MONDian Dark Matter

Work in progress with Djordje Minic and Jack Ng

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- Milgrom (1983):

$$F = \begin{cases} m\,a_N = \frac{G\,M\,m}{r^2} & a \gg a_0 \\ m\sqrt{a_N\,a_0} = \frac{\sqrt{G\,M\,a_0}\,m}{r} & a \ll a_0 \end{cases}$$

- Numerical coincidence:

$$a_0 \sim 1.2 \times 10^{-10}\,ms^{-2} \sim H_0 \sim \sqrt{\frac{\Lambda}{3}}$$

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- fits hundreds of galaxy rotation curves very well — both trends and shapes
- automatic Tully-Fisher relation ($M \sim v^4$)

A Stupid Question

- Could there be some kind of dark matter that behaves like MOND in the non-relativistic limit?

- Erik Verlinde (arXiv:1001.0785):

$$
\begin{aligned}
F_{entropic} &= T\,\frac{\Delta S}{\Delta x} && \text{first law of thermodynamics} \\
&= T\,(2\,\pi\,k_B\,m) && \text{similar argument by Bekenstein} \\
&= \left(\frac{a}{2\,\pi\,k_B}\right)(2\,\pi\,k_B\,m) && \text{Unruh temperature formula} \\
&= m\,a
\end{aligned}
$$

What is a?

- Consider a spherical holographic screen of radius r with temperature T

$$\begin{aligned} a &= 2\,\pi\,k_B\,T \qquad \text{Unruh temperature formula} \\ &= 2\,\pi\,k_B\left(\frac{2\,E}{N\,k_B}\right) \qquad \text{equipartition energy:}\ \ E = \frac{1}{2}\,N\,k_B\,T \\ &= 4\pi\left(\frac{2\,E}{A/G}\right) \qquad \text{t'Hooft and Susskind:}\ \ N = \frac{A}{G} \\ &= \frac{G\,M}{r^2} \qquad \text{using } E = M \text{ and } A = 4\,\pi\,r^2 \end{aligned}$$

Generalization to our Universe with a positive Λ?

- Inertial observer (Hawking):

$$T_\Lambda = \frac{1}{2\,\pi\,k_B}\,a_0 \quad ; \quad a_0 = \sqrt{\frac{\Lambda}{3}}$$

- Non-inertial observer (Deser):

$$T_{\Lambda+a} = \frac{1}{2\,\pi\,k_B}\sqrt{a^2 + a_0^2}$$

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- Net temperature measured by the non-inertial observer (due to some matter sources that cause acceleration a):

$$\begin{aligned} T' &= T_{\Lambda+a} - T_\Lambda \\ &= \frac{1}{2\,\pi\,k_B}\left(\sqrt{a^2 + a_0^2} - a_0\right) \end{aligned}$$

Theoretical Suggestion for Dark Matter?

- Consider a spherical holographic screen of radius r with temperature T'
- Entropic force in de Sitter space:

$$F_{entropic} = m\left(\sqrt{a^2 + a_0^2} - a_0\right) = m\,\frac{G\,M_{\rm total}}{r^2}$$

- What is $M_{\rm total}$?

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- What is $M_{\rm total}$?
- If $M_{\rm total} = M_{\rm ordinary}$, then RHS suggests that $F_{entropic} = m\,a_N$
  $\Rightarrow$ no modification of gravity
  $\Rightarrow$ a in LHS has to be a_N

$$\Rightarrow \quad \sqrt{a_N^2 + a_0^2} - a_0 = a_N \quad \Rightarrow \quad 2\,a_0\,a_N = 0$$

A Proposal

- Recall from last slide:

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$M_{\rm total}$ has to be $M_{\rm total} = M_{ordinary} + M_{dark}$ theoretically and observationally

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- Propose that:

$$M_{dark} = 2\left(\frac{a_0}{a}\right)^2 M_{ordinary}$$

So What?

- We can write:

$$F_{Entropic} = m \left(\sqrt{a^2 + a_0^2} - a_0 \right) = m\, a_N \left[1 + 2 \left(\frac{a_0}{a} \right)^2 \right]$$

- When $a \ll a_0$:

$$F_{Entropic} \approx m \frac{a^2}{2\, a_0} \approx m\, a_N\, 2 \left(\frac{a_0}{a} \right)^2$$

$$\Rightarrow a \approx \left(4\, a_N\, a_0^3 \right)^{1/4}$$

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- Reproducing MOND:

$$F_{Entropic} \approx m\,\frac{a^2}{2\,a_0} \approx m\sqrt{a_N\,a_0}, \quad \text{when } a \ll a_0$$

DarkMatter-MOND Duality?

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- We can also write:

$$F_{Entropic} = m\,\frac{G\,(M_{ordinary} + M_{dark})}{r^2}$$

$\Rightarrow$ NO MODIFICATION OF GRAVITY but dark matter

- Macroscopic statement:
  If we have some kind of dark matter which has the profile $M_{dark} = 2\left(\frac{a_0}{a}\right)^2 M_{ordinary}$, *it could behave as if there were a modification of gravity but no dark matter*
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- A microscopic theory? Could dS_4/CFT_3 correspondence help?