Warped Penguins

Yuhsin Tsai

Based on arXiv:1004.2037
In collaboration with Csaba Csáki, Yuval Grossman, Flip Tanedo

Cornell University

PHENO 10

The Penguins

- Penguin Diagrams are loop-induced processes.
- We focus on $\mu \to e\,\gamma$,

References

FCNC in RS

R.Kitano (00);S.J. Huber (03); K.Agashe, G.Perez, A.Soni (04)(05); G.Moreau, K.Agashe, A.E.Belchman, F. Petriello (06); J.I.Silva-Marcos (06); A.L. Fitzpatrick, L.Randall, G.Perez (08); K. Agashe, A. Azatov, and L. Zhu (09)...

Feynman Rules in 5D

L.Randall, M. D. Schwartz (01); M. Puchwein, Z. Kunszt (04); R. Catino, A. Pomarol (04)...

Reduceing the Flavor Constraints

J. Santiago (08); M. Blanke, A. J. Buras, B. Duling, S. Gori, A. Weiler (09); A. J. Buras, B. Duling, and S. Gori (09); M. E. Albrecht, M. Blanke, A. J. Buras, B. Duling, K. Gemmler (09); K. Agashe (09); C. Csaki, C. Delaunay, C. Grojean, Y. Grossman (09)...

Problem: $\mu \to e\,\gamma$ in RS is divergent!?

Previous Analyses

$\mu \to e\,\gamma$ with brane higgs loop is UV sensitive!

- Tree-level process: $\mathcal{M}_{\rm tree} + \mathcal{M}_{\rm loop}(\Lambda_c)$.
- Loop-induced process: $\mathcal{M}_{\rm loop}(\Lambda_c) + \ldots$.
- Cannot get a physical result.

Problem: $\mu \to e\,\gamma$ in RS is divergent!?

We Claim

One loop $\mu \to e\,\gamma$ in RS is finite!

- Can get a physical result for the amplitude.
- Gives interesting bounds on Yukawa.

The divergence in the KK picture

Naive NDA		KK	SM
Loop Integral	$(\sum)\ d^4k$	5	4
Fermi-Propg	$(1/\not{k})^2$	-2	-2
Higgs-Propg	$(1/k)^2$	-2	-2
Gauge Symm	$(\not{k}/k^2) \to (\not{p}/k^2)$	-1	-1
k-Power		0	-1

Hard to see the finiteness in KK!

- Infinitely large mass matrix including the KK-towers.
- Complications from summing the KK-modes.

The divergence in the KK picture

Naive NDA		KK	SM
Loop Integral	$(\sum)\ d^4k$	5	4
Fermi-Propg	$(1/\not{k})^2$	-2	-2
Higgs-Propg	$(1/k)^2$	-2	-2
Gauge Symm	$(\not{k}/k^2) \to (\not{p}/k^2)$	-1	-1

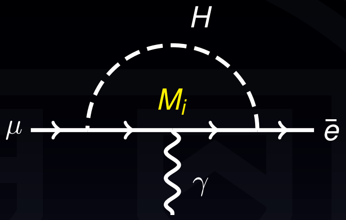
Yukawa	$y \propto M_i^{-1}$	-1	-1
k-Power		-1	-2

Hard to see the finiteness in KK!

- Infinitely large mass matrix including the KK-towers.
- Complications from summing the KK-modes.

The divergence in the KK picture

Naive NDA		KK	SM
Loop Integral	$(\sum)\ d^4k$	5	4
Fermi-Propg	$(1/\not{k})^2$	-2	-2
Higgs-Propg	$(1/k)^2$	-2	-2
Gauge Symm	$(\not{k}/k^2) \rightarrow (\not{p}/k^2)$	-1	-1
Yukawa	$y \propto M_i^{-1}$	-1	-1
k-Power		-1	-2



Show the finiteness with a full 5D calculation!

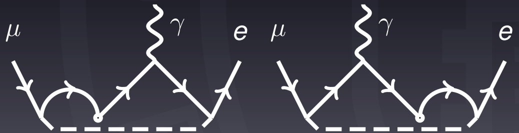
- No need to do the KK-sum.
- The only cutoff is the 4D momentum.

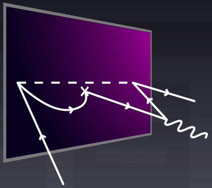
Results of the 5D Calculation

Finiteness & Branching ratio:

- The finiteness comes from the Lorentz invariance & chiral BC.
- The branching ratio given by the explicit 5D-loop calculation is

$$\mathrm{Br}(\mu \to e\,\gamma) \simeq 2 \cdot 10^{-7} \left(\frac{3\mathrm{TeV}}{M_{KK}}\right)^4 \left(\frac{Y_*}{2}\right)^4$$





The Yukawa bounds from FCNC

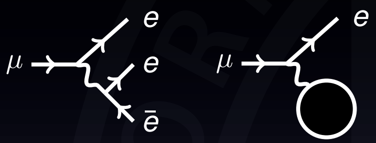
How to use this result?

$$\mathrm{Br}(\mu \to \mathrm{e}\,\gamma) \simeq 2 \cdot 10^{-7} \left(\frac{3\mathrm{TeV}}{\mathrm{M_{KK}}}\right)^4 \left(\frac{\mathrm{Y}_*}{2}\right)^4$$

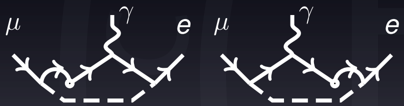
We can use the flavor bounds on the Yukawa coupling.

Use the branching ratio to constraint the Yukawa

There are two different FCNC processes:



gives a lower bound on Y.



gives an upper bound on Y.

They constraint the Yukawa from both sides!

The Yukawa bounds

Combine the two results

There is a tension between the $\mu \to e$ & $\mu \to e\gamma$ bounds!

For $M_{kk} = 3\,\mathrm{TeV}$,

$$Y_*(\mu \to e) > 3.7 \qquad Y_*(\mu \to e\gamma) < 0.2$$

These flavor bounds are important for model building.

The Yukawa bounds

Combine the two results

There is a tension between the $\mu \to e$ & $\mu \to e\gamma$ bounds!

For $M_{kk} = 3\,\mathrm{TeV}$,

$$Y_*(\mu \to e) > 3.7 \qquad Y_*(\mu \to e\gamma) < 0.2$$

These flavor bounds are important for model building.

To relax the tension: Custodial Symmetry, A_4 Symmetry, ...

Match the 5D & KK results.

Finiteness & Amplitude

Match the 5D & KK results.

Finiteness & Amplitude

Match the 5D & KK results.

Finiteness & Amplitude

Matching the KK & 5D results

Calculating the amplitude in KK picture, if we do

$$\mathcal{M} = \sum_{n=1}^{N} \int_0^{\infty} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

the result is different from the 5D result.

Matching the KK & 5D results

Calculating the amplitude in KK picture, if we do

$$\mathcal{M} = \sum_{n=1}^{N} \int_0^{\infty} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

the result is different from the 5D result.

5D $\neq$ KK !?

Matching the KK & 5D results

The reason is,

Wrong UV limit!

To match the 5D & KK, the momentum cutoff $\lesssim$ KK cutoff.

One way to do this is

$$\lim_{N\to\infty} \sum_{n=1}^{N} \int_0^{N\,M_{KK}} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

This gives the same result as in 5D. $\quad$ 5D $=$ KK

Conclusion

- One loop $\mu \to e\gamma$ in RS is finite.
- A tension between the $\mu \to e$ & $\mu \to e\gamma$ Yukawa bounds.
- The nontrivial UV limit in KK calculation is important for matching the KK & 5D results.

$\mu \to e\,\gamma$ in 5D

The diagrams of $\mu \to e\,\gamma$:

- Get the leading order result by doing mass insertions.
- Rotate all the flavor mixing to the Yukawa.
- It is an L-R operator: $a\,\bar{L}\sigma^{\mu\nu}F_{\mu\nu}E$
- From the gauge symmetry:

$$\epsilon_\mu \mathcal{M}^\mu \sim \epsilon_\mu \bar{u}\,(\,p^\mu + p_e^\mu - (m_\mu + m_e)\gamma^\mu\,)\,u$$

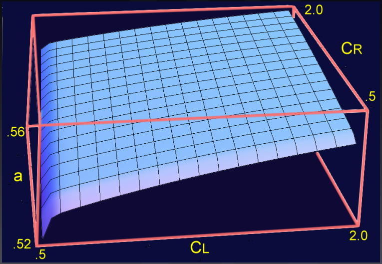
- To get a, we only need to calculate the coefficient of p^μ.

The Result

The result of the amplitude can be written as follows:

$$a_{k\ell} \times R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} \left(f_{L_i} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j} f_{-E_j} \right) \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

The loop integral gives $a_{k\ell} \simeq 0.5$.



$$\mathrm{Br}(\mu \to e\,\gamma)_{\text{thy}} > 8.2 \cdot 10^{-7} a^2 \left(\frac{3\mathrm{TeV}}{M_{KK}}\right)^4 \left(\frac{Y_*}{2}\right)^4$$

$$\mathrm{Br}(\mu \to e\,\gamma)_{\text{exp}} < 1.2 \cdot 10^{-11}$$

for $M_{kk} = 3\,\mathrm{TeV}$, we have

$$Y_* < 0.2$$

The fermion propagator in 5D

$$\left(-\not{p} + i\gamma^5\partial_5 + m\right)\Delta(p,z,z') = i\delta(z-z')$$

Trick:

$$\left(p^2 - \partial_5^2 + m^2\right)F(p,z,z') = i\delta(z-z')$$

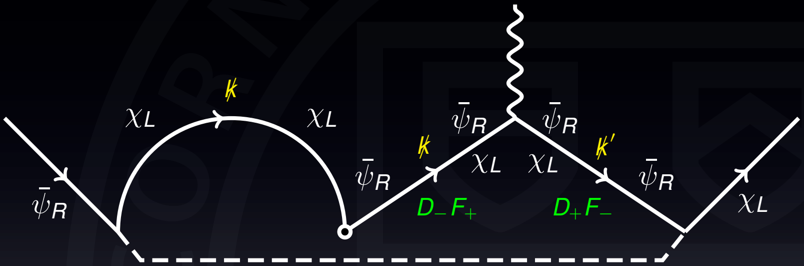
$$\Delta(p,z,z') = \left(\not{p} - i\gamma^5\partial_5 + m\right)F(p,z,z')$$

In flat XD case:

$$F(p,z,z') = A(p,z')\cos(p\,z) + B(p,z')\sin(p\,z)$$

The chiral BC

The chiral BC constraints the form of the amplitude!



$$\Psi_L = \begin{pmatrix} \chi_L \\ \bar\psi_L \end{pmatrix} \qquad \Psi_R = \begin{pmatrix} \chi_R \\ \bar\psi_R \end{pmatrix}$$

$$\Delta = \begin{pmatrix} \Delta_{\psi\chi} & \Delta_{\psi\psi} \\ \Delta_{\chi\chi} & \Delta_{\chi\psi} \end{pmatrix} = \begin{pmatrix} D_+ F_- & \sigma^\mu p_\mu F_+ \\ \bar\sigma^\mu p_\mu F_- & D_- F_+ \end{pmatrix}$$

How to calculate the 5D loop?

Feynman's trick with Bessel functions?? We don't know that...

However, we can

- Tayler expand the propagator into powers of the external momentum (p^μ, q^μ). $\frac{1}{(k^2+2k\cdot q)} = \frac{1}{k^2}\left(1 - \frac{2k\cdot q}{k^2} + \ldots\right)$.
- Isolate the p^μ terms. Get the coefficient. $\not{p}\gamma^\mu = 2p^\mu - \gamma^\mu \not{p}$.
- Solve the numerical integral, get the coefficient of $\mu \to e\gamma$.
  $a \times R'^2 \frac{e}{16\pi^2}\frac{v}{\sqrt{2}}\left(f_L YY^\dagger Y f_E\right)\bar{u}\not{\epsilon}u;\quad a = \int dx \int dy\,(\text{scalar function})$.

Why is $\mu \to e\,\gamma$ finite?

Lorentz invariance $+$ chiral BC gives the finiteness:

The propagators are composed of two parts

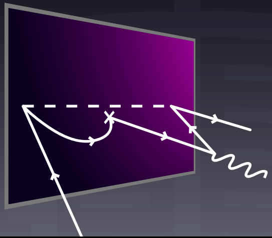
- The Dirac operator part (gives the γ^μ structure).
- The Bessel function part (contains the 5D profile).

In the UV limit, the photon vertex is pulled back to the IR brane.

- Can just look at the Dirac operator part.

Each loop contains two sectors:

- photon emission
- mass insertion



Why is $\mu \to e\,\gamma$ is finite?

- γ emission: Lorentz inv $+$ chiral BC gives $\left(\not{k}\gamma^\mu\not{k} - k^2\gamma^\mu\right)$
- mass insertion: The chiral BC gives $\not{k}$.
- Combining the two m-insertion amp, the leading order becomes

$$\mathcal{M}_{(a)} + \mathcal{M}_{(b)} \sim \not{k}\left(\not{k}\gamma^\mu\not{k} - k^2\gamma^\mu\right) + \left(\not{k}\gamma^\mu\not{k} - k^2\gamma^\mu\right)\not{k} = 0$$

- From NDA, only the leading order term can be divergent.

$\mu \to e\,\gamma$ is finite!