THERMODYNAMICS OF P-ADIC STRINGS

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T. Biswas, J. Cembranos, J. Kapusta PRL104:021601 (2010)

T. Biswas, J. Cembranos, J. Kapusta arXiv:1005.0430 [hep-th]

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Non-local Theories

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p-adic string model

- **The action given by:**

$$S = \frac{m_s^D}{g_p^2}\int d^Dx\left[-\frac{1}{2}\varphi\,\mathrm{e}^{-\Box/m_p^2}\varphi+\frac{1}{p+1}\varphi^{p+1}\right]$$

where

$$\frac{1}{g_p^2} \equiv \frac{1}{g_o^2}\frac{p^2}{p-1} \qquad m_p^2 \equiv \frac{2m_s^2}{\ln p}$$

P. Freund, M. Olson PLB 199, 186 (1987)
P. Freund, E Witten PLB 199, 191 (1987)
P. Frampton, Y. Okada PRL 60, 484 (1988)

describes the open string tachyon

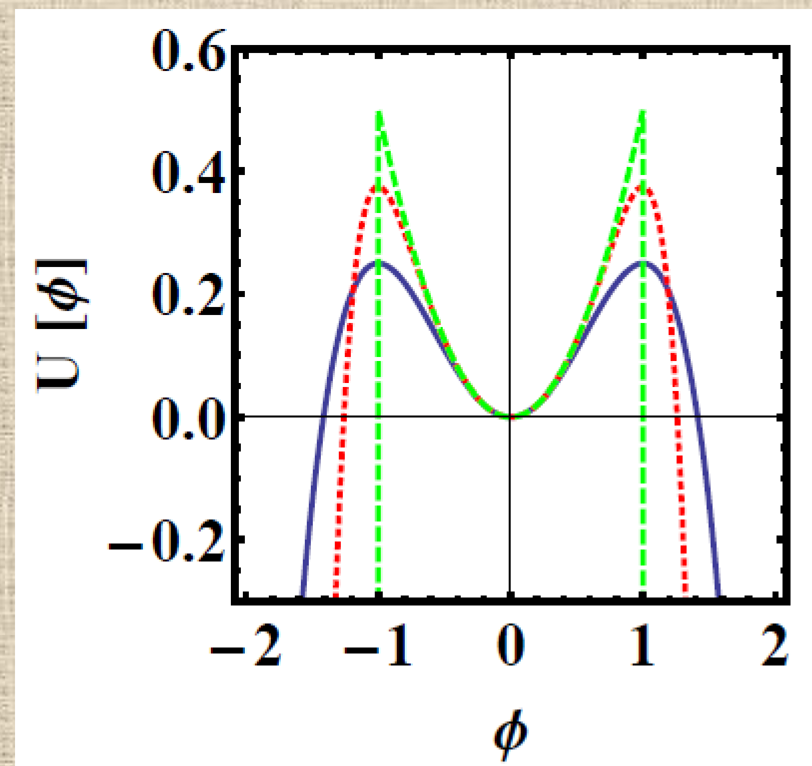
- **m_s is the string mass scale**
- **g_o is the open string coupling**
- **p is a prime number (may be generalized to other values)**

p-adic potential

- **We can talk about the p-adic potential as given by a constant field:**

$$U = (m_s^D/g_p^2)(\tfrac{1}{2}\varphi^2 - \tfrac{1}{p+1}\varphi^{p+1})$$

- **But the kinetic is not the standard one!!**



$p = 3,\, 7$ and $p \to \infty$

Free energy

- **The action for D=4 and p=3 is given by:**

$$S = \int_0^\beta d\tau \int d^3x \left[-\frac{1}{2}\phi(\mathrm{x},\tau) \mathrm{e}^{-(\partial^2/\partial\tau^2+\nabla^2)/M^2} \phi(\mathrm{x},\tau) - \lambda\phi^4(\mathrm{x},\tau) \right]$$

with

$$\phi \equiv \frac{m_s^2}{g_3}\varphi\,, \;\; \lambda \equiv -\frac{1}{18}\frac{g_o^2}{m_s^4}\,, \;\; M^2 \equiv \frac{2m_s^2}{\ln 3}\,.$$

- **To perform the functional integral, we use the Fourier transform**

$$\phi(\mathrm{x},\tau) = \frac{1}{\sqrt{\beta V}} \sum_n \sum_{\mathbf{k}} \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{x}+\omega_n\tau)} \phi_n(\mathbf{k})$$

Fourier transformation

- **The Matsubara frequency:** $\omega_n = 2\pi n T$
- **After integration in the imaginary time, we get the free action:**

$$S_0 = -\frac{1}{2}\sum_n \sum_{\mathbf{k}} D_0^{-1}(\omega_n, \mathbf{k})\phi_n^*(\mathbf{k})\phi_n(\mathbf{k})$$

- **We have used** $\phi_n^*(\mathbf{k}) = \phi_{-n}(-\mathbf{k})$
- **The action defines the free propagator:**

$$D_0(\omega_n, \mathbf{k}) = \mathrm{e}^{-(\omega_n^2+\mathbf{k}^2)/M^2}$$

  - **Difference with the standard field theory:**

$$\frac{1}{p^2} = \frac{1}{\mathbf{p}^2 + \omega_n^2} \longrightarrow D_0(\omega_n, \mathbf{p}) \equiv \mathrm{e}^{-p^2/M^2} = \mathrm{e}^{-(\mathbf{p}^2+\omega_n^2)/M^2}$$

Partition function

- **The partition function of the free theory is**

$$
\begin{aligned}
Z_0 &= N' \prod_n \prod_{\mathbf{k}} \left[\int_{-\infty}^{\infty} dA_n(\mathbf{k}) \mathrm{e}^{-\frac{1}{2} D_0^{-1}(\omega_n, \mathbf{k}) A_n^2(\mathbf{k})} \right] \\
&= N' \prod_n \prod_{\mathbf{k}} \left[2\pi D_0(\omega_n, \mathbf{k}) \right]^{1/2} .
\end{aligned}
$$

- **Taking the logarithm:**

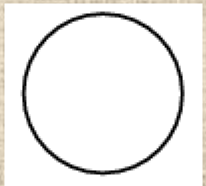
$$
\ln Z_0 = \ln N' + \tfrac{1}{2} \ln(2\pi) \sum_n \sum_{\mathbf{k}} + \tfrac{1}{2} \sum_n \sum_{\mathbf{k}} \ln[D_0(\omega_n, \mathbf{k})]
$$

  - **The 2 first terms are T independent and the normalization is choosen to cancel.**

Free energy: Zero order

- **The result is**

$$\ln Z_0 = -\tfrac{1}{2}\sum_n \sum_{\mathbf{k}} \frac{\omega_n^2 + \mathbf{k}^2}{M^2}$$



- **We can express the sum as a contour integral:**

$$T\sum_n f(k_0 = i\omega_n) = \frac{1}{4\pi i}\int_{-i\infty}^{i\infty} dk_0\,[f(k_0) + f(-k_0)]$$
$$+\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon} dk_0\,[f(k_0) + f(-k_0)]\,\frac{1}{e^{\beta k_0} - 1}$$

$$f(k_0) = \quad (-k_0^2 + \mathbf{k}^2)/M^2$$

$$k_0 = ik_4$$

  - No singularities in imaginary axis.
- **First integral: Vacuum contribution**
  - Zero by applying standard regularization
- **Second integral: Finite Temperature contribution**
  - Zero because $f(k_0)$ is analytic

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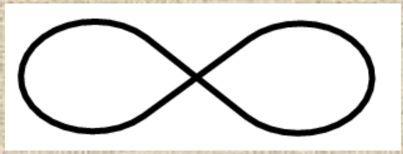
Free energy: First order

- The computation and Feynman rules are identical to a standard scalar quantum field theory:

$$\ln Z_1 = 3(-\lambda)\beta V \left[T \sum_n \int \frac{d^3k}{(2\pi)^3} D_0(\omega_n, \mathbf{k}) \right]^2$$



- Due to the exponential nature of the bare propagator, it is convergent in both the IR and UV

$$\sum_n \int \frac{d^3k}{(2\pi)^3} D_0^N(\omega_n, \mathbf{k}) = \left(\frac{M}{2\sqrt{N\pi}} \right)^3 \varsigma\left(\frac{2\sqrt{N}\pi T}{M} \right)$$

$$\varsigma(x) = \sum_{n=-\infty}^{\infty} e^{-n^2 x^2} = \vartheta_3(0, e^{-x^2})$$

  - Pressure: $$P_1 = -3\lambda \left(\frac{M^6 T^2}{2^6 \pi^3} \right) \varsigma^2 \left(\frac{2\pi T}{M} \right)$$

Free energy: First order

- **The third Jacobi elliptic theta function verifies:**

$$\varsigma(x) = \sum_{n=-\infty}^{\infty} e^{-n^2 x^2} = \frac{\sqrt{\pi}}{x} \sum_{m=-\infty}^{\infty} e^{-\frac{m^2\pi^2}{x^2}} = \frac{\sqrt{\pi}}{x}\,\varsigma\left(\frac{\pi}{x}\right)$$

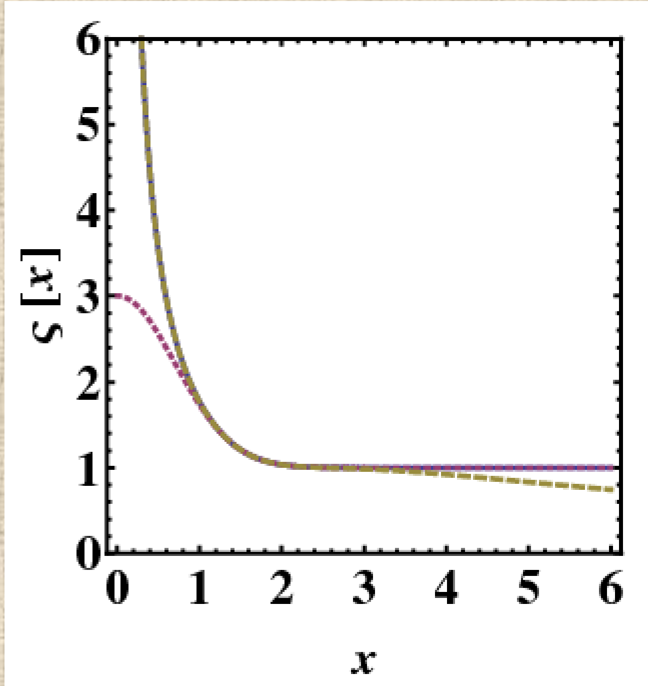
- **Asymptotic limits:**

$$\varsigma(x) \to \frac{\sqrt{\pi}}{x}\left[1 + 2\,e^{-\pi^2/x^2}\right], \quad x \ll 1,$$

$$\varsigma(x) \to 1 + 2\,e^{-x^2}, \quad x \gg 1,$$

- **High and low temperature Approximations:**

$$\ln Z_1 \to \begin{cases} = -\Lambda V/T\,, & T \ll M \\ = -4\pi\Lambda T V/M^2\,, & T \gg M \end{cases}$$



$\Lambda \equiv 3\lambda T_c^8$

$T_c \equiv M/2\sqrt{\pi}$

Thermal duality

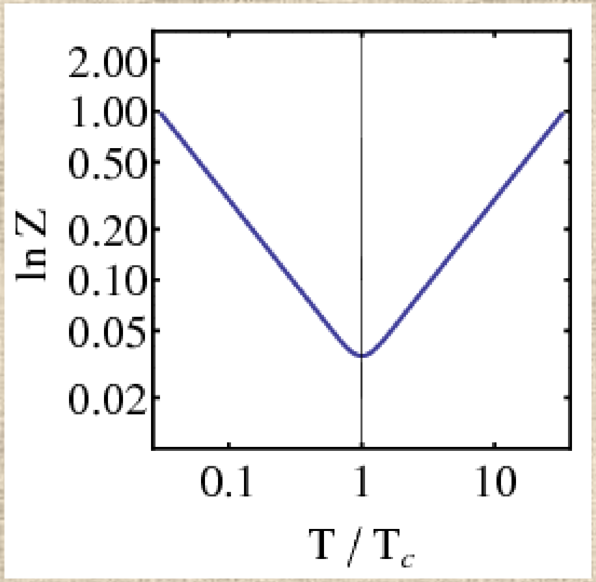
- **The third Jacobi elliptic theta function verifies:**

$$\varsigma(x) = \sum_{n=-\infty}^{\infty} e^{-n^2 x^2} = \frac{\sqrt{\pi}}{x} \sum_{m=-\infty}^{\infty} e^{-\frac{m^2 \pi^2}{x^2}} = \frac{\sqrt{\pi}}{x} \varsigma\left(\frac{\pi}{x}\right)$$

  - **n: Standard thermal modes**
  - **Higher n more suppressed at high temperature**
  - **m: Inverse thermal modes**

- **Thermal duality:**

$$Z_1(T) = Z_1\left(\frac{T_c{}^2}{T}\right)$$



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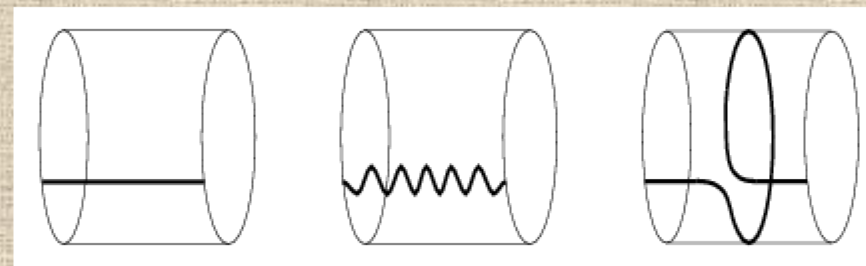
Thermal duality in string theory

- **Due to the compact nature of one dimension, there is not only the standard contribution of Matsubara thermal modes, but also the topological contributions of wrapped strings.**



$$Z_1(T) = Z_1\left(\frac{T_c^{\,2}}{T}\right)$$

$$2\pi T \longleftrightarrow \frac{m_s^2}{\pi T}$$

$$T_c = m_s/\pi\sqrt{2}$$

$$n \longleftrightarrow n_W$$

- **Hagedorn Transition:** $T_H = T_c/a$
  - **Bosonic string:** $a = \sqrt{2}$
  - **Type II superstring:** $a = 2$
  - **Heterotic string:** $a = 1 + 1/\sqrt{2}$

Ghost states

- **The lowest order non-zero contribution to the partition function gives rise to a first order contribution to the self energy by:**

$$D^{-1} = D_0^{-1} + \Pi$$

$$\begin{aligned} \Pi_1 &= 12\lambda T \sum_n \int \frac{d^3k}{(2\pi)^3} D_0(\omega_n, \mathbf{k}) \\ &= 12\lambda T \left(\frac{M}{2\sqrt{\pi}}\right)^3 \zeta\left(\frac{2\pi T}{M}\right) \end{aligned}$$

- **We note the reappearance of a pole**
  - **Possible interpretation: massive closed string states.**
- **It can be avoided by adding a counter term:** $-\frac{1}{2}\gamma\phi^2$

that cancels the self-energy contribution

  - **At first order:** $\gamma = -\frac{3\lambda M^4}{4\pi^2}$

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Self Energy

- **The counter term also contributes to the pressure at order lambda:**

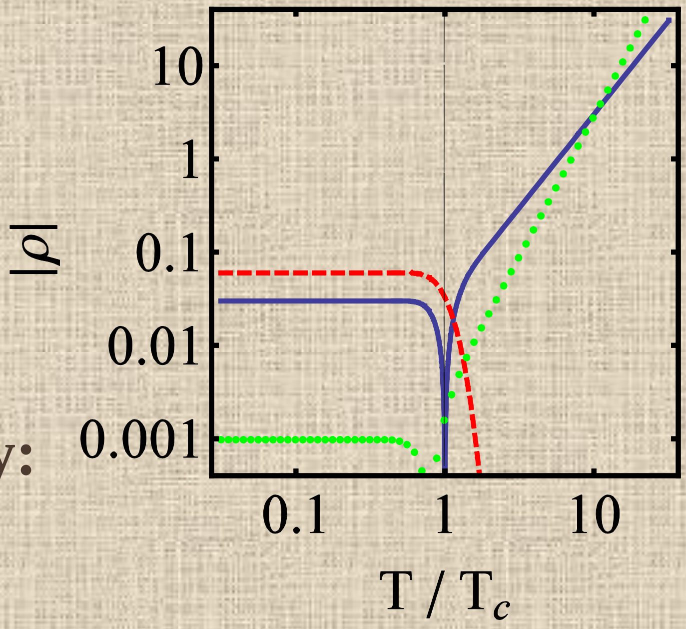
$$-\frac{1}{2}\gamma T \sum_n \int \frac{d^3k}{(2\pi)^3} D_0(\omega_n, \mathbf{k}) = \frac{3\lambda M^4}{8\pi^2} T \left(\frac{M}{2\sqrt{\pi}}\right)^3 \varsigma\left(\frac{2\pi T}{M}\right)$$

- **That implies that the total pressure may be written as:**

$$P_1 = -3\lambda \left(\frac{M^2}{4\pi}\right)^4 \frac{2\sqrt{\pi}T}{M} \varsigma\left(\frac{2\pi T}{M}\right) \left[\frac{2\sqrt{\pi}T}{M} \varsigma\left(\frac{2\pi T}{M}\right) - 2\right]$$

  - **A negative value of lambda leads to a positive vacuum energy:**

$$\Lambda = -3\lambda \left(\frac{M^2}{4\pi}\right)^4$$



Vacuum energy for general dimension

- **The p-adic string model can be formulated in arbitrary space-time dimension.**
- **The low temperature limit of this pressure fixes the vaccum energy:**

$$\Lambda_{\rm vac} = -\frac{p-1}{2}p!!\lambda\left(\frac{\zeta\left(\frac{1}{MR}\right)}{2\pi R}\right)^{d(p+1)/2}\left(\frac{M^2}{4\pi}\right)^{p+1}$$

- **In the 4 dimensional space:**
  - **For R M << 1:**

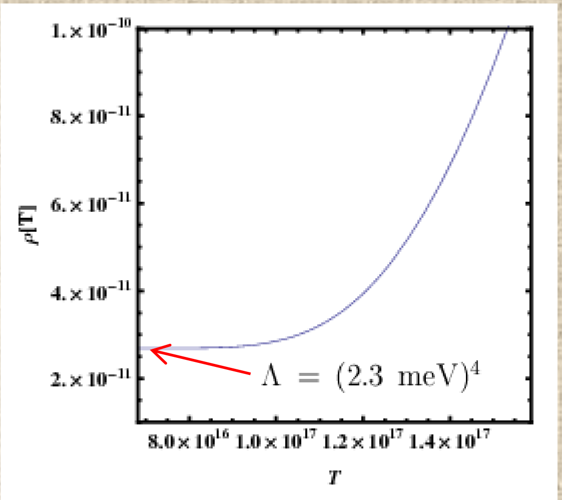
$$\frac{\Lambda}{M_p^4} = Q\left(\frac{m_s}{M_p}\right)^{p+3} \text{ with } Q \equiv \frac{(p-1)p!!}{2^{\frac{5+p}{2}}\pi^{4p-2}(p+1)(\ln p)^{p+1}}\left(\frac{p-1}{p^2}\right)^{\frac{p-1}{2}}$$

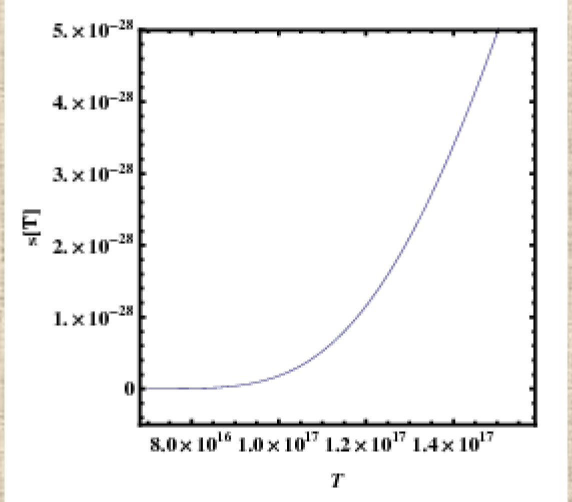
  - **For R M >> 1:**

$$\frac{\Lambda}{M_p^4} = Q_>\left(\frac{m_s}{M_p}\right)^{2} g_o^{p+1} \text{ with } Q_> \equiv \frac{(p-1)p!!}{2^8(p+1)(\ln p)^{\frac{5(p+1)}{2}}(2\pi)^{\frac{5p-7}{2}}}\left(\frac{p-1}{p^2}\right)^{\frac{p-1}{2}}$$

Cosmological Constant

- **The vacuum energy is generally suppressed by the ration between the string scale and the Planck scale.**
  - **This vacuum energy may be of phenomenological interest for inflationary studies in the early Universe.**
  - **Or may be interpreted as dark energy for the late evolution.**
    - **A very large p and/or a very small coupling are needed.**





$p = 7 \quad m_s = 385 \text{ PeV} \quad g_0 \ll 1$

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arXiv:1005.0430 [hep-th]

Conclusions

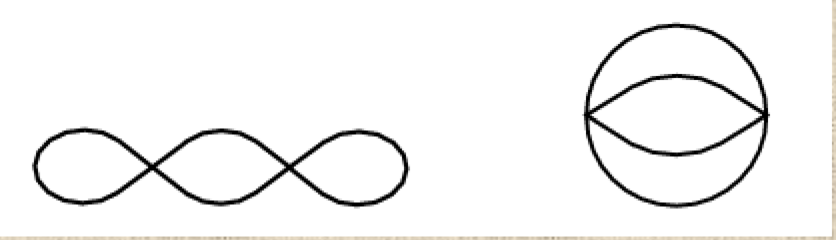
- **We have analyzed the main thermodynamical properties of p-adic string models, that describe the tachyon phenomenology in bosonic string theory.**
  - **We have reproduced known results of string theory**
    - **Thermal duality (leading order, p=3)**
    - **Temperature dependence of radiative corrections**
    - **...**
- **P-adic models constitute a motivated example of non-local field theories.**
  - **We have developed a basic approach to this study:**
    - **Free theory: physical degrees of freedom.**
    - **Self-energy: Ghost states**
    - **...**

BACK-UP SLIDES

Thermodynamics of p-adic strings

Free energy: Second order

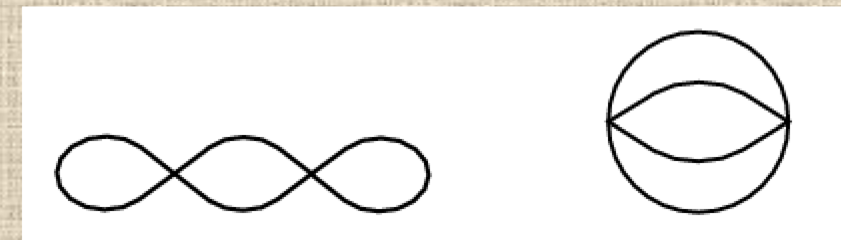
- **There are two contributions at second order:**



Necklace Diagram Sunset Diagram

Necklace contribution

- **There are two contributions at second order:**



- **Necklace contribution:**
  - Can be computed as

$$P_{2,\mathrm{necklace}} = 36\lambda^2 \left[T \left(\frac{M}{2\sqrt{\pi}} \right)^3 \varsigma\left(\frac{2\pi T}{M} \right) \right]^2 \left[T \left(\frac{M}{2\sqrt{2\pi}} \right)^3 \varsigma\left(\frac{2\sqrt{2}\pi T}{M} \right) \right]$$

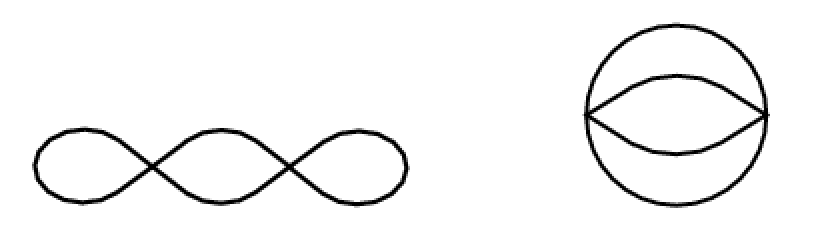
  - For high temperatures:

For $T \gg M$, $\varsigma \to 1$,

$$P_1 \sim (M^3T)(\lambda M^3 T) \text{ while } P_{2,\mathrm{necklace}} \sim (M^3T)(\lambda M^3 T)^2$$

Necklace contribution

- **There are two contributions at second order:**



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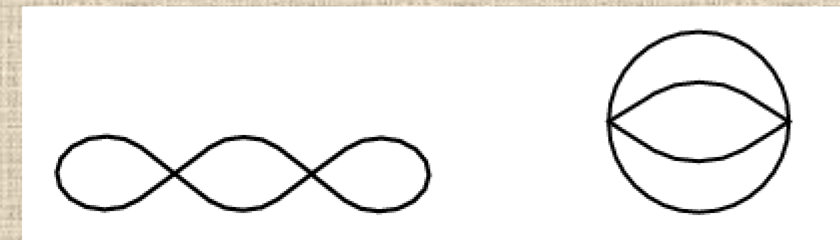
  - **For low temperatures:**

$$T \ll M,\ \varsigma \to M/T$$

$$P_1 \sim M^4(\lambda M^4) \text{ while } P_{2,\text{necklace}} \sim M^4(\lambda M^4)^2$$

Sunset contribution

- **There are two contributions at second order:**



- **Sunset contribution:**
  - **It is proportional to:**

$$\left[T\sum_{n_1}\int\frac{d^3k_1}{(2\pi)^3}D_0(\omega_{n_1},\mathbf{k}_1)\right]\cdots\left[T\sum_{n_4}\int\frac{d^3k_4}{(2\pi)^3}D_0(\omega_{n_4},\mathbf{k}_4)\right]$$
$$\times(2\pi)^3\delta(\mathbf{k}_1+\cdots\mathbf{k}_4)\beta\delta_{n_1+\cdots n_4,0}$$

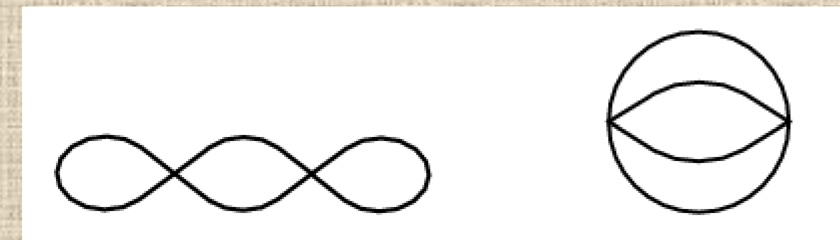
  - **And the pressure can be written in terms of the third Jacobi elliptic theta function:**

$$P_{\mathrm{sunset}}=\frac{3}{2}\lambda^2\left(\frac{M}{2\sqrt{\pi}}\right)^9\chi(T,M)$$

$$\chi(T,M)=\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\left[\theta_3\left(\tfrac{1}{2}\phi,e^{-x^2}\right)\right]^4$$

Sunset contribution

- **There are two contributions at second order:**



- **Sunset contribution:**
  - **It verifies:**

$$\theta_3(u, \mathrm{e}^{-x^2}) = \frac{\sqrt{\pi}}{x}\mathrm{e}^{-u^2/x^2}\theta_3\left(\frac{i\,\pi\,u}{x^2}, \mathrm{e}^{-\pi^2/x^2}\right)$$

  - **It also allows an interpretation in terms of inverse modes, but they need to be weighted in a different way.**

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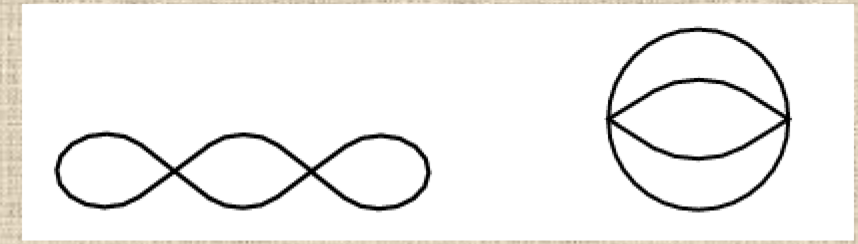
Sunset contribution

- **There are two contributions at second order:**



- **Sunset contribution:**

$$P_{\rm sunset} = \frac{3}{2}\lambda^2\left(\frac{M}{2\sqrt{\pi}}\right)^9 \chi(T,M) \qquad \chi(T,M) = \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\left[\theta_3\left(\tfrac{1}{2}\phi,\, {\rm e}^{-x^2}\right)\right]^4$$

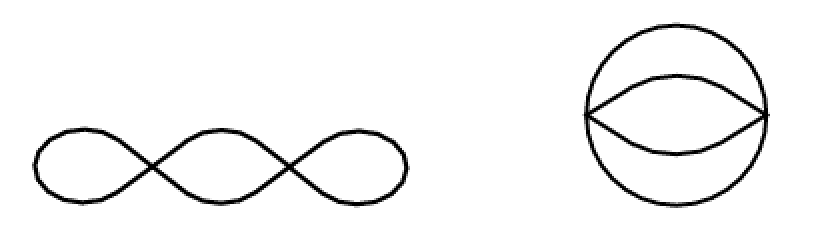
  - **For high temperatures:**

$$\chi(T \gg M, M) = T^3$$

$$P_{\rm sunset}(T \gg M) = \frac{3}{2}\lambda^2\left(\frac{M}{2\sqrt{\pi}}\right)^9 T^3$$

Sunset contribution

- **There are two contributions at second order:**



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$$\chi(T,M) = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \left[\theta_3\left(\tfrac{1}{2}\phi, \mathrm{e}^{-x^2}\right)\right]^4$$

  - **For low temperatures:**

$$\chi(T \ll M, M) = \frac{1}{2}\left(\frac{M}{2\sqrt{\pi}}\right)^3$$

$$P_{\text{sunset}}(T \ll M) = \frac{3}{4}\lambda^2 \left(\frac{M^2}{4\pi}\right)^6$$

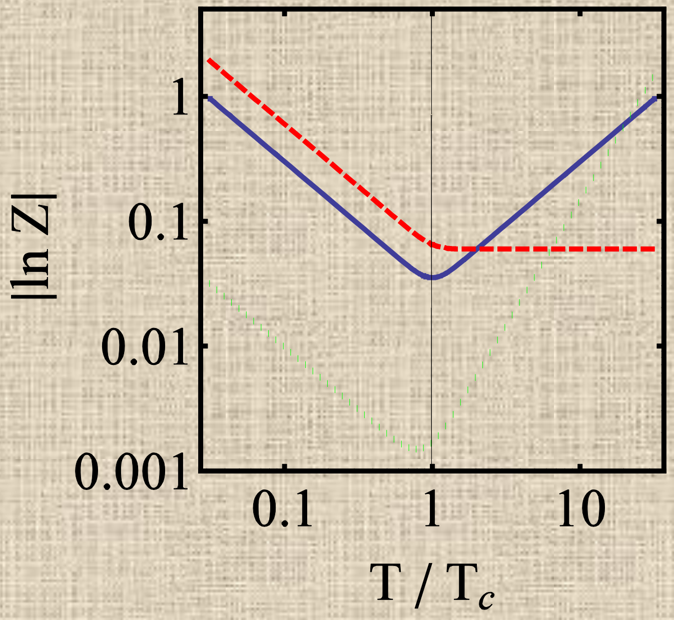
Perturbative computation

- **These perturbative analyses suggest some general power counting arguments:**
  - **For low temperatures, an l-loop graph is suppressed as**

$$g_o^{2(l-1)}$$

  - **For high temperatures, the expansion parameter is**

$$(g_o^2 T/m_s)^{l-1}$$



Perturbative pressure

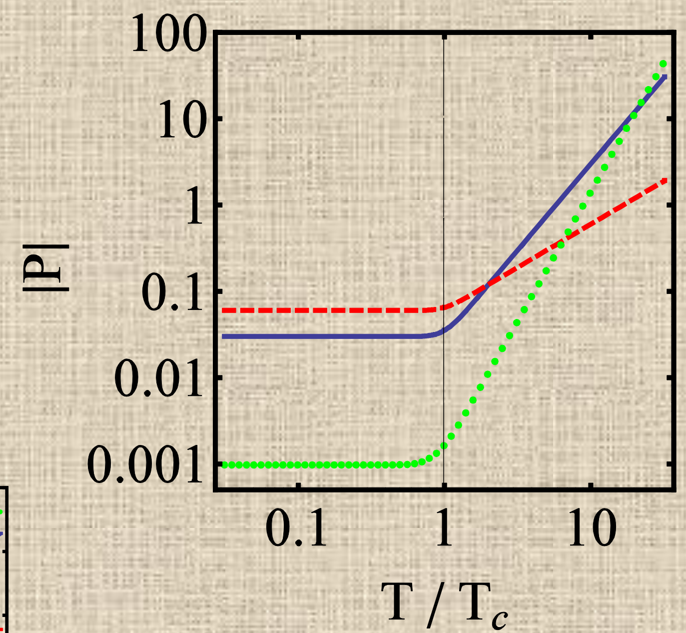
- **These perturbative computation is extended to any thermodynamical property:**

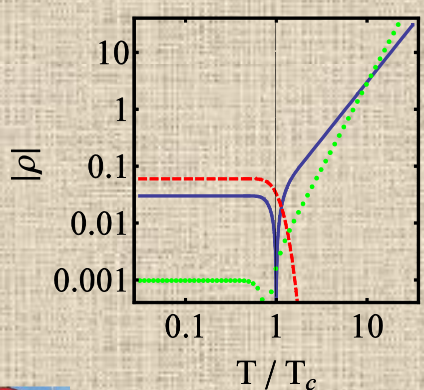
$$\ln Z \to \begin{cases} = -\Lambda V/T\,, & T \ll M \\ = -4\pi\Lambda T V/M^2\,, & T \gg M \end{cases}$$

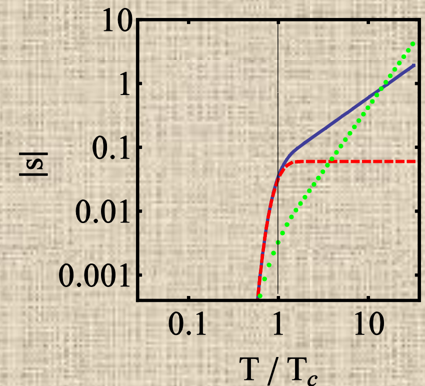
$$P = \frac{\partial(T\ln Z)}{\partial V} = \begin{cases} -\Lambda\,, & T \ll M \\ -4\pi\Lambda T^2/M^2\,, & T \gg M \end{cases}$$

$$s = \frac{\partial(T\ln Z)}{V\partial T} = \begin{cases} 0\,, & T \ll M \\ -8\pi\Lambda T/M^2\,, & T \gg M \end{cases}$$

$$\rho = \frac{T^2}{V}\frac{\partial(\ln Z)}{\partial T} = \begin{cases} \Lambda\,, & T \ll M \\ -4\pi\Lambda T^2/M^2\,, & T \gg M \end{cases}$$







Perturbative entropy

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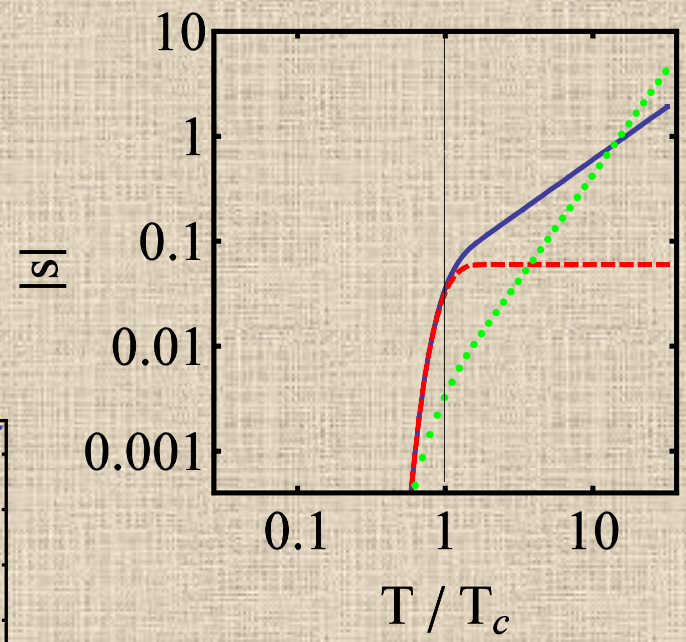
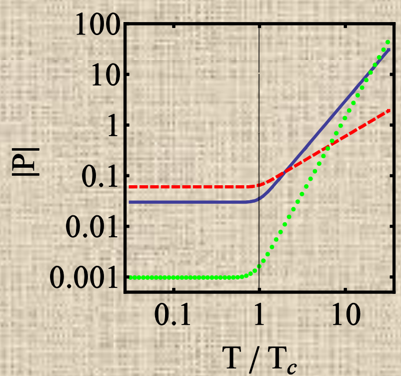
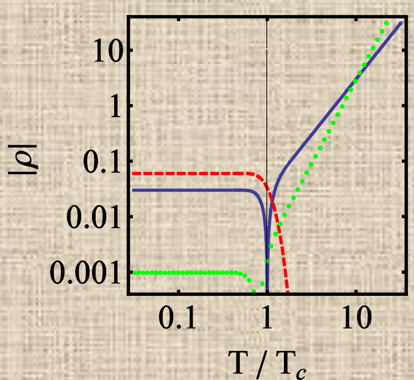






Perturbative energy

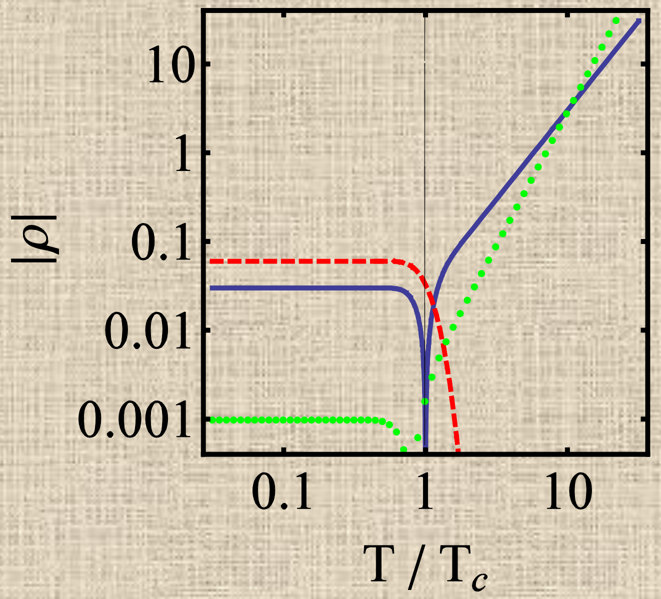
- **These perturbative computation is extended to any thermodynamical property:**

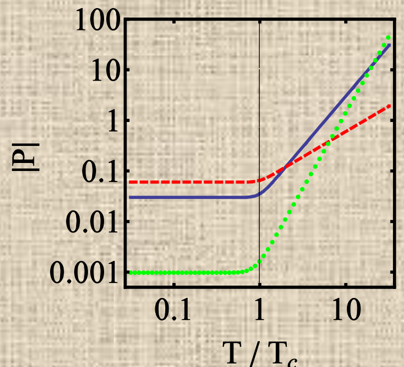
$$\ln Z \rightarrow \begin{cases} = -\Lambda V/T\,, & T \ll M \\ = -4\pi\Lambda T V/M^2\,, & T \gg M \end{cases}$$

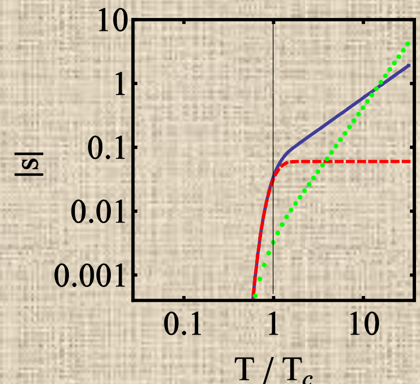
$$P = \frac{\partial(T\ln Z)}{\partial V} = \begin{cases} -\Lambda\,, & T \ll M \\ -4\pi\Lambda T^2/M^2\,, & T \gg M \end{cases}$$

$$s = \frac{\partial(T\ln Z)}{V\partial T} = \begin{cases} 0\,, & T \ll M \\ -8\pi\Lambda T/M^2\,, & T \gg M \end{cases}$$

$$\rho = \frac{T^2}{V}\frac{\partial(\ln Z)}{\partial T} = \begin{cases} \Lambda\,, & T \ll M \\ -4\pi\Lambda T^2/M^2\,, & T \gg M \end{cases}$$

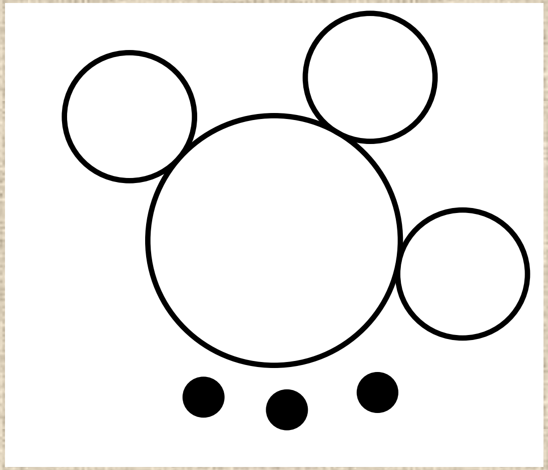






Ring diagrams

- **In ordinary field theories with massless particles, one generally finds infrared divergences in these diagrams, that becomes more severe with increasing number of loops:**

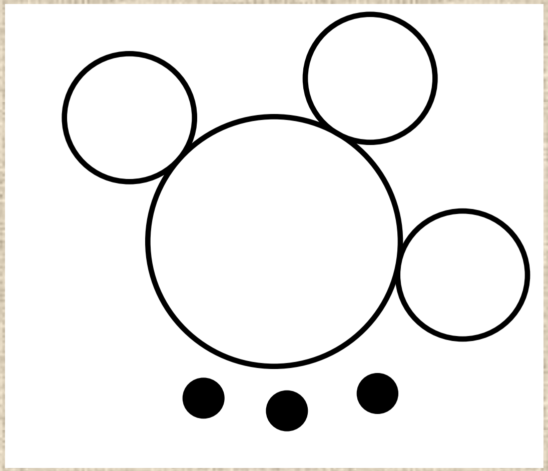


$$P_{\rm ring} = \frac{1}{2} T \sum_n \int \frac{d^3k}{(2\pi)^3} \sum_{l=2}^{\infty} \frac{1}{l} \left[-\Pi_1 D_0(\omega_n, \mathbf{k})\right]^l$$
$$= -\frac{1}{2} T \sum_n \int \frac{d^3k}{(2\pi)^3} \left[\ln\left(1 + \Pi_1 D_0\right) - \Pi_1 D_0\right]$$

- standard case: Non analytic result coming from the n=0 in the Matsubara summation
  - proportional to $\lambda^{3/2}$

Ring diagrams

- In ordinary field theories with massless particles, one generally finds infrared divergences in these diagrams, that becomes more severe with increasing number of loops:

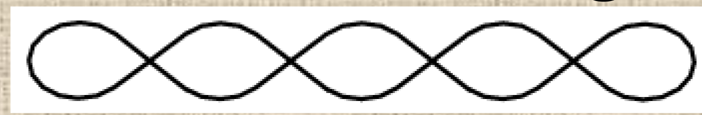


$$
\begin{aligned}
P_{\rm ring} &= \tfrac{1}{2} T \sum_n \int \frac{d^3k}{(2\pi)^3} \sum_{l=2}^{\infty} \frac{1}{l} \left[-\Pi_1 D_0(\omega_n, \mathbf{k}) \right]^l \\
&= -\tfrac{1}{2} T \sum_n \int \frac{d^3k}{(2\pi)^3} \left[\ln\left(1 + \Pi_1 D_0\right) - \Pi_1 D_0 \right]
\end{aligned}
$$

- p-adic case: individual diagrams are already convergent.
  - No need to sum the series, that converges even much rapidly than a logarithm.

Necklace diagrams

- **One can sum the infinite string of diagrams:**



- **With the result:**

$$P_{\rm necklace} = 3\sum_{l=0}^{\infty}(-\lambda)^{l+1}12^l\left[T\sum_n\int\frac{d^3k}{(2\pi)^3}D_0(\omega_n,\mathbf{k})\right]^2$$

$$\times\left[T\sum_n\int\frac{d^3k}{(2\pi)^3}D_0^2(\omega_n,\mathbf{k})\right]^l$$

$$= -3\lambda\left[T\left(\frac{M}{2\sqrt{\pi}}\right)^3\varsigma\left(\frac{2\pi T}{M}\right)\right]^2\sum_{l=0}^{\infty}\left[-12\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^3\varsigma\left(\frac{2\sqrt{2}\pi T}{M}\right)\right]^l$$

$$= \frac{-3\lambda\left[T\left(\frac{M}{2\sqrt{\pi}}\right)^3\varsigma\left(\frac{2\pi T}{M}\right)\right]^2}{1+12\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^3\varsigma\left(\frac{2\sqrt{2}\pi T}{M}\right)}.$$

- **With the standard self-energy insertion:**

$$P_{\rm necklace} = \frac{-3\lambda\left(\frac{M^2}{4\pi}\right)^4\frac{2\sqrt{\pi}T}{M}\varsigma\left(\frac{2\pi T}{M}\right)\left[\frac{2\sqrt{\pi}T}{M}\varsigma\left(\frac{2\pi T}{M}\right)-2\right]}{1+12\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^3\varsigma\left(\frac{2\sqrt{2}\pi T}{M}\right)}$$

Necklace diagrams

- **This expression has a maximum temperature where the denominator vanishes**

$$P_{\mathrm{necklace}} = \frac{-3\lambda\left(\frac{M^2}{4\pi}\right)^4 \frac{2\sqrt{\pi}T}{M}\varsigma\left(\frac{2\pi T}{M}\right)\left[\frac{2\sqrt{\pi}T}{M}\varsigma\left(\frac{2\pi T}{M}\right)-2\right]}{1+12\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^3\varsigma\left(\frac{2\sqrt{2}\pi T}{M}\right)}$$

- **We can interpret this fact as arising due to the potential being unbounded from below for the large values of the field.**

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General even powered potential

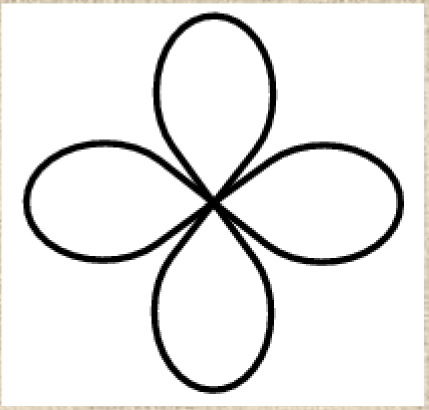
- **The above analysis can be extended to an interaction term of the form:**

$$-\lambda\phi^{2N} \qquad N=(p+1)/2$$

  - **In this case the energy dimension of lambda is -4(N-1).**

- **First order:**

$$\ln Z_1 = (2N-1)!!(-\lambda)\beta V\left[T\sum_n\int\frac{d^3k}{(2\pi)^3}D_0(\omega_n,\mathbf{k})\right]^N$$



- **Self energy:**

$$\Pi_1 = 2N(2N-1)!!\lambda\left[T\sum_n\int\frac{d^3k}{(2\pi)^3}D_0(\omega_n,\mathbf{k})\right]^{N-1}$$

- **Counter term:** $-\frac{1}{2}\gamma\phi^2$

$$\gamma = -2N(2N-1)!!\lambda\left(\frac{M^2}{4\pi}\right)^{2(N-1)}$$

Pressure: First order

- **The total pressure at first order is given by:**

$$P_1 = -(2N-1)!!\lambda\left(\frac{M^2}{4\pi}\right)^{2N}\frac{2\sqrt{\pi}T}{M}S\left(\frac{2\pi T}{M}\right)\left[\left(\frac{2\sqrt{\pi}T}{M}S\left(\frac{2\pi T}{M}\right)\right)^{N-1}-N\right]$$

- **It implies a vacuum energy:**

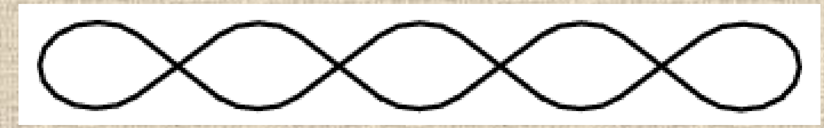
$$\epsilon_{\rm vac} = -(N-1)(2N-1)!!\lambda\left(\frac{M^2}{4\pi}\right)^{2N}$$

- **In contrast, the high temperature limit is given by:**

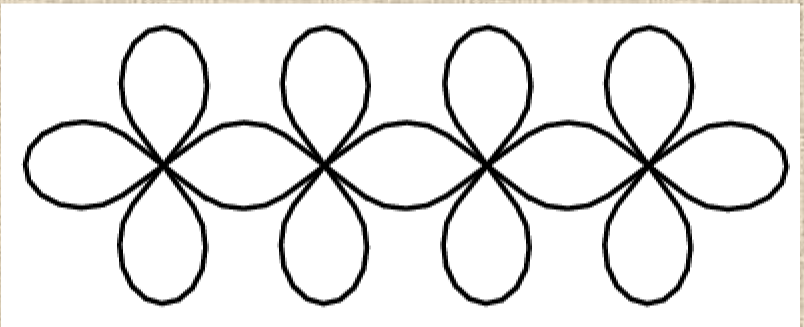
$$P_1 = -(2N-1)!!\lambda\left(\frac{M}{2\sqrt{\pi}}\right)^{3N}T^N$$

Necklace diagrams

- **Necklace diagrams are obtained by connecting each vertex with two legs.**
  - **N=2: The end vertices have one closed loop attached to them while interior vertices have none.**



  - **N>2: The end vertices have N-1 closed loops attached to them while the interior vertices have N-2 closed loops attached.**



Necklace pressure

- **The pressure can be computed as:**

$$P_{\text{necklace}} = (-\lambda)(2N-1)!!\left[T\sum_n \int \frac{d^3k}{(2\pi)^3} D_0(\omega_n,\mathbf{k})\right]^N \times$$

$$\sum_{l=0}^{\infty}\left\{(-\lambda)4(2N-1)!!\left[T\sum_n \int \frac{d^3k}{(2\pi)^3} D_0(\omega_n,\mathbf{k})\right]^{N-2} T\sum_n \int \frac{d^3k}{(2\pi)^3} D_0^2(\omega_n,\mathbf{k})\right\}^l$$

$$= \frac{-(2N-1)!!\lambda\left[T\left(\frac{M}{2\sqrt{\pi}}\right)^3 S\left(\frac{2\pi T}{M}\right)\right]^N}{1+4(2N-1)!!\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^3 S\left(\frac{2\sqrt{2}\pi T}{M}\right)\left[T\left(\frac{M}{2\sqrt{\pi}}\right)^3 S\left(\frac{2\pi T}{M}\right)\right]^{N-2}}$$

- **And taking into account the self-energy corrections on the loops attached to the end vertices:**

$$P_{\text{necklace}} = \frac{-(2N-1)!!\lambda\left(\frac{M^2}{4\pi}\right)^{2N}\frac{2\sqrt{\pi}T}{M} S\left(\frac{2\pi T}{M}\right)\left[\left(\frac{2\sqrt{\pi}T}{M} S\left(\frac{2\pi T}{M}\right)\right)^{N-1} - N\right]}{1+4(2N-1)!!\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^3 S\left(\frac{2\sqrt{2}\pi T}{M}\right)\left[T\left(\frac{M}{2\sqrt{\pi}}\right)^3 S\left(\frac{2\pi T}{M}\right)\right]^{N-2}}$$

Necklace maximum

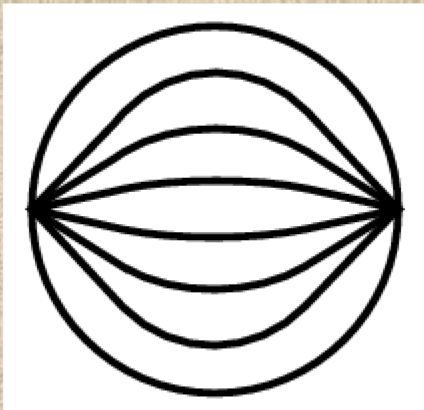
- **And taking into account the self-energy corrections on the loops attached to the end vertices:**

$$P_{\mathrm{necklace}} = \frac{-(2N-1)!!\lambda\left(\frac{M^2}{4\pi}\right)^{2N}\frac{2\sqrt{\pi}T}{M}S\left(\frac{2\pi T}{M}\right)\left[\left(\frac{2\sqrt{\pi}T}{M}S\left(\frac{2\pi T}{M}\right)\right)^{N-1}-N\right]}{1+4(2N-1)!!\lambda T\left(\frac{M}{2\sqrt{2\pi}}\right)^{3}S\left(\frac{2\sqrt{2}\pi T}{M}\right)\left[T\left(\frac{M}{2\sqrt{\pi}}\right)^{3}S\left(\frac{2\pi T}{M}\right)\right]^{N-2}}$$

- **There is again a maximum temperature determined by the vanishing of the denominator, that may be related with the fact that the potential is not bounded from below.**

Sunset diagrams

- The sunset diagram has two vertices, and every leg of one vertex is connected to a leg of the other one.



- It is proportional to

$$\left[T\sum_{n_1}\int\frac{d^3k_1}{(2\pi)^3}D_0(\omega_{n_1},\mathbf{k}_1)\right]\cdots\left[T\sum_{n_{2N}}\int\frac{d^3k_{2N}}{(2\pi)^3}D_0(\omega_{n_{2N}},\mathbf{k}_{2N})\right]$$
$$\times(2\pi)^3\delta(\mathbf{k}_1+\cdots\mathbf{k}_{2N})\beta\delta_{n_1+\cdots n_{2N},0}\,.$$

- And for the p-adic case:

$$P_{\text{sunset}}=\lambda^2\frac{(2N)!}{2(2N)^{3/2}}\left(\frac{M}{2\sqrt{\pi}}\right)^{3(2N-1)}\chi(T,M)$$

Sunset diagrams

- **The pressure can be computed as:**

$$P_{\text{sunset}} = \lambda^2 \frac{(2N)!}{2(2N)^{3/2}} \left(\frac{M}{2\sqrt{\pi}} \right)^{3(2N-1)} \chi(T, M)$$

- **With**

$$\chi(T, M) = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \left[\theta_3 \left(\tfrac{1}{2}\phi, \mathrm{e}^{-x^2} \right) \right]^{2N}$$

  - **High temperature limit:**

$$\chi(T \gg M) = T^{2N-1}$$

$$P_{\text{sunset}}(T \gg M) = \lambda^2 \frac{(2N)!}{2(2N)^{3/2}} \left(\frac{M}{2\sqrt{\pi}} \right)^{3(2N-1)} T^{2N-1}$$

  - **Low temperature limit:**

$$\chi(T \ll M) = \frac{1}{\sqrt{2N}} \left(\frac{M}{2\sqrt{\pi}} \right)^{2N-1}$$

$$P_{\text{sunset}}(T \ll M) = \lambda^2 \frac{(2N)!}{8N^2} \left(\frac{M^2}{4\pi} \right)^{2(2N-1)}$$

General dimension

- **The p-adic string model can be formulated in arbitrary space-time dimension.**
- **We will proceed in two steps:**
  - **1.- Compute the D dimensional thermodynamics**
  - **2.- Compactify d=D-4 dimensions on d circles of radius R.**
- **The results simplify in two different limits:**
  - **A.- RM << 1**
  - **B.- RM >> 1**

General dimension

- **The p-adic thermal action for D = d + 4 dimensions is given by**

$$S = \int_0^\beta d\tau \int_0^{2\pi R} d^d y \int d^3x \left[-\frac{1}{2}\phi e^{-(\partial^2/\partial\tau^2 + \nabla_y^2 + \nabla_x^2)/M^2}\phi - \lambda\phi^{p+1} \right]$$

with $\lambda \equiv -\dfrac{m_s^{\frac{-D(p-1)}{2}} g_p^{p-1}}{p+1}$

- **The above computations can be generalized by adding the contribution from the entire Kaluza-Klein tower of scalar modes:**

$$\sum_n \int \frac{d^3k}{(2\pi)^3} e^{-(\omega_n^2 + \mathbf{k}^2)/M^2} \to \sum_n \sum_{\{n_i\}} \int \frac{d^3k}{(2\pi)^3} e^{-(\omega_n^2 + \sum_i \omega_{n_i}^2 + \mathbf{k}^2)/M^2}$$
$$\equiv \sum_n \sum_{\{n_i\}} \int \frac{d^3k}{(2\pi)^3} D_d(\{\omega_n\}, \mathbf{k})$$

$$\omega_{n_i} = \frac{n_i}{R}$$

General dimension: leading order

- **Partition function:**

$$\begin{aligned}\ln Z_1 &= (2N-1)!!(-\lambda)\beta(2\pi R)^d V \left[T(2\pi R)^{-d} \sum_{\{n_i\}} \int \frac{d^3k}{(2\pi)^3} D_d(\{\omega_{n_i}\}, \mathbf{k}) \right]^N \\ &= (2N-1)!!(-\lambda)\beta(2\pi R)^d V \left[\left(\frac{M}{2\sqrt{\pi}} \right)^3 T\varsigma(x) \left(\frac{\varsigma(\frac{1}{MR})}{2\pi R} \right)^d \right]^N\end{aligned}$$

$$x = \frac{2\pi T}{M}$$

- **Pressure:**

$$P_1 = (2N-1)!!(-\lambda) \left[\left(\frac{M}{2\sqrt{\pi}} \right)^3 T\varsigma(x) \left(\frac{\varsigma(\frac{1}{MR})}{2\pi R} \right)^d \right]^N$$

- **Self energy:**

$$\begin{aligned}\Pi_1 &= 2N(2N-1)!!\lambda \left[T(2\pi R)^{-d} \sum_{\{n_i\}} \int \frac{d^3k}{(2\pi)^3} D_d(\{\omega_{n_i}\}, \mathbf{k}) \right]^{N-1} \\ &= 2N(2N-1)!!\lambda \left[T\varsigma(x) \left(\frac{M}{2\sqrt{\pi}} \right)^3 \left(\frac{\varsigma(\frac{1}{MR})}{2\pi R} \right)^d \right]^{N-1}\end{aligned}$$

General dimension: leading order

- **Counter term**

$$-\tfrac{1}{2}\gamma\phi^2 \qquad \gamma = -\frac{3\lambda M^4}{4\pi^2}(2\pi R)^{-d}\varsigma^d\left(\frac{1}{RM}\right)$$

- **Counter term pressure:**

$$\begin{aligned} P_c &= -\tfrac{1}{2}\frac{\gamma T}{(2\pi R)^d}\sum_n\int\frac{d^3k}{(2\pi)^3}D_d(\{\omega_n\},\mathbf{k}) \\ &= \frac{3\lambda M^4 T}{8\pi^2(2\pi R)^d}\left(\frac{M}{2\sqrt{\pi}}\right)^3\varsigma_d\left(\frac{2\pi T}{M},\frac{1}{RM}\right) \end{aligned}$$

- **Total pressure:**

$$P_1 = -3\lambda\left(\frac{M^2}{4\pi}\right)^4\frac{2\sqrt{\pi}T}{M}\varsigma\left(\frac{2\pi T}{M}\right)\frac{\varsigma^{2d}\left(\frac{1}{RM}\right)}{(2\pi R)^{2d}}\left[\frac{2\sqrt{\pi}T}{M}\varsigma\left(\frac{2\pi T}{M}\right)-2\right]$$

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