Unparticle Solution to Hierarchy

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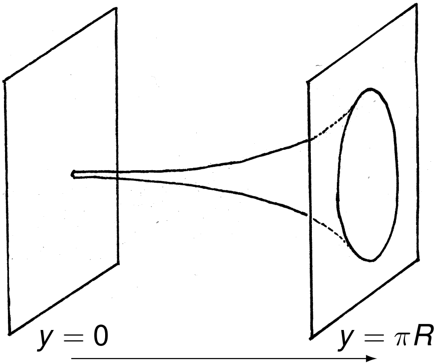
Randall Sundrum One (RS1)

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

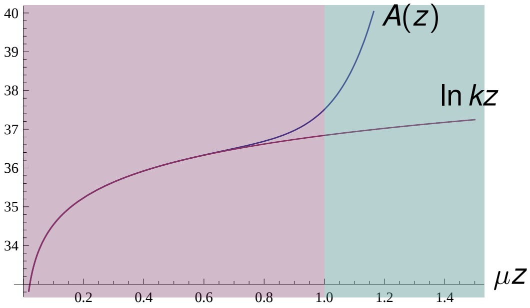


- Need to stabilize R distance (Goldberger-Wise)
- Why not use GW scalar to replace IR brane?

BG Soft-Wall Geometry

$$ds^2 = e^{-2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

$$A(z) = \ln kz + \frac{2}{3}(\mu z)^\nu$$



Planck Weak Hierarchy

- Need to achieve $\mu/k \sim 10^{-16}$
- What sets μ/k?
- Consider:
  - μ sets scale where scalar back-reaction strong
  - Must fix field at one location
  - Boundary condition on UV brane fixes field
- Boundary potential:

$$\lambda_{\text{UV}} = W(\eta_0) + \partial_\eta W(\eta_0)(\eta - \eta_0) + m_{\text{UV}}(\eta - \eta_0)^2$$

- Boundary conditions require $\eta_0 = \langle \eta \rangle_0$

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Planck Weak Hierarchy

- The value of the field at the UV brane is

$$\eta_0 = \pm\sqrt{3}\left(\frac{\nu+1}{\nu}\right)\left[\sqrt{\frac{2}{3}\frac{\nu}{\nu+1}\left(\frac{\mu}{k}\right)^{\nu}+\left(\frac{2}{3}\frac{\nu}{\nu+1}\left(\frac{\mu}{k}\right)^{\nu}\right)^{2}} + \sinh^{-1}\sqrt{\frac{2}{3}\frac{\nu}{\nu+1}\left(\frac{\mu}{k}\right)^{\nu}}\right]$$

- Not easily inverted...

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Planck Weak Hierarchy

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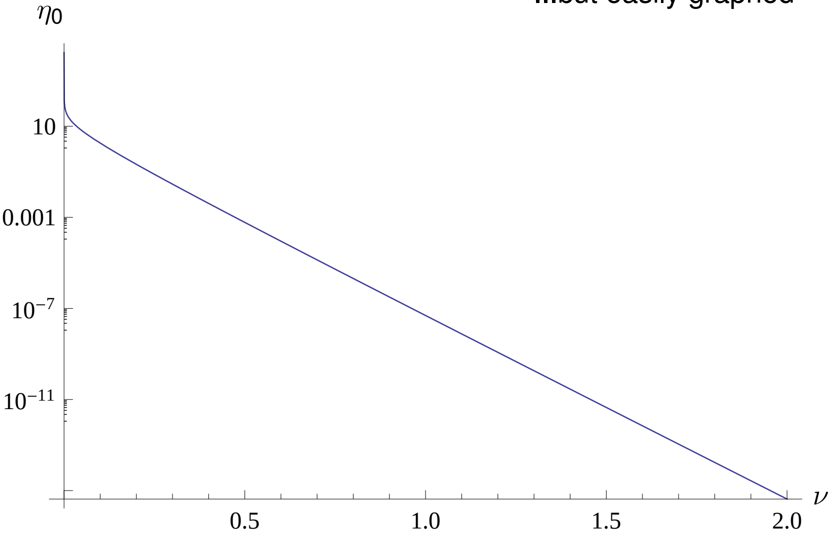

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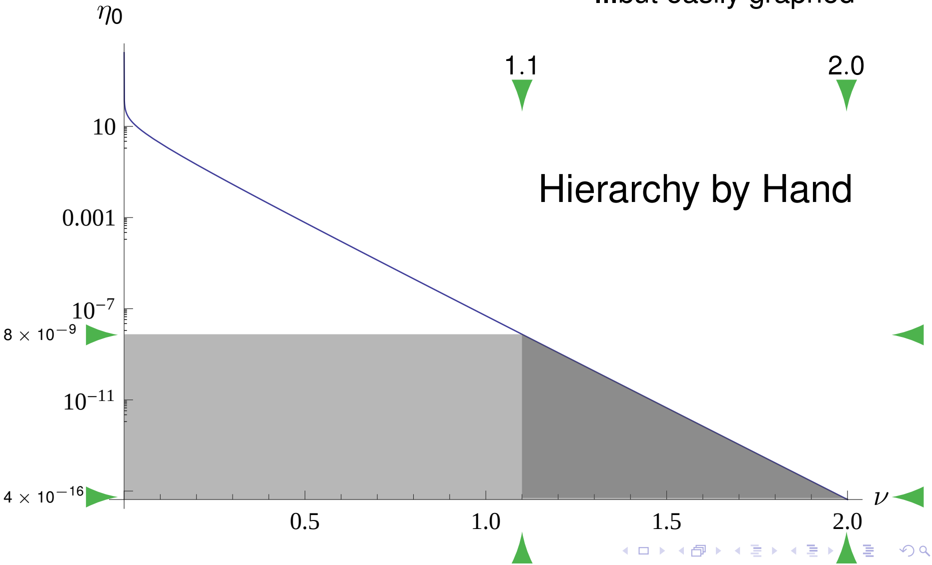

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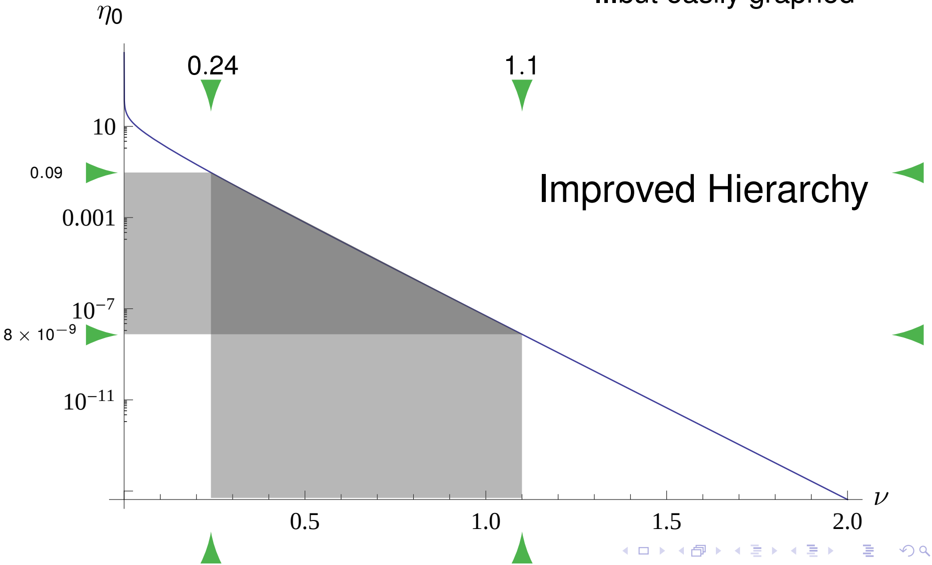

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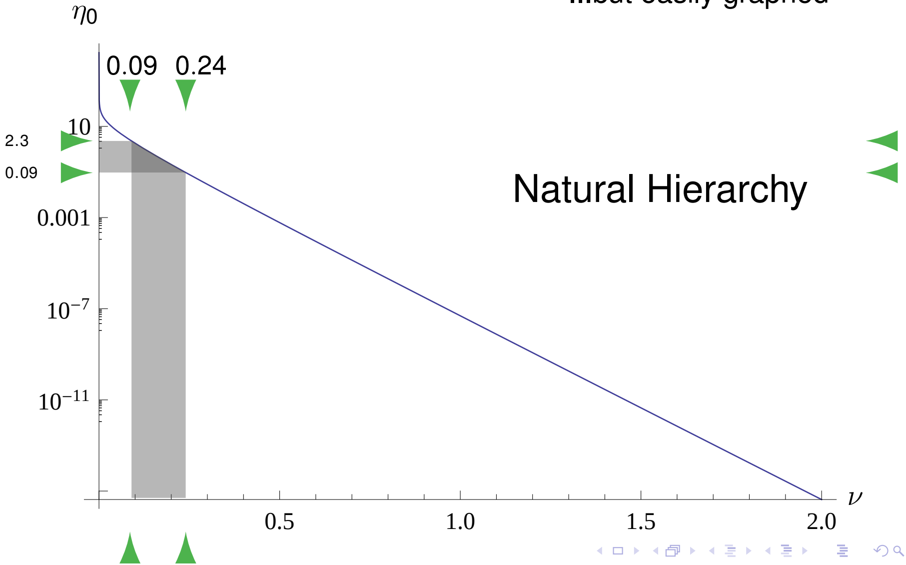

Scalar's Potential

Of course, ν has other consequences...
Look at potential

$$V(\eta) = -12k^2 - k^2\nu\left(1 - \frac{\nu}{8}\right)\eta^2 + \cdots$$

Gives η's mass as

$$m_\eta^2 = -2k^2\nu\left(1 - \frac{\nu}{8}\right)$$

AdS/CFT correspondence says operator dimension is

$$\Delta = 2 + \sqrt{4 + \frac{m_\eta^2}{k^2}} = 2 + \frac{1}{2}|4 - \nu|$$

Operator Dimension

The breakdown is

Hierarchy by Hand	$\nu > 1$	$\Delta > \frac{5}{2}$
Improved Hierarchy	$\nu \sim 1$	$\Delta \sim \frac{5}{2}$
Natural Hierarchy	$0 < \nu < 1$	$2 < \Delta < \frac{5}{2}$

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Fractional Dimension!

Fluctuations Parameterized

$$ds^2 = e^{2(F-A(z))}\left[\left((1-2F)\eta_{\mu\nu} + h_{\mu\nu}\right)dx^\mu dx^\nu + 2A_\mu dx^\mu dz + dz^2\right]$$

$$\eta = \langle\eta\rangle + \tilde{\eta}$$

- Consider Just Scalar Modes
  - gravi-scalar, F
  - scalar tower of η
- Suitable field redefinition permits writing as Schrödinger equation

$$\left(-\partial_z^2 + V_{\mathrm{SE}}(z)\right)\psi = m^2\psi$$

- Start with $m = 0$ modes

No Massless Scalars

- There are no massless modes
- Consider theory without UV brane
  - Theory invariant under $A(z) \to A(z) + C$
  - Look again at parameterization of fluctuations:

$$ds^2 = e^{2(F - A(z))}\left[\left((1 - 2F)\eta_{\mu\nu} + h_{\mu\nu}\right)dx^\mu dx^\nu + 2A_\mu dx^\mu dz + dz^2\right]$$

  - F Goldstone boson
  - But μ/k changes under shift: $\frac{\mu}{k} \to \frac{\mu}{k} e^{-C}$
  - Fixing μ/k breaks symmetry
- With UV brane, μ/k determined, so no massless modes
- What about massive modes?

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Scalar Modes - Massive

- Massive modes dynamical variable

$$v = -\sqrt{2}e^{-3A(z)/2}\frac{\langle\eta\rangle'}{A'(z)}\left(-\frac{1}{2}F + \frac{A'(z)}{\langle\eta\rangle'}\tilde{\eta}\right)$$

- Schrödinger Potential Behavior

$$\nu > 1 \quad z \to \infty \Rightarrow V_{\text{SE}} \to \infty$$

$$\nu = 1 \quad z \to \infty \Rightarrow V_{\text{SE}} \to \mu^2$$

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Continuum
$m^2 > 0$
(Unparticles)

Summary

- Examined Planck weak Hierarchy for Batell-Gherghetta Soft-Wall
- Found natural hierarchy for $\nu < 1$
- $\nu < 1$ corresponds to fractional-dimension operators in dual theory
- $\nu < 1$ implies a continuum of modes without a mass gap in the $5D$ theory
- Thus, natural hierarchy implies unparticles