Muon g-2 and a peculiar extra U(1)

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g-2 of the muon

- **Radiative corrections (loop corrections)** :
  different magnitude of MDMs for charged leptons (electron, muon and tauon)

Assuming QED is correct,
a_e provides the most accurate value α.

- **Current measurements:**

D. Hanneke, *et al*, PRL 100, 120801 (2008)

$$a_e(\text{exp}) = 11596521.8073(0.0028)\times 10^{-10} \quad \text{Havard}$$

$$a_\mu(\text{exp}) = 11659208.0(6.33)\times 10^{-10} \quad \text{BNL E821}$$

G.W. Benett, *et al*, PRD 73, 072003 (2006)

The electron *g-2* is 350 times more precisely measured.
However, it is much less sensitive to new physics,
since the effect of mass is suppressed by the factor $(m_\mu / m_e)^2 \approx 43000$

Electron *g-2* is a test of QED, but not for other aspects of SM

SM predictions of the muon *g*-2

$$a_\mu(\text{QED}) = 11658471.8113(162)\times 10^{-10}$$

$$a_\mu(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857388(44)\left(\frac{\alpha}{\pi}\right)^2 + 24.050509(2)\left(\frac{\alpha}{\pi}\right)^3 + 126.04\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

Schwinger Sommerfield Laporta &Remiddi Kinoshita Milstein *et al*

$$a_\mu(\text{EW}) = 15.4(0.2)\times 10^{-10}$$

$$a_\mu(\text{hadrons}) = 693.07(6.89)\times 10^{-10}$$

- **The discrepancy from the SM prediction :**

$$\Delta a_\mu \equiv a_\mu(\exp) - a_\mu(\text{SM}) = 27.7(9.3)\times 10^{-10}$$

About 3.0σ deviation from the SM prediction

We consider that this deviation comes from new physics beyond the standard model

Theoretical Frame Work

- **Extend the SM with an exotic lepton triplet *E* per family**
  : $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$

Barr, Dorsner, PRD 72 015011 (2005)

- **Anomaly cancellations**
  : additional 6 anomaly cancellations necessary

These constraints provide gauge charges of fermions

$U(1)_Y^2 \times U(1)_X$, $U(1)_Y \times U(1)_X^2$, $U(1)_X^3$, $SU(2)^2 \times U(1)_X$, $SU(3)^2 \times U(1)_X$, gravity $\times U(1)_X$

- **Gauge invariance with an extra U(1)** : renomalizable Lagrangians

- **Symmetry breakings :** 2 steps symmetry breaking

$$SU(2)_L \times U(1)_Y \times U(1)_X \xrightarrow[<\eta>]{} SU(2)_L \times U(1)_Y \xrightarrow[<\phi>]{} U(1)_{em}$$

- An additional singlet necessary.
- Assumed the symmetry breaking near the weak scale

Usual EW symmetry breaking

Fermion gauge charges

TABLE II: Fermion gauge charges. T_3 is the weak isospin, Y is the hypercharge, X is the extra $U(1)_X$ charge, and $Q = T_3 + Y$ is the electric charge. The charges for the right handed fermions can also be assigned in the identical way. ($f_L^c \equiv (f_L)^c$ in this letter, so $(f^c)_L$ implies the antiparticle of f_R).

	u_L	d_L	$(u^c)_L$	$(d^c)_L$	ν_L	ℓ_L	$(\ell^c)_L$	E_L^+	E_L^0	E_L^-
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	-1
Y	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	0	0
X	1	1	-1	-1	1	1	1	-1	-1	-1

		U		u^c		d^c		L		e^c		E		
$U(1)_Y^2 \times U(1)_X$	:	$3[2\left(\frac{1}{6}\right)^2$	$+$	$-\left(-\frac{2}{3}\right)^2$	$+$	$-\left(\frac{1}{3}\right)^2]$	$+$	$2\left(-\frac{1}{2}\right)^2$	$+$	$(1)^2$	$+$	0	$=$	0
$U(1)_Y \times U(1)_X^2$	:	$3[2\left(\frac{1}{6}\right)$	$+$	$\left(-\frac{2}{3}\right)$	$+$	$\frac{1}{3}]$	$+$	$2\left(-\frac{1}{2}\right)$	$+$	1	$+$	0	$=$	0
$SU(3)^2 \times U(1)_X$	:	2	$+$	-1	$+$	-1	$+$	0	$+$	0	$+$	0	$=$	0
$U(1)_X^3$	:	$3[2(1)^3$	$+$	$(-1)^3$	$+$	$(-1)^3]$	$+$	$2(1)^3$	$+$	1^3	$+$	$3(-1)^3$	$=$	0

$SU(2)^2 \times U(1)_X$ and gravity$\times U(1)_X$

Yukawa potential

E, χ : bidoublet representation

$$E_L = \begin{pmatrix} \frac{1}{\sqrt{2}}E^0 & E^+ \\ E^- & -\frac{1}{\sqrt{2}}E^0 \end{pmatrix}_L, \quad \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix}$$

- **Leptons** :

A Majorana combination

$$y_1 \mathrm{Tr}(\overline{E_L^c} E_L)\eta + y_2 \overline{L_L}\phi \ell_R + y_3 \overline{L_L^c} i\sigma_2 \chi L_L + y_4 \overline{L_L^c} i\sigma_2 E_L \phi + y_5 \mathrm{Tr}(\overline{E_L}\chi)\ell_R + h.c.$$

Gauge charges of the Higgses by combinations with leptons

$$y_1 \mathrm{Tr}\left(\overline{E_L^c} E_L\right)\eta_{(2)} + y_2 \overline{L_L}\phi_{(2)}\ell_R + y_3 \overline{L_L^c} i\sigma_2 \chi_{(-2)} L_L$$

$$+ y_4 \overline{L_L^c} i\sigma_2 E_L \phi_{(0)} + y_5 \mathrm{Tr}\left(\overline{E_L}\chi_{(0)}\right)\ell_R + h.c.$$

- Higgs doublet and triplet can have two distinct extra U(1) charges
- Z_2 symmetry: E is odd and all other particles are even $\Rightarrow$ E^0 is a dark matter candidate

Higgs gauge charges

TABLE III: Higgs gauge charges. T_3 is the weak isospin, Y is the hypercharge, X is the $U(1)_X$ charge, and $Q = T_3 + Y$ is the electric charge.

	ϕ^+	ϕ^0	η	χ^{++}	χ^+	χ^0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	-1
Y	$\frac{1}{2}$	$\frac{1}{2}$	0	1	1	1
X	$(0, 2)$	$(0, 2)$	2	$(0, -2)$	$(0, -2)$	$(0, -2)$

- **Quarks** $$y_6 \overline{Q_L} \widetilde{\phi_{(0)}} u_R + y_7 \overline{Q_L} \phi_{(0)} d_R + h.c.$$

  : only couples to a Higgs, respect MFV(minimal flavor violation)

- Leptons and quarks interact with two distinctive Higgs doublets
  -> different from the standard 2HDM

Neutrino mass generation

- **Mass matrix of neutral leptons**

$$y_1 \mathrm{Tr}\left(\overline{E_L^c} E_L\right)\eta_{(2)} + y_2 \overline{L_L}\phi_{(2)}\ell_R + y_3 \overline{L_L^c} i\sigma_2 \chi_{(-2)} L_L$$

$$+ y_4 \overline{L_L^c} i\sigma_2 E_L \phi_{(0)} + y_5 \mathrm{Tr}\left(\overline{E_L}\chi_{(0)}\right)\ell_R + h.c.$$

$$\mathcal{M}_{\nu E} = \begin{pmatrix} m_\nu & y_4\langle\phi_{(0)}\rangle \\ y_4\langle\phi_{(0)}\rangle & M_E \end{pmatrix}$$

$$m_\nu \equiv y_3 \langle\chi_{(-2)}\rangle$$

$$M_E \equiv y_1 \langle\eta\rangle$$

- The coupling y_4 must be very small since $<\phi_{(0)}>$ is of the order of 100 GeV
- Under Z_2 symmetry, there is no y_4 coupling.
  y_3 and/or $<\chi_{(-2)}>$ are sized for the neutrino mass
  This VEV has to very small to avoid large radiative corrections to the ρ parameter
- We predict Majorana-type massive neutrinos.

Higgs potential

- **Higgs potential with the gauge invariance with the extra U(1)**

$$V \supset V_{2HDM} + \left\{\mu_1 \phi^T_{(0)} \chi^\dagger_{(0)} \phi_{(0)} + \mu_2 \phi^\dagger_{(2)} \chi^\dagger_{(-2)} \phi_{(0)} + h.c.\right\}$$
$$+ \left\{\lambda_1 \phi^\dagger_{(2)} \phi_{(0)} \mathrm{Tr}\left(\chi^\dagger_{(-2)} \chi_{(0)}\right) + \lambda_2 \phi^\dagger_{(2)} \sigma^a \phi_{(0)} \mathrm{Tr}\left(\chi^\dagger_{(-2)} \sigma^a \chi_{(0)}\right) + h.c.\right\}$$

- V_{2HDM} : only Higgs doublets involved
  functional form is the same as the 2HDM with Z_2 symmetry

- **Two complex trilinear couplings**
  :neutrino masses and Baryogensis via Leptogenesis

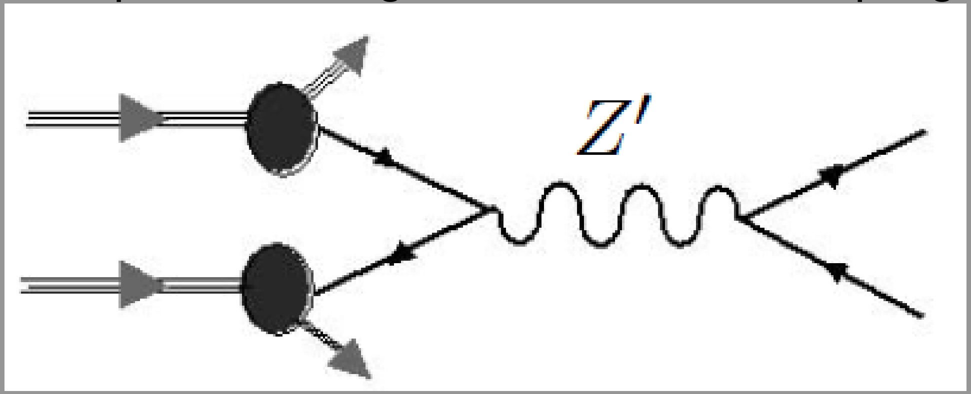
  Ma, Sarkar, PRL 80 5716 (1998)

- **Two complex quartic couplings**
  : often mentioned for EDM of fermions

  Heo,Keung, PLB 661 259 (2008)

Z' searches at the Hadron Colliders

- **The decay into charged leptons :** $p\bar{p}(p) \rightarrow Z' \rightarrow \ell^+\ell^-$
  explore new regions in mass and couplings at Hadron colliders



➢ Partonic cross section

$$\hat{\sigma}(Z') = \frac{\pi z_f'^2 g_{Z'}^2}{48} \delta(\hat{s} - M_{Z'}^2)$$

For pp collision, proton PDF replaced

➢ Hadronic cross section in the $\ell^+\ell^-$ channel

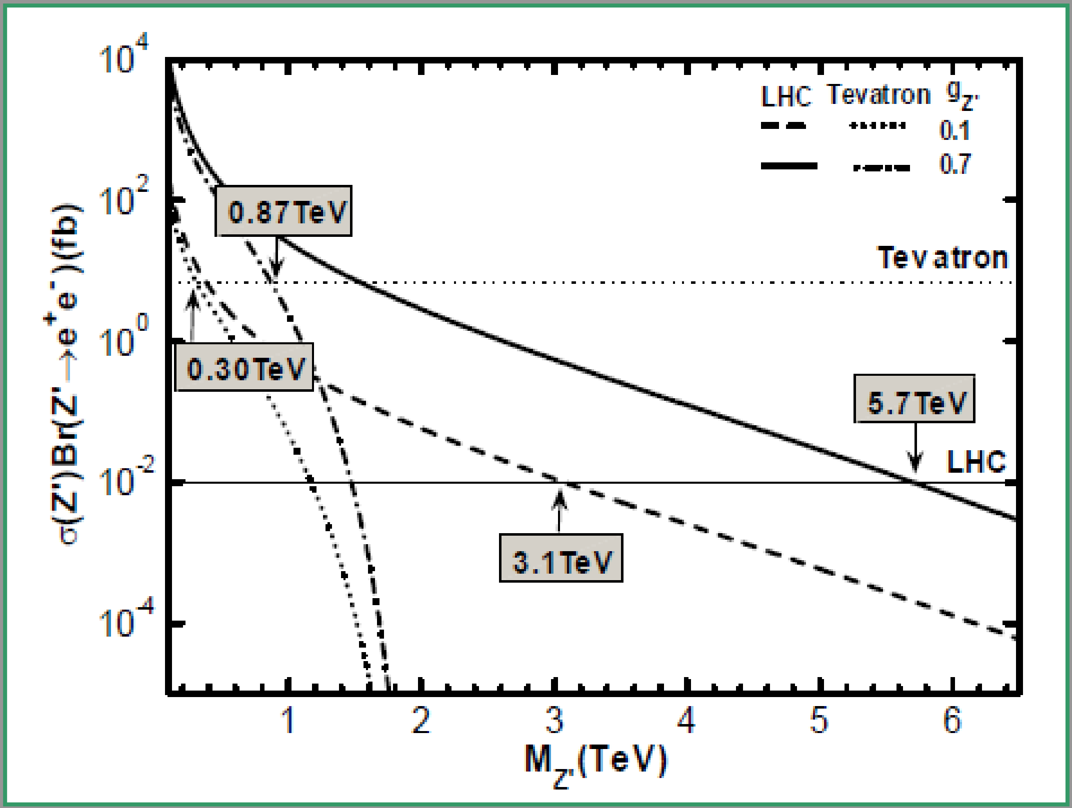
$$\sigma(Z') \cdot Br_{\ell^+\ell^-} = K \frac{\pi z_f'^2 g_{Z'}^2}{48s} \sum_{q,\bar{q}} \int_{\frac{m_{Z'}^2}{s}}^{1} \frac{dx}{x} \left(f_q^p(x) f_{\bar{q}}^{\bar{p}}\left(\frac{M_{Z'}^2}{xs}\right) + f_{\bar{q}}^p(x) f_q^{\bar{p}}\left(\frac{M_{Z'}^2}{xs}\right)\right) \cdot Br_{\ell^+\ell^-}$$

➢ K is QCD correction factor (~1.3) : ratio of higher order over leading order

M. Carena,*et al* , PRD 70 093009 (2004)

➢ Branching ratio : depend how many decay channels are open.
Two more channels are possible in this model,
but still within 4~5%, though the channels are open or not

Z' discovery limit at the Tevatron and LHC



The predictions:

MRST LO PDFs are used

MRST, PLB 531 216 (2002)

Intersections of the curves

: lower mass limits of $M_{Z'}$

$\Rightarrow$ The lower limits of the extra U(1) symmetry breaking: 200 ~ 800 GeV

The horizontal lines:

Tevatron ($\sqrt{s} = 1.96$ TeV and $L = 1.3\ \text{fb}^{-1}$)

LHC($\sqrt{s} = 14$ TeV and $L = 100\ \text{fb}^{-1}$)

g-2 MDM generation Lagrangian at one loop

Not proper interaction terms

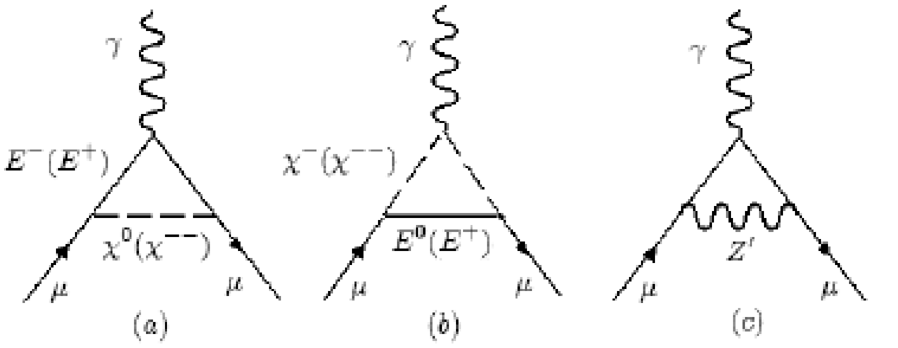
$$y_1 \mathrm{Tr}\left(\overline{E_L^c} E_L\right)\eta_{(2)} + y_2 \overline{L_L}\phi_{(2)}\ell_R + y_3 \overline{L_L^c} i\sigma_2 \chi_{(-2)} L_L$$

$$+ y_4 \overline{L_L^c} i\sigma_2 E_L \phi_{(0)} + y_5 \mathrm{Tr}\left(\overline{E_L}\chi_{(0)}\right)\ell_R + h.c.$$

This coupling is very small from neutrino mass constraint

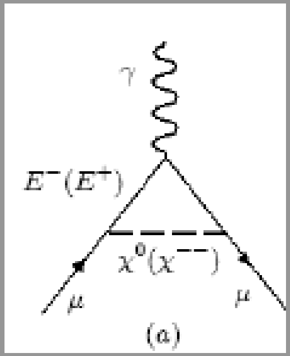
Introduce new Yukawa coupling in mass eigenstates

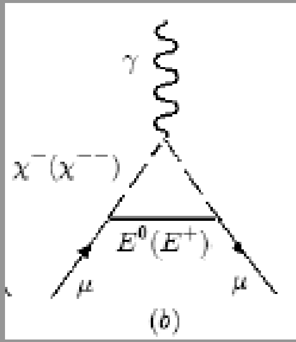
$$-y\left(\overline{E_L^0}\chi_\ell^+\mu_R + \overline{E_L^-}\chi_\ell^0\mu_R - \overline{E_L^+}\chi_\ell^{++}\mu_R + h.c.\right)$$

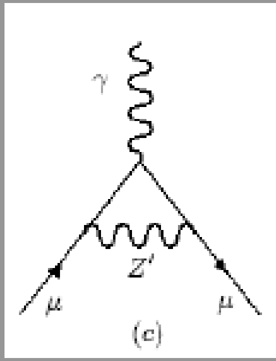


The one-loop contributions to a_μ involving the extra particles, E, χ, and Z'

MDM generation at one loop







- Negligible contribution, since the particles (E^+, E^-) on the line which are hooked up by the photon have opposite charges.

- $$\Delta a_\mu = \frac{3y^2}{8\pi^2}\left(\frac{m_\mu}{M_E}\right) f\left(\frac{M_\chi^2}{M_E^2}\right)$$

Corresponding one loop function :

$$f(z) = \int_0^1 dx \frac{(1-x)x}{zx+1-x} = \frac{1-z^2+2z\ln z}{2(1-z)^3}$$

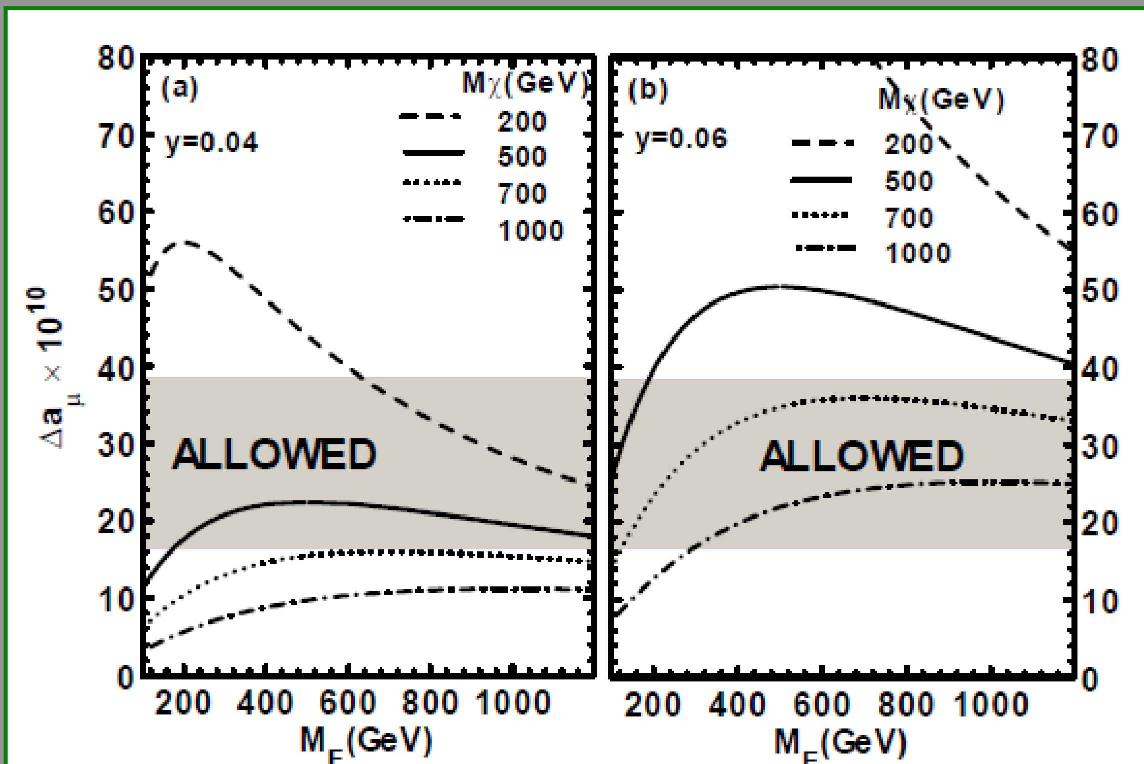
Asymptotic behaviors :

$$\longrightarrow \begin{cases} \frac{1}{6} & \text{as } z = 1 \\ \frac{1}{2z} - \frac{\ln z}{z^2} & \text{for } z \gg 1 \\ \frac{1}{2} + z\ln z & \text{for } z \ll 1 \end{cases}$$

- $$\Delta a_\mu \simeq \frac{g_{z'}^2}{12\pi^2}\frac{m_\mu^2}{M_{z'}^2} < 10^{-10}$$, negligible

$$\Delta a_\mu^{(one)} = \frac{3y^2}{8\pi^2}\left(\frac{m_\mu}{M_E}\right) f\left(\frac{M_\chi^2}{M_E^2}\right)$$

- Very sensitive to the Yukawa coupling *y*



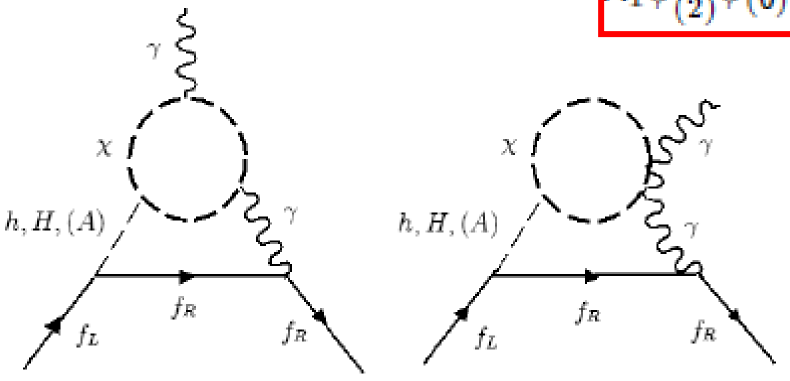
Δa_μ as a function of M_E at the one loop for various values of M_χ

- Predictions in the allowed band around $y = 0.05$ for $0.1\text{TeV} < M_E, M_\chi < 1\text{TeV}$.
- $y > 0.06$ region is possible for $M_E, M_\chi > 1\text{TeV}$.

MDM generation at two loop

Higgs

$$\lambda_1 \phi^\dagger_{(2)} \phi_{(0)} \mathrm{Tr}\left(\chi^\dagger_{(-2)} \chi_{(0)}\right) + \lambda_2 \phi^\dagger_{(2)} \sigma^a \phi_{(0)} \mathrm{Tr}\left(\chi^\dagger_{(-2)} \sigma^a \chi_{(0)}\right) + h.c.$$



Two-loop contributions to a_μ (d_e) (mirror graphs are not displayed.)

Two distinctive extra U(1) charges
: the exact Z_2 symmetry in standard 2HDM
-> no mixing between CP-even and -odd Higgses.

CP-even Higgs (h, H) : MDM
CP-odd Higgs (A) : EDM

Yukawa

$$y_1 \mathrm{Tr}\left(\overline{E^c_L} E_L\right) \eta_{(2)} + y_2 \overline{L_L} \phi_{(2)} \ell_R + y_3 \overline{L^c_L} i\sigma_2 \chi_{(-2)} L_L$$

$$+ y_4 \overline{L^c_L} i\sigma_2 E_L \phi_{(0)} + y_5 \mathrm{Tr}\left(\overline{E_L} \chi_{(0)}\right) \ell_R + h.c.$$

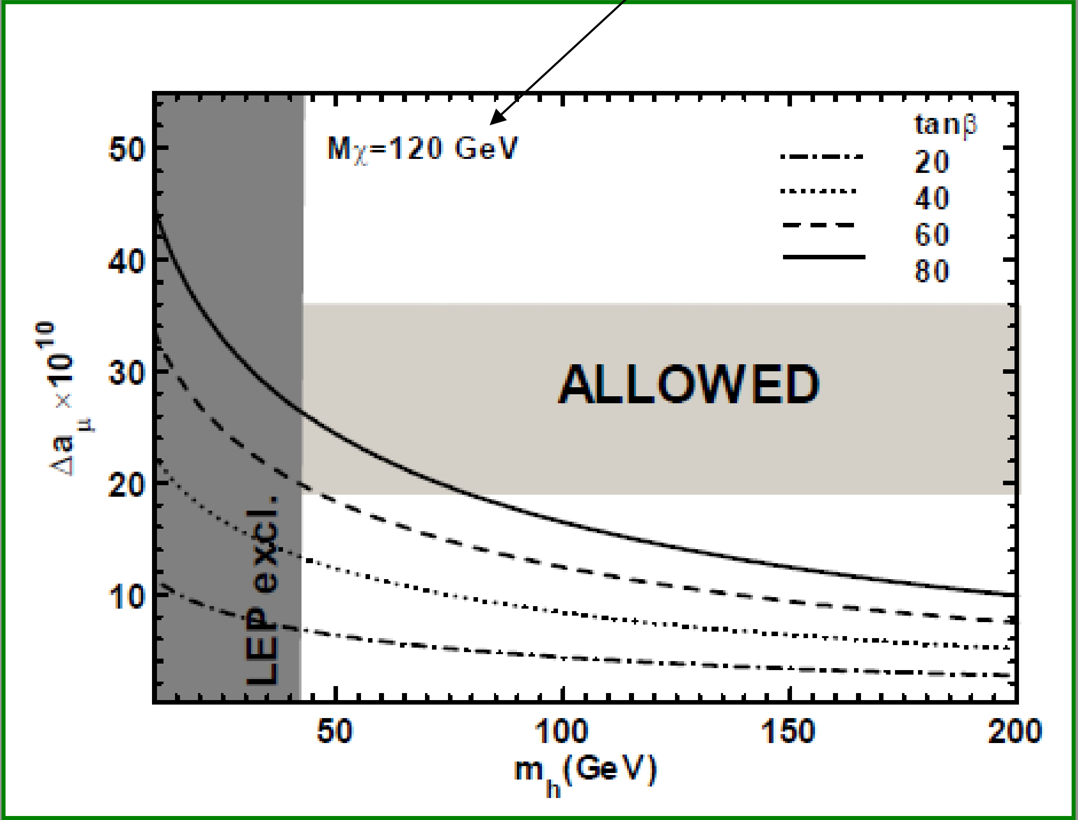
Z_2 symmetry conserved Yukawa Interaction terms

$$\Delta a_\mu^{(\text{two})} \simeq -\sum_{\mathcal{H},\chi} \frac{\alpha m_\mu^2}{16\pi^3} \frac{Q_\chi^2 r_{\mathcal{H}} \lambda_+}{m_{\mathcal{H}}^2} \left[F\left(\frac{M_{\chi l}^2}{m_{\mathcal{H}}^2}\right) + F\left(\frac{M_{\chi h}^2}{m_{\mathcal{H}}^2}\right) \right]$$

Lower limit of the doubly charged scalar from the Tevatron and the LEP

CDF Col., PRL 95 071801 (2005)

L3 Col., PLB 576 18 (2003)



Δa_μ as a function of m_h at the two loop for various values of $\tan\beta$

- $Q_\chi^2 = 5$ due to singly and doubly charged scalars in the inner loop
- The Yukawa coupling λ_+ : the same size as the SM Higgs quartic couling for the SM Higgs of 120 GeV
- Predictions barely reside in the allowed band, but still possible parameter space

Conclusion

- **A model (Lagrangian) with a peculiar extra U(1) was built.**
  - An extra U(1) with an exotic lepton triplet per family in anomaly free gauge

- **The sizable g-2 MDM could be generated.**
  - g-2 MDM at one and two loops, but the allowed parameter space is very narrow for two loop