right\rangle} \approx 1800\mathrm{km/s}\left(\frac{m_\nu}{0.1\mathrm{eV}}\right)^{-1}(1+z)$$

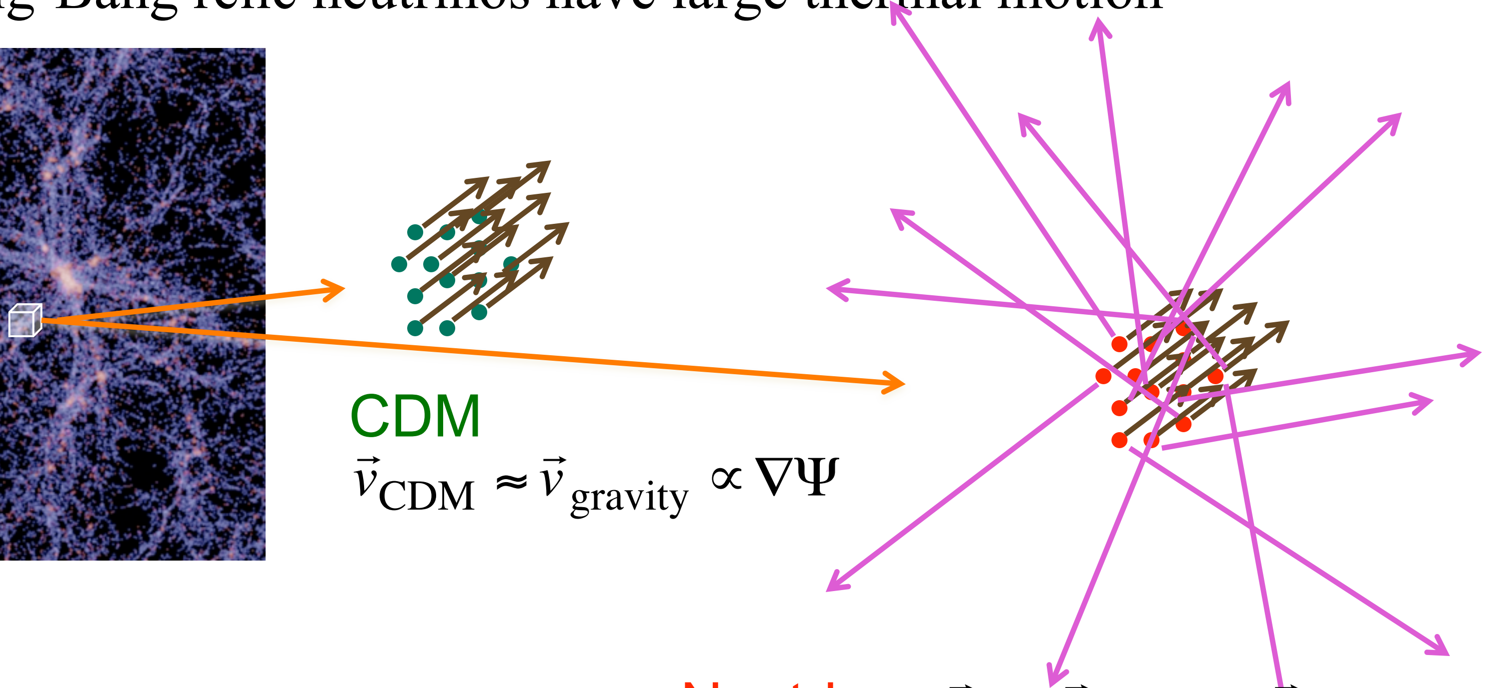
- Structure formation at relevant redshifts is induced by gravity of *total matter*

$$\delta_m = \frac{\bar\rho_c\delta_c + \bar\rho_b\delta_b + \bar\rho_\nu\delta_\nu}{\bar\rho_c + \bar\rho_b + \bar\rho_\nu} \equiv f_c\delta_c + f_b\delta_b + f_\nu\delta_\nu \Rightarrow \nabla^2\Phi = 4\pi G\bar\rho_m a^2\delta_m$$

C+H DM model (CDM + Neutrinos)

- Structure formation in the real universe is caused by CDM + neutrinos
- The Big-Bang relic neutrinos have large thermal motion





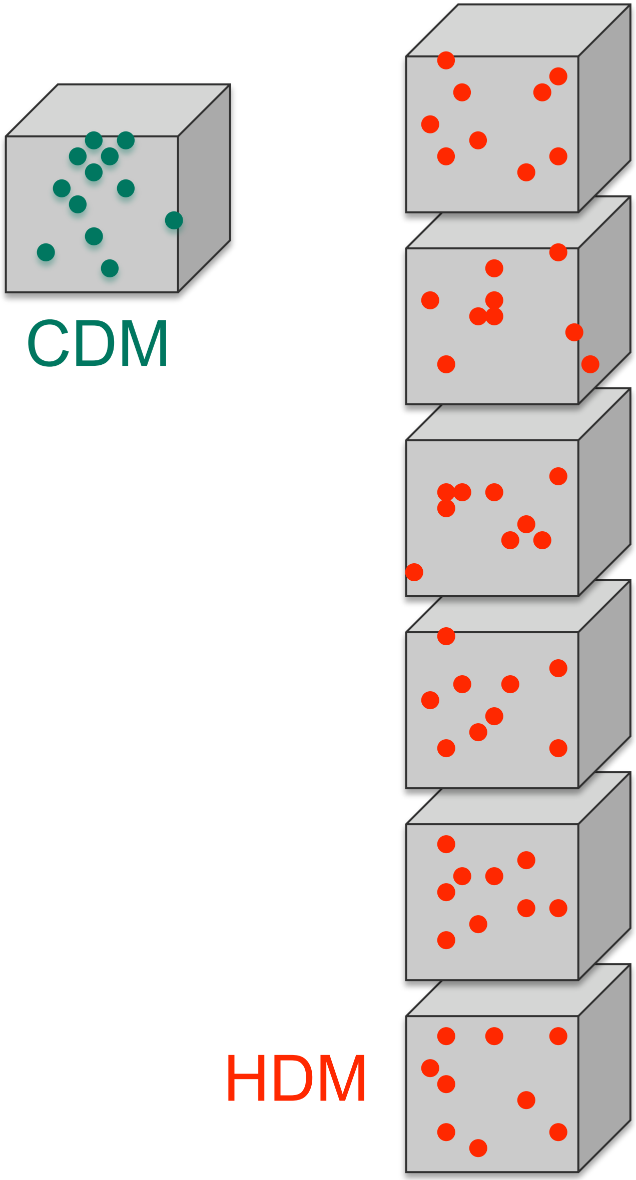
CDM
$\vec{v}_{\rm CDM} \approx \vec{v}_{\rm gravity} \propto \nabla\Psi$

Neutrinos $\vec{v}_\nu \approx \vec{v}_{\rm gravity} + \vec{v}_{\rm thermal}$

$$\sigma_\nu(z) = \sqrt{\left\langle \frac{p^2}{2m_\nu} \right\rangle} \approx 1800 \mathrm{km/s} \left(\frac{m_\nu}{0.1\mathrm{eV}} \right)^{-1} (1+z)$$

The r.m.s. thermal velocity > the velocity dispersion of galaxy clusters (~1000km/s): neutrinos can't much cluster on small scales (free-streaming)

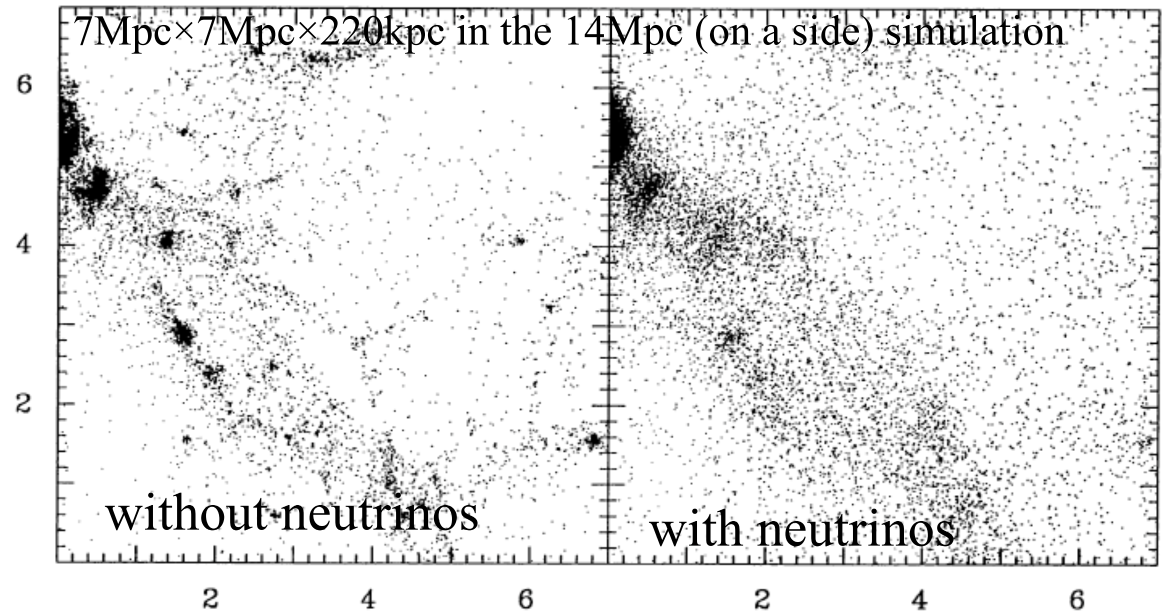
Particle-based simulations for C+HDM model



- The pioneer work by Klypin et al. (1993)
- PM code: each simulation uses 7 sets of N-body particles
  - CHDM model: Ω_cb=0.7, Ω_ν=0.3 ($m_{\nu,tot}$~7eV)
  - One set is for CDM (128^3)
  - 6 sets are for neutrinos (6 × 128^3 particles): more particles are used to better sample the FD phase-space density of neutrinos at each grid
  - For the initial conditions of neutrinos, the random thermal velocities are added to each particle

$$\vec{v}_\nu \approx \vec{v}_{\mathrm{gravity}} + \vec{v}_{\mathrm{thermal}}$$

  - Follow the trajectories of each particles according to the gravitational force

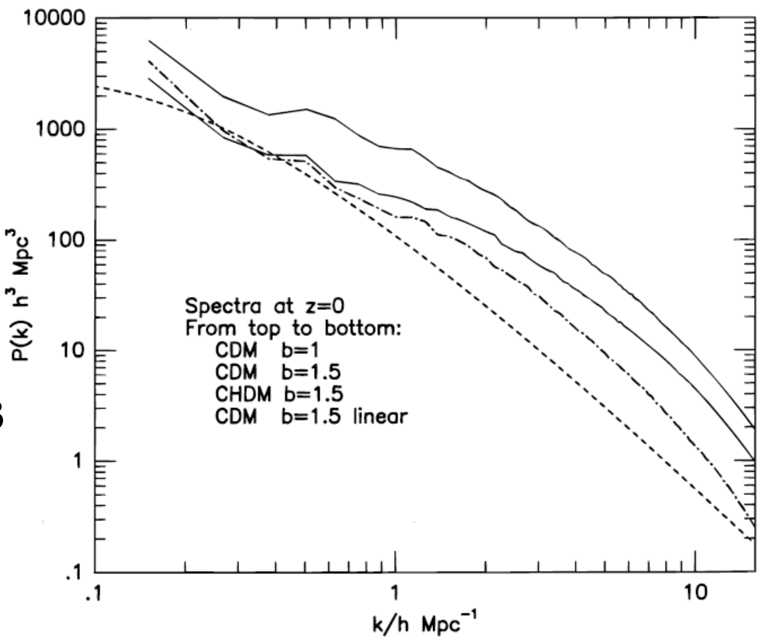


Klypin et al. 93

FIG. 3.—Distribution of "cold" (*left-hand panel*) and "hot" (*right-hand panel*) particles in a slice of 220 kpc × 7 Mpc × 7 Mpc in the 14 Mpc simulation. We show 85% of "cold" particles (1.3×10^4 particles) and 16.6% of "hot" particles (1.2×10^4 particles). Note that while all large condensations in the plot are seen both in "cold" and "hot" particles (for example, the large group in the top left-hand corner, a few big "galaxies" in the filament, and the filament itself), small halos of "cold" particles do not find counterparts in "hot" particles (for example, two "galaxies" in the void at the right middle part). The resolution for this simulation is 55 kpc, which is about 4 times smaller than the distance between small tick marks.

Ω_cb=0.7(Ω_c=0.6),
Ω_ν=0.3 ($m_{\nu,tot}$~7eV)

$P(k)h^3$



Klypin et al. 97

100Mpc simulation
256^3 CDM particles, 2×256^3 neutrinos

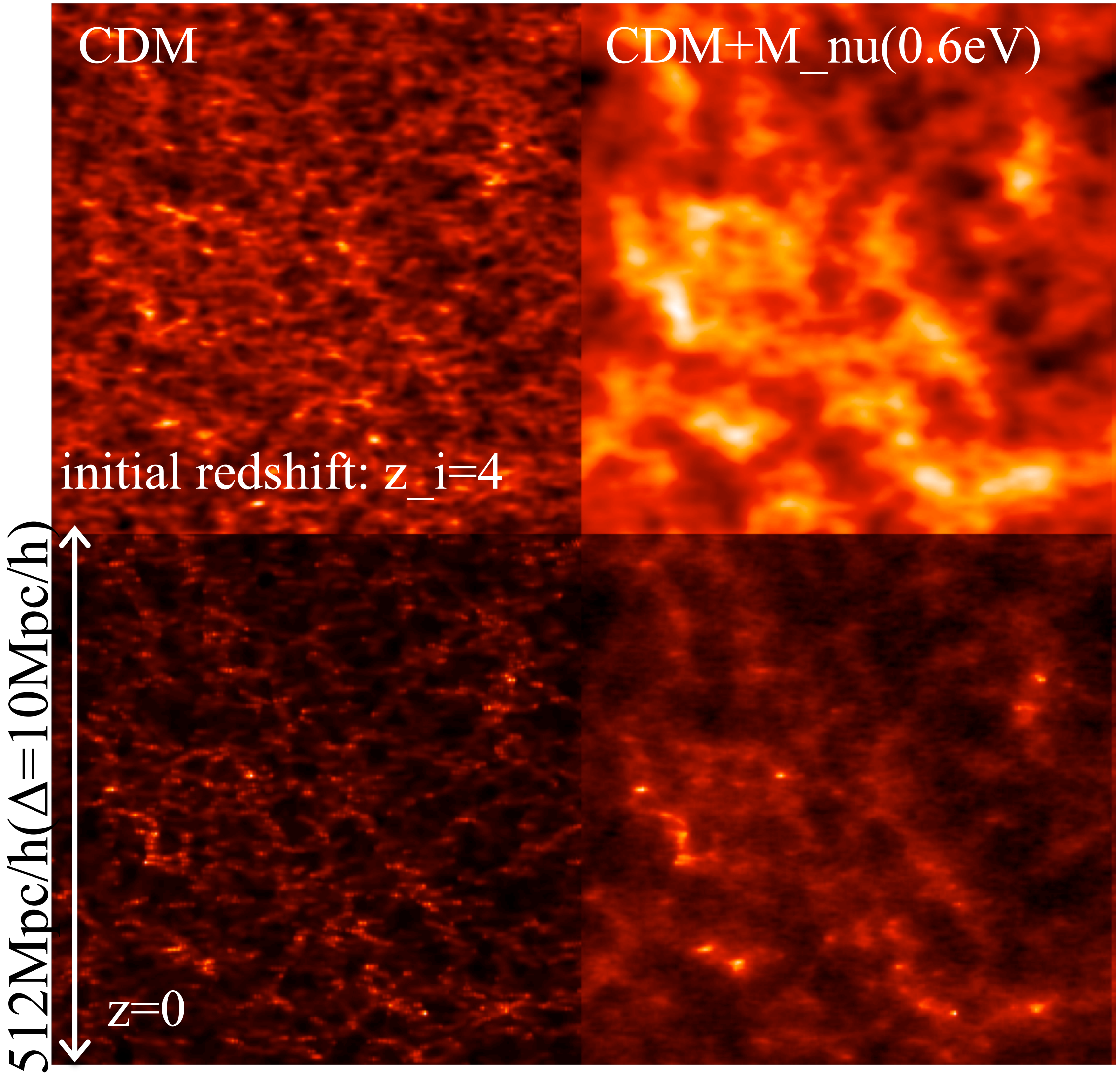
FIG. 5.—Comparison of power spectra for CDM and $CHDM_1$ simulations in the nonlinear regime at the final moment ($z = 0$). The spectrum of

Recent attempt

Brandbyge, Hannestad+ 08

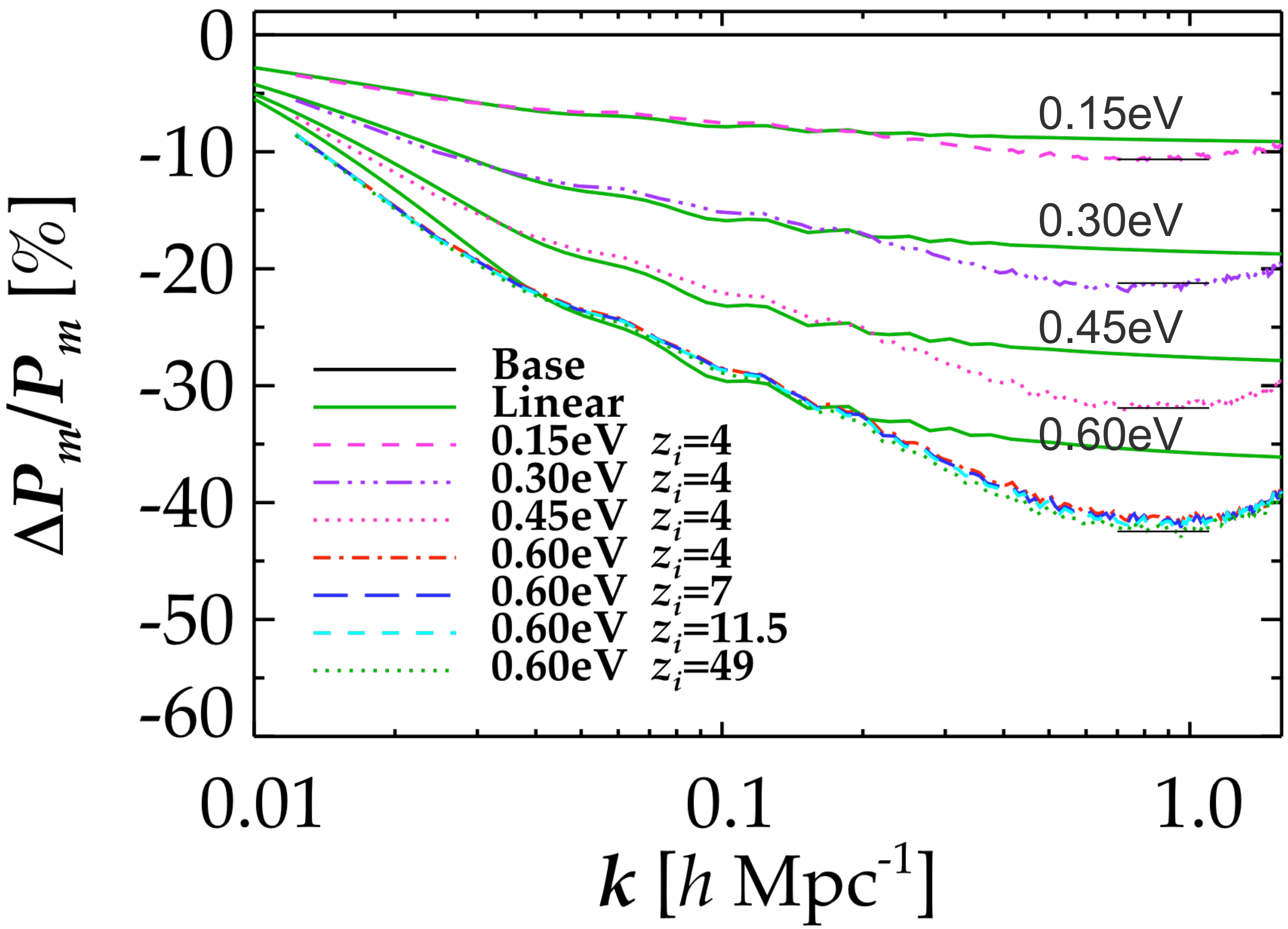
- More realistic: assume smaller mass scales of neturinos
  - $\Omega_m=\Omega_{cdm}+\Omega_b+\Omega_\nu=0.3$, $\Omega_\Lambda=0.7$, $h=0.7$
  - Right panel: $\Omega_\nu=0.013(0.6\text{eV})$, $f_\nu=0.043$
- Issues need to be more carefully studied
  - Initial redshift (z_i=4)
  - Initial conditions: how to generate the initial velocity fields of each component according to the different TFs
  - Discrete effect, especially for small neutrino masses
  - Numerical artifacts?

256^3 CDM + 512^3 neutrinos



Suppression of P(k) in NL regime

Brandbyge, Hannestad+ 08



In linear theory, the suppression at scales much smaller than the free-streaming scale is given as

$$\frac{\Delta P_m}{P_m} \approx -8\,\frac{\Omega_{\nu 0}}{\Omega_{m0}}$$

Hu, Eisenstein, Tegmark 98

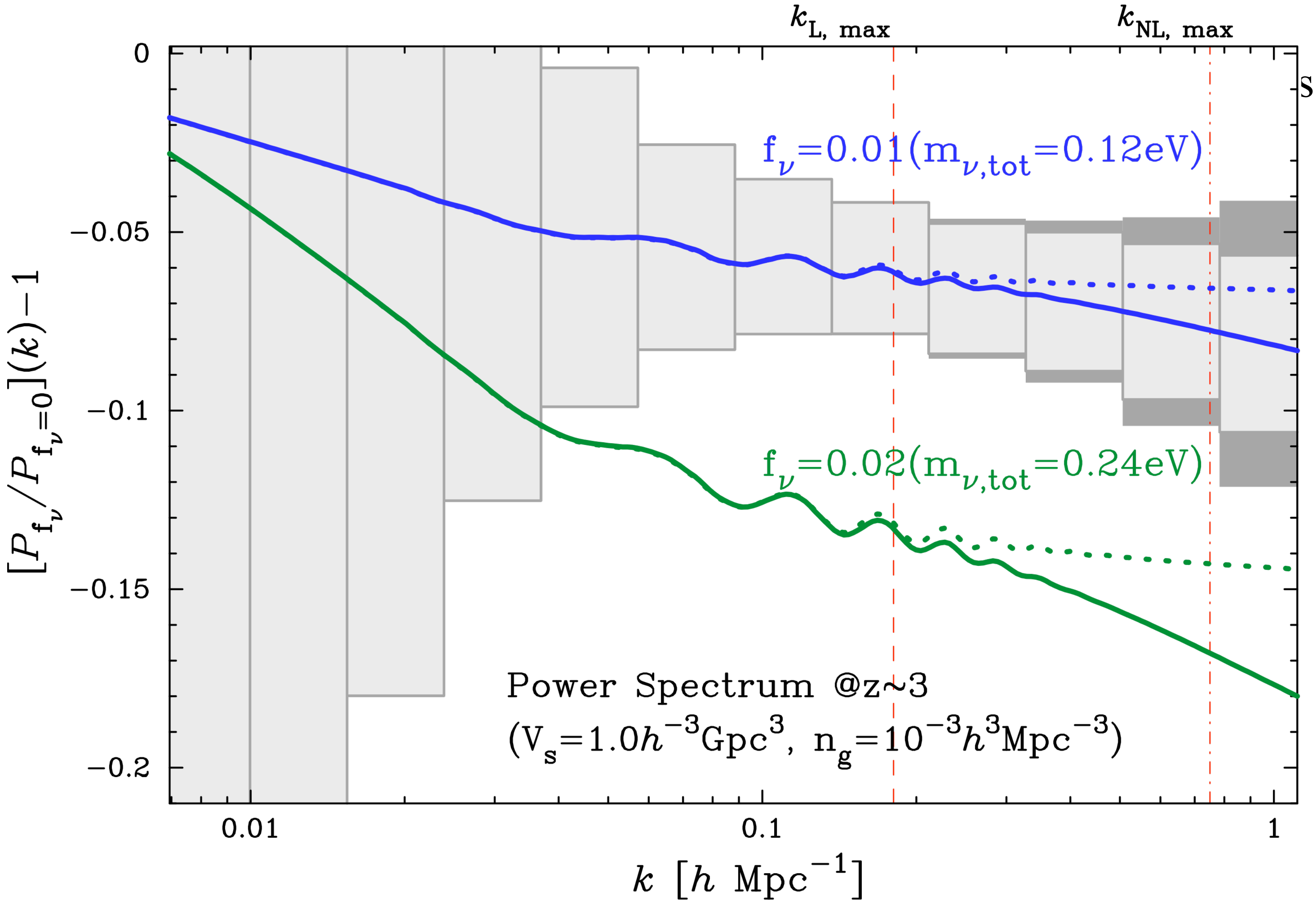
In the weakly nonlinear regime

$$\frac{\Delta P_m}{P_m} \approx -10\,\frac{\Omega_{\nu 0}}{\Omega_{m0}}$$

- *The suppression in power spectrum amplitudes is enhanced in the weakly nonlinear regime, compared to the linear theory prediction (also Saito, MT, Taruya PRL 08)*

PT approach for C+HDM model

Saito, MT, Taruya PRL 08, PRD 09; Wong 08



Hybrid (particle + grid) simulations for C+HDM

Brandbyge & Hannestad 08

- Avoid the difficulty of sampling phase-space PDF with a finite number of N-body particles
- Solve the *linearized* Boltzmann equation for neutrino perturbations at each grid
  - Can use publicly available code of the Boltzmann solver (e.g. CAMB)

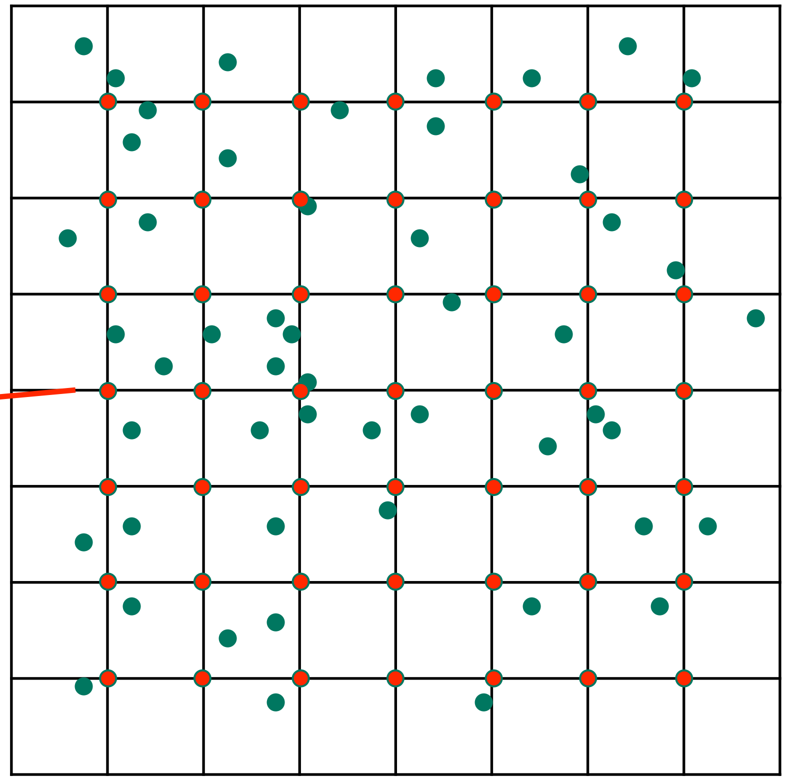
$$\frac{\partial \delta f}{\partial t} + \frac{\vec{p}}{m_\nu a} \cdot \nabla \delta f - a m_\nu \nabla \Phi \cdot \frac{\partial \delta f}{\partial \vec{p}} = 0$$

$$\nabla^2 \Phi = 4\pi G a^2 \bar{\rho}_m \left(f_{cb} \delta_{cb} + f_\nu \delta_\nu \right)$$

  - Obtain the neutrino perturbations from the moments

$$\delta_\nu = \frac{1}{\bar{\rho}_\nu} \int d^3\vec{p}\ \delta f$$

$$\left(\vec{v}_\nu = \frac{1}{\bar{\rho}_\nu} \int d^3\vec{p}\ \delta f \vec{p},\ \ \sigma_{ij} = \frac{1}{\bar{\rho}_\nu} \int d^3\vec{p}\ \delta f p_i p_j, \ldots \right)$$

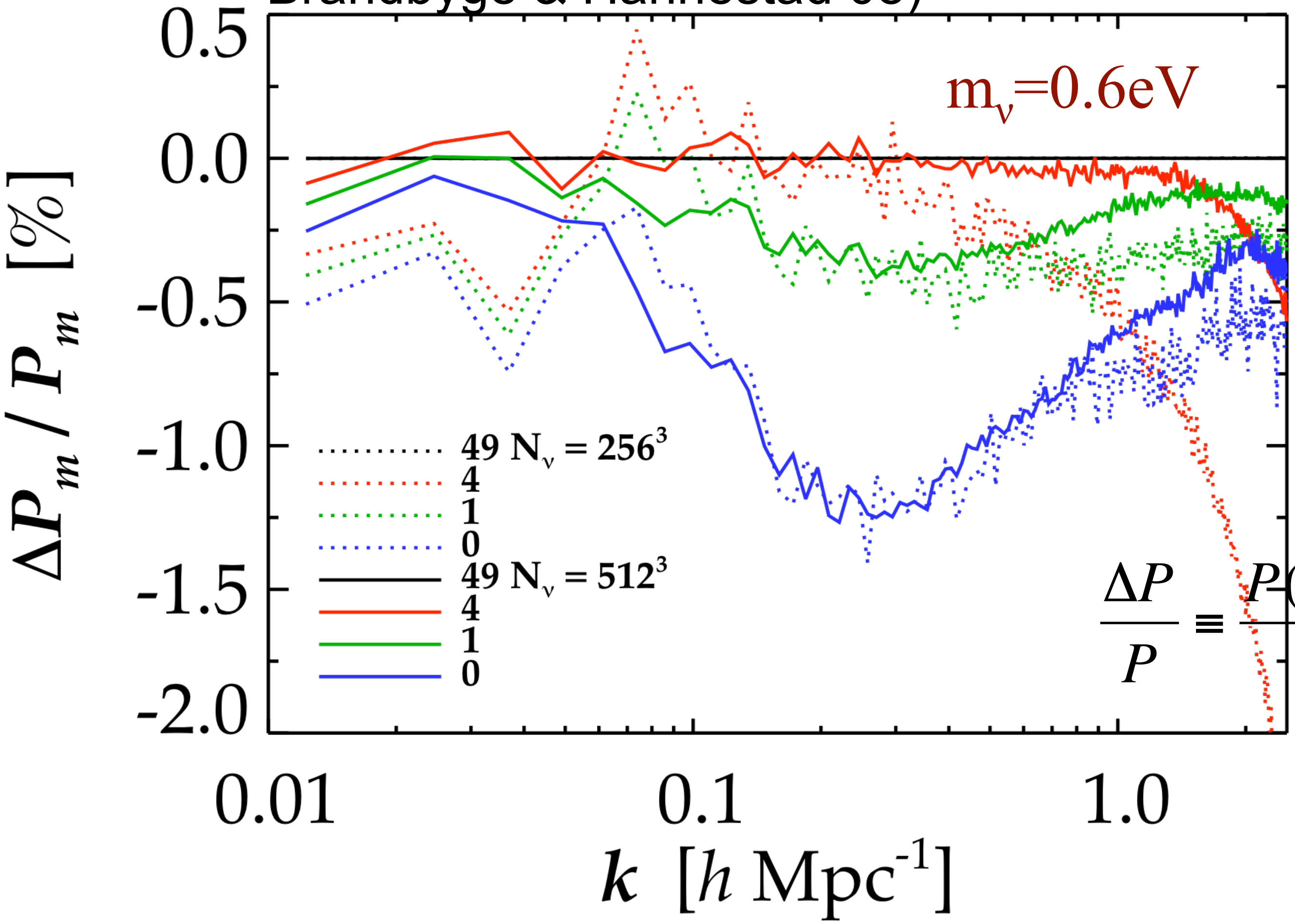


CDM: particles

Neutrinos: grids

Hybrid simulations (contd.)

Brandbyge & Hannestad 08)

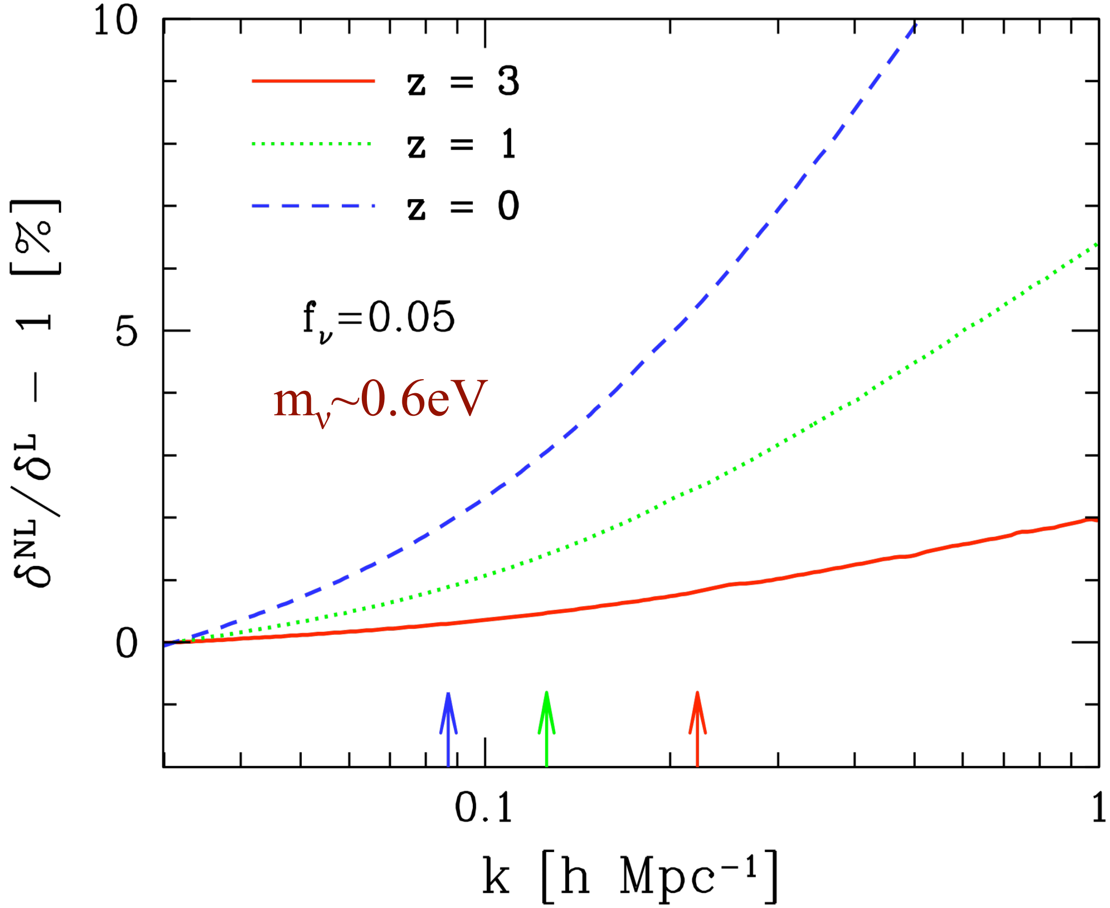


$$\frac{\Delta P}{P} \equiv \frac{P(\text{hybrid}) - P(\text{particle})}{P}$$

- Only includes the linear-order neutrino perturbations (note the phase information of neutrino density perturbations is included)
- Potentially promising method, but needs to be further studied

Improving PT approach

Saito, MT, Taruya 09; also Wong 08; Lesgourgues et al 09



- Include the contribution of nonlinear CDM+baryon density perturbation

$$\frac{\partial \delta f}{\partial t} + \frac{\vec{p}}{m_\nu a} \cdot \nabla \delta f - a m_\nu \nabla \Phi \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

$$\nabla^2 \Phi = 4\pi G a^2 \bar{\rho}_m \left(f_{cb}\delta_{cb} + f_\nu \delta_\nu \right)$$

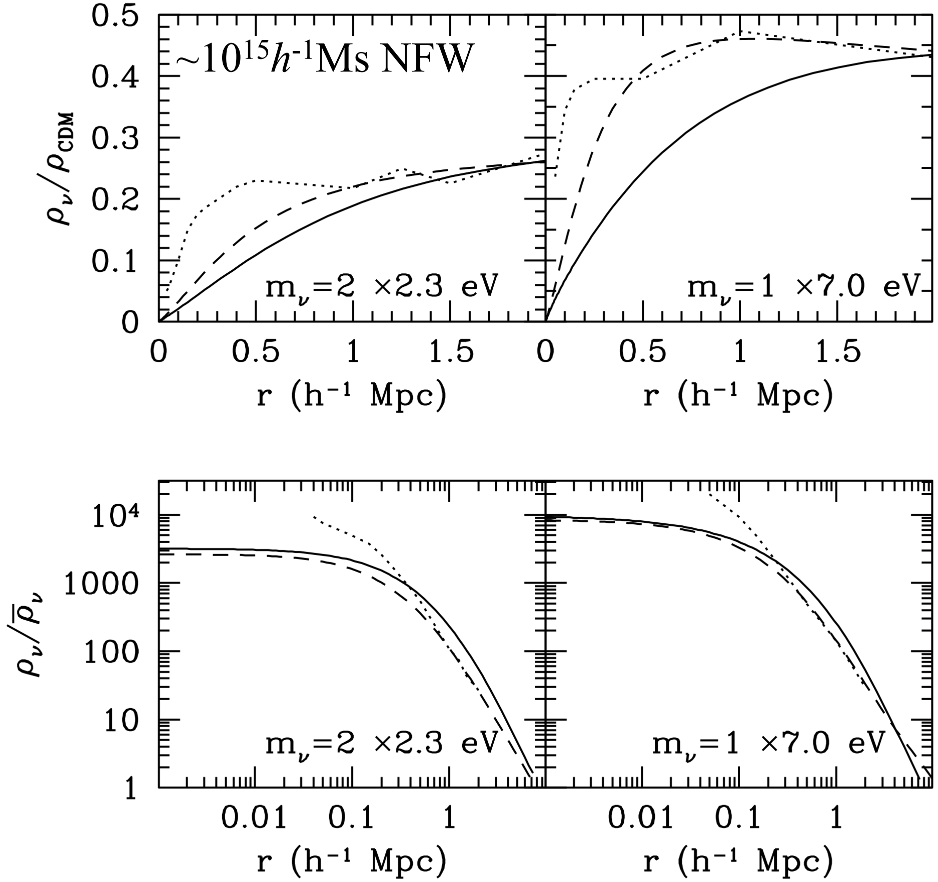
Substitute the density perturbation, δ_{cb}, including the 1-loop correction

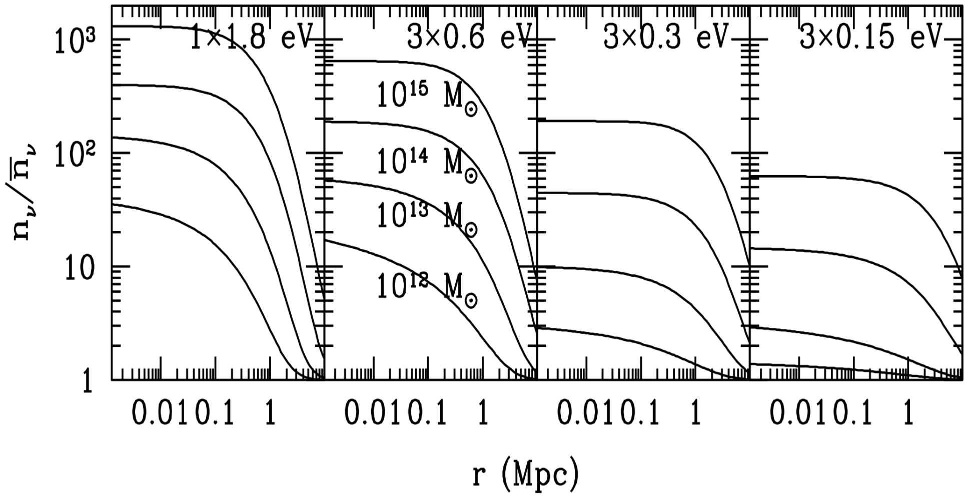
- Note: $P_m(k) \leftarrow f_\nu^2 \langle \delta_\nu^2 \rangle$

- The PT theory predictions can be used to test the simulation results
- The nonlinear correction due to the CDM perturbations may be readily included in a numerical solver of the linearized Boltzmann equations of neutrino perturbations

Neutrino clustering in strongly nonlinear regime (around halos)

Singh & Ma 02





$$\frac{\vec{p}}{m_\nu a}\cdot\nabla\delta f - a m_\nu \nabla\Phi_{NFW}\cdot\frac{\partial(f_0+\delta f)}{\partial\vec{p}} = 0$$

$$\sigma_\nu(z=0) = \sqrt{\left\langle\frac{p^2}{2m_\nu}\right\rangle} \approx 90\mathrm{km/s}\left(\frac{m_\nu}{2\mathrm{eV}}\right)^{-1}$$

$$\sigma_\nu(\mathrm{halos}) \sim 1500\mathrm{km/s}\left(\frac{M_{\mathrm{halo}}}{10^{15}h^{-1}Ms}\right)^{1/3}$$

Dashed: neutrinos from Botlzmann eq.
Dotted: from simulations of Kofman et al (96)

- Estimate a static solution of the neutrino distribution given a spherically symmetric CDM profile

Updated SDSS constraints

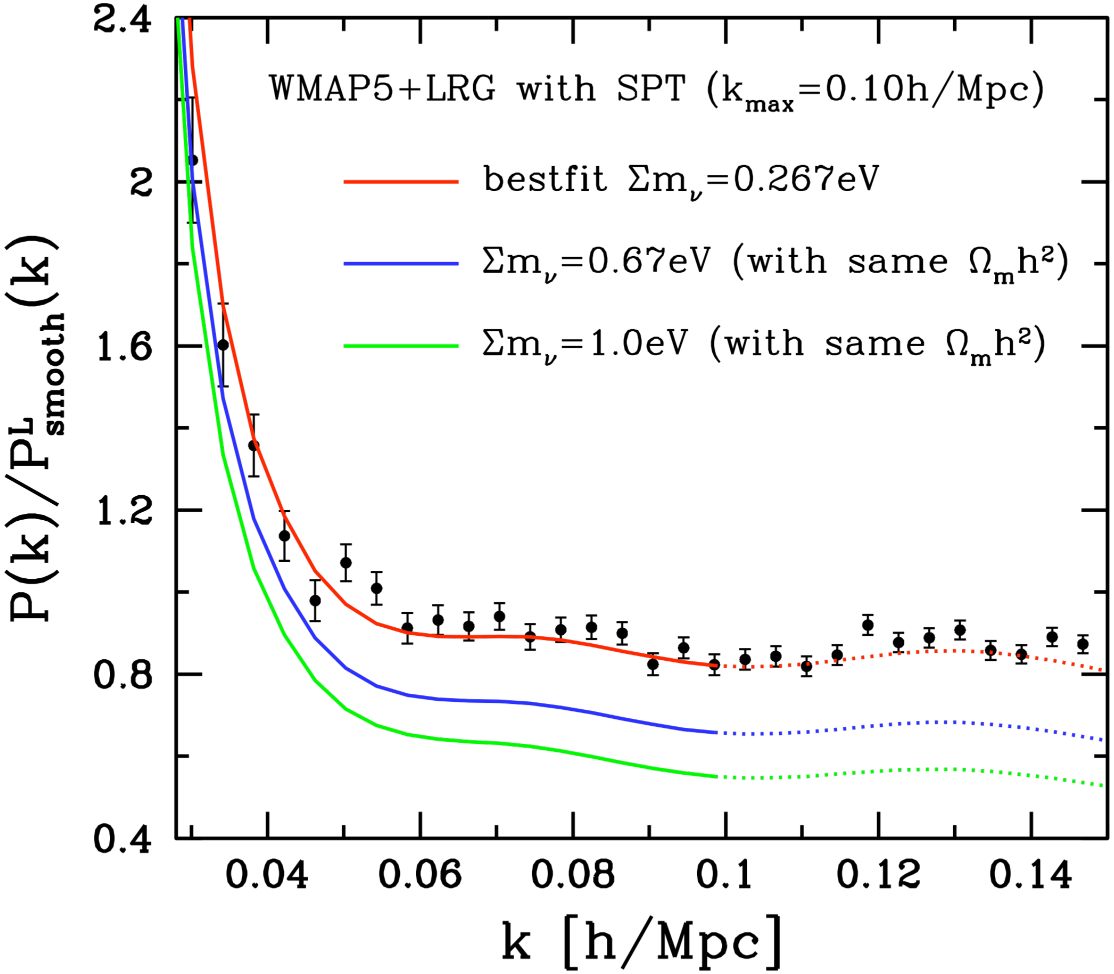
Saito et al. in prep.

$$P_m(k) = \left\langle \left(\frac{\delta\rho_m}{\bar{\rho}_m} \right)^2 \right\rangle = \left\langle \left\{ f_{cb}\left(\delta_{cb}^{(1)} + \delta_{cb}^{(2)} + \delta_{cb}^{(3)} \right) + f_\nu \delta_\nu^{(1)} \right\}^2 \right\rangle$$

$$P_g(k) = b_1^2 \left[P_m(k) + b_2 P_{b2,m}(k) + b_2^2 P_{b22} \right] + N$$

$$P_{b2,\delta}(k) \equiv 2 \int \frac{d^3q}{(2\pi)^3} P_m^L(q) P_m^L(|\boldsymbol{k}-\boldsymbol{q}|) \mathcal{F}_\delta^{(2)}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}),$$

$$P_{b22}(k) \equiv \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} P_m^L(q) [P_m^L(|\boldsymbol{k}-\boldsymbol{q}|) - P_m^L(q)].$$



- Comparing the perturbation predictions with the SDSS measurements
  - Include parameters of nonlinear galaxy bias in the model predictions
  - Fitting the model to the measured power spectrum with varying all the parameters
  - Derive the neutrino mass constraint, marginalized over other parameters
- $m_{\nu,tot}$<0.67eV(95% CL), similar to the results in Reid et al. 09)

Summary & Discussion

- Nonlinear structure formation for CDM cosmologies is now better understood
  - N-body simulations: most powerful tool, 1% accuracy achieved up to k~1/Mpc
  - Refined perturbation theory approach: complementary to N-body simulations, remarkable agreement with the simulations up to k~0.1/Mpc
  - Issues: baryonic effects
- Very important to study nonlinear structure formation for a C+HDM model, assuming relevant scales of neutrino masses (<0.6eV) to attain the full potential of planned surveys (galaxy, WL, cluster)
- Simulating structure formation for C+H models is still an open issue
  - Particle based simulations: two kinds of N-body particles (computationally expensive?)
  - Hybrid simulations: N-particles for CDM, grids for neutrinos where the Boltzmann eqns. need to be solved at each grid (nonlinear corrections to the Boltzmann eqns.?)
- Improving the PT approach for C+HDM models
  - Complementary to simulation based studies
- Neutrino perturbations are small anyway, probably possible to develop reasonably accurate hybrid methods
  - Simulations + PT approach + halo models?