Asymptotic Safety at the LHC

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Outline

Large extra dimensions

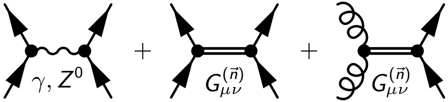
Compactified large extra dimensions [Antoniadis, Arkani-Hamed, Dimopoulos, Dvali]

- Planck mass $M_{\rm P} \sim 10^{16}$ TeV not fundamental [$M_{\rm P} = (2\pi r)^{\delta/2} M_\star^{\delta/2+1}$]

Effective theory of KK gravitons [Giudice, Rattazzi, Wells; Han, Lykken, Zhang; Hewett,...]

- Tower of massive KK gravitons [KK mass: $m = |\vec{n}|/r$]
- Real emission $\Rightarrow$ missing energy [sum over KK tower restricted by kinematics]
- Virtual exchange $\Rightarrow$ higher rates [sum over all KK states]

$$\mathcal{A}(pp \to \mu^+\mu^-) \sim$$



- But...UV divergence in coefficient of dim-8 operator (at tree level)

$$\mathcal{L}_8 = \mathcal{S}(s) T_{\mu\nu} T^{\mu\nu} \quad \text{with} \quad \mathcal{S}(s) = \frac{S_{\delta-1}}{M_\star^{2+\delta}} \int dm \frac{m^{\delta-1}}{s - m^2}$$

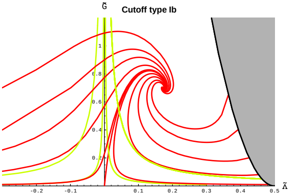
Divergence of $\mathcal{S}(s)$

Cut-off methods [Giudice & Strumia; Giudice, Strumia, Plehn; Kachelriess & Plümacher,...]

- Effective theory cut-off (or NDA slightly better) $\mathcal{S}(s)_{\text{eff}} = \frac{4\pi}{\Lambda_{\text{eff}}^4}$

String theory [Cullin, Peskin, Perelstein; Figy, Han, Benkali...]

- String resonances occur at $\sqrt{n} M_S$
- Exponential suppression above M_S



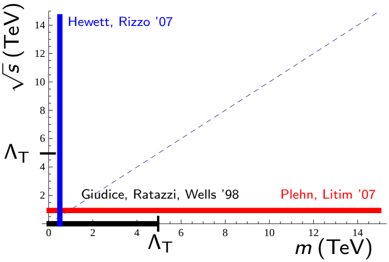
Asymptotic safety [Weinberg; Hewett, Rizzo; Litim, Plehn]

- Non-gaussian UV fixed point found [anomalous dimension $\eta = -2$] [Reuter et al.]
- Renormalization group improved running Newtons "constant"

$$G_4 \to G_4(\mu^2) = \frac{G_4}{\mu^2} \text{ for } \mu \to \infty$$

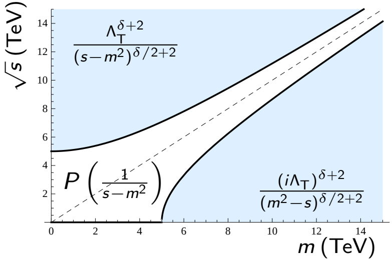
- Gravitational fixed point persists in $D > 4$ [$\eta = -D + 2$] [Fischer, Litim]

Current situation



$$\mathcal{S}(s) \to \frac{G_D}{\mu^{\delta/2+1}} \int_0^\infty dm \frac{m^{\delta-1}}{s - m^2}$$

Renormalization scale in a compactified theory

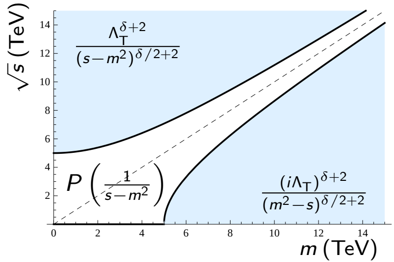


Analogy from AS in D-dimensions [EG, Plehn, Litim]

- Assumption #1: ren. scale set as $|q^2|$ in s-channel propagator
- Assumption #2: η set by short distance physics $[\eta_4 \rightarrow \eta_D = -D+2]$

Asymptotic safety completion

Renormalization scale in a compactified theory



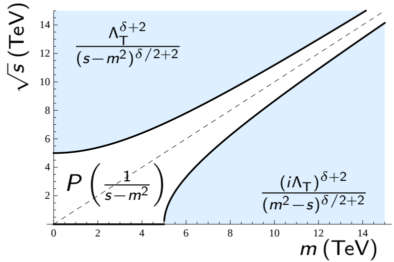
Analogy from AS in D-dimensions [EG, Plehn, Litim]

– Physics assumption #1: ren. scale set as $|q^2|$ in s-channel propagator

$$\Delta(q^2) = \begin{cases} \frac{1}{q^2} & |q^2| < \Lambda_T^2 \\ \frac{\Lambda_T^{-\eta}}{(q^2)^{1-\eta/2}} & |q^2| > \Lambda_T^2 \end{cases} \longrightarrow \begin{cases} \frac{1}{p^2-m^2} & |p^2 - m^2| < \Lambda_T^2 \\ \frac{\Lambda_T^{-\eta}}{(p^2-m^2)^{1-\eta/2}} & |p^2 - m^2| > \Lambda_T^2 \end{cases}$$

Asymptotic safety completion

Renormalization scale in a compactified theory

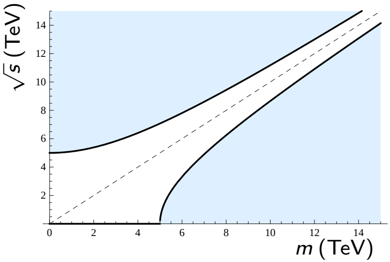


Advantages

- KK mass integration is convergent [for fixed s always $\sim \int^\infty dm/m^5$]
- Smooth transition at boundaries [up to a phase, required for realistic smoothly running η]
- But what about s behaviour for $\sqrt{s} > \Lambda_T$

Large $\sqrt{s}$ behaviour in a modified theory

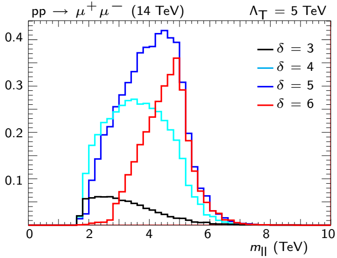
- Off-diagonal (blue) regions well-behaved in UV $\mathcal{S}(s > \Lambda_{\mathrm{T}}) \sim 1/s^2$
- Euclidean computation gives same $\mathcal{S}(s_E > \Lambda_{\mathrm{T}}) \sim 1/s_E^2$
- Width gives (at least) $1/s^2$ suppression at the pole ($m\Gamma \sim m^4/M_{\mathrm{P}}^2$)
- Still need to match the central region at energy $\sim \Lambda_{\mathrm{T}}$



$\mathcal{S}(s)$	Single graviton	Whole tower
Coupling	$1/M_{\mathrm{P}}$	$1/M_\star$
UV in $\sqrt{s}$	$1/s^{\delta/2+2}$	$1/s^2$
Dimension of FP	D	4

LHC rates

- Signal strongly depends on number of extra dimensions
- Non-trivial interference effects due to phase factor $e^{i\delta\pi/2}$ for $m > \sqrt{s}$



Summary

- Virtual graviton exchange not well defined (negative view)
- Virtual graviton exchange sensitive to quantum gravity (positive view)
- Asymptotic safety offers a possible completion with distinctive results
- Asymptotic Safety in D dimensions maps non-trivially into 4 dimensions
- Complicated signal dependence on number of extra dimensions
- Effects of FP scaling KK modes noticeable even for $\sqrt{s} < \Lambda_T$

Implementation of asymptotic safety

Smooth transition [Hewett,Rizzo]

– Running coupling

$$\frac{1}{\Lambda_T^{\delta+2}} \to \frac{1}{\Lambda_T^{\delta+2}} F(\mu^2)$$

$$= \frac{1}{\Lambda_T^{\delta+2}} \left[1 + \frac{\omega}{8\pi} \left(\frac{\sqrt{s}}{\Lambda_T} \right)^{n+2} \right]^{-1}$$

– Renormalization scale $\mu^2 = s$ [valid for $m^2 \ll s$, Improved unitarity]

– Problem: Still need to cut-off KK integral.

Abrubt transition [Litim, Plehn]

– Modified propagator [aquires full AS anomolous dimension at $\mu = \Lambda_T$]

$$\frac{1}{s+m^2} \to \frac{\Lambda_T^{n+2}}{(s+m^2)^{n/2+2}}$$

Renormalization scale $\mu^2 = s + m^2$ [in computation expand in s/m^2]

Numerical results with smooth transition

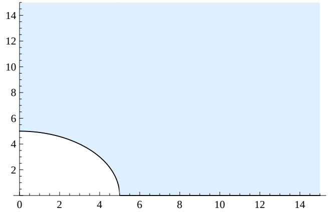
- Anomalous dimension becomes function of renormalization scale
- Fit parameters b and l to flow equation for g_k

$$\ln \frac{k}{k_0} = \ln \left[l + \frac{1}{b} \tan \left[\frac{\pi}{2}(-1 + 8Dg) \right] \right]$$

$$\ln \frac{k}{k_0} = \frac{1}{D-2} \ln \frac{g_k}{g_0} - \frac{1}{\theta_{NG}} \ln \frac{g_\star - g_k}{g_\star - g_0}.$$

- $\approx 20\%$ variation for total rate

$$\eta_k = \frac{2(D-2)(D+2)g_k}{2(D-2)g_k - 1}$$



At the pole

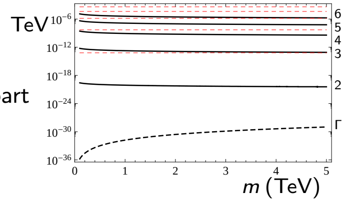
Breit-Wigner decay width vs. spacing

- Decay width to SM very small: $\Gamma \sim 10^{-32} m^3 \text{ GeV}^{-2}$
- Lower bound on KK spacing

$$\Delta m \geq \frac{1}{\sqrt{r}} \left[\sqrt{m} - \sqrt{m - \frac{1}{r}} \right]$$

- Integrated propagator: keep only real part

$$\Delta(s) = \int dm \frac{m^{\delta-1}}{s - m^2 + im\Gamma} \rightarrow P \int dm \frac{m^{\delta-1}}{s - m^2}$$



Realistic model