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About a peculiar extra $U(1)$: Z' discovery limit, muon anomalous magnetic moment, and electron electric dipole moment

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The model (Lagrangian) with a peculiar extra $U(1)$ [S. M. Barr and I. Dorsner, Phys. Rev. D **72**, 015011 (2005); S. M. Barr and A. Khan, Phys. Rev. D **74**, 085023 (2006)] is clearly presented. The assigned extra $U(1)$ gauge charges give a strong constraint to build Lagrangians. The Z' discovery limits are estimated and predicted at the Tevatron and the LHC. The new contributions of the muon anomalous magnetic moment are investigated at one and two loops, and we predict that the deviation from the standard model may be explained. The electron electric dipole moment could also be generated because of the explicit CP-violation effect in the Higgs sector, and a sizable contribution is expected for a moderately sized CP phase [argument of the CP-odd Higgs], $0.1 \leq \sin\delta \leq 1$ [$6° \leq \arg(A) \leq 90°$].

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I. INTRODUCTION

An extra $U(1)$ [or a few extra $U(1)'$s] may arise in the context of grand unified theories [1] or superstring theories [2], or they may generically emerge as simple extensions of the standard model (SM). Therefore, the models with an extra $U(1)$ [or a few extra $U(1)'$s] have been extensively considered. Recently, Barr and Dorsner [3] suggested another possibility for an extra $U(1)$ gauge, which satisfies all anomaly constraints in a maximally economical way, whatever its origin[1] is. In the standard model, all the possible anomalies from triangle diagrams of three gauge bosons must be canceled if the Ward identities of the gauge theory are to be satisfied. The existence of an extra $U(1)$ brings six additional anomaly cancellation conditions, $U(1)_Y^2 \times U(1)_X$, $U(1)_Y \times U(1)_X^2$, $U(1)_X^3$, $SU(2)^2 \times U(1)_X$, $SU(3)^2 \times U(1)_X$, gravity $\times\, U(1)_X$. These anomaly cancellations are nontrivial,[2] but Barr and Dorsner showed a remarkably trivial solution [3] with a single extra lepton triplet per family. These gauge anomalies are exactly canceled for the fermion gauge charges listed at Table I.

In this paper the model (Lagrangian) is clearly presented. With an extra lepton triplet, an additional Higgs singlet is necessary to provide masses of the exotic leptons and the extra gauge boson Z'. The Higgs singlet would involve the extra $U(1)$ gauge symmetry breaking, and we assume that the symmetry is broken near the weak scale. A Higgs triplet[3] with the required gauge charges is added to explain our interesting phenomenology. The new gauge boson Z' is the intrinsic particle that explains the existence of an extra $U(1)$, so its discovery limits at the Tevatron and LHC are estimated and predicted. The muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, has been a powerful tool to account for new physics because of its importance. We investigate the contributions involving the new particles at the one- and two-loop levels. One can also see the explicit CP violation that generates the electric dipole moment (EDM) of the electron d_e, and a sizable contribution is expected via the Barr-Zee two-loop mechanism for a moderate size of the CP phase (argument of the CP-odd Higgs), $0.1 \leq \sin\delta \leq 1$ [$6° \leq \arg(A) \leq 90°$].

II. MODEL DESCRIPTION (LAGRANGIAN)

With the new particle content, the Yukawa potential for the lepton sector can have the following enlarged form without the *ad hoc* imposition of lepton number conservation.

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[1]The origin is close to the Pati-Salam model with an extra $U(1)$ [3].

[2]The general analyses about these cancellations can be found in Ref. [4].

[3]The Higgs triplet is introduced because of the need to induce the CP-violating interaction in this work. We can add additional scalars, such as nongauged Higgs singlets or doublets, in other ways, but adding the gauged scalars is more generic. If we only consider neutrino mass generation without electric dipole and dark matter (we need to impose the discrete Z_2 symmetry; see the next section) phenomenology, the additional Higgs triplet is unnecessary. The interactions induced by the Higgs triplet could also give a significant contribution to the anomalous magnetic moment of the muon (see Sec. IV).

TABLE I. Fermion gauge charges. T_3 is the weak isospin, Y is the hypercharge, X is the extra $U(1)_X$ charge, and $Q = T_3 + Y$ is the electric charge. The charges for the right-handed fermions can also be assigned in an identical way. [$f_L^c \equiv (f_L)^c$ in this paper, so $(f^c)_L$ implies the antiparticle of f_R].

	u_L	d_L	$(u^c)_L$	$(d^c)_L$	ν_L	ℓ_L	$(\ell^c)_L$	E_L^+	E_L^0	E_L^-
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	-1
Y	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	0	0
X	1	1	-1	-1	1	1	1	-1	-1	-1

TABLE II. Higgs gauge charges. T_3 is the weak isospin, Y is the hypercharge, X is the $U(1)_X$ charge, and $Q = T_3 + Y$ is the electric charge.

	ϕ^+	ϕ^0	η	χ^{++}	χ^+	χ^0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	-1
Y	$\frac{1}{2}$	$\frac{1}{2}$	0	1	1	1
X	(0, 2)	(0, 2)	2	(0, -2)	(0, -2)	(0, -2)

$$y_1\,\mathrm{Tr}(\overline{E_L^c}E_L)\eta + y_2\overline{L_L}\phi\ell_R + y_3\overline{L_L^c}i\sigma_2\chi L_L + y_4\overline{L_L^c}i\sigma_2 E_L\phi + y_5\,\mathrm{Tr}(\overline{E_L}\chi)\ell_R + \mathrm{H.c.}, \tag{1}$$

where η and χ denote a Higgs singlet and a Higgs triplet, $\phi = (\phi^+, \phi^0)^T$ is a Higgs doublet, and $L = (\nu_\ell, \ell)^T$ is the lepton doublet. The bidoublet representation is taken for the additional lepton triplet, and the Higgs triplet is also taken in the form of a 2×2 matrix transforming under $SU(2)$ as $\chi \to U\chi U^\dagger$.

$$E_L = \begin{pmatrix} \frac{1}{\sqrt{2}}E^0 & E^+ \\ E^- & -\frac{1}{\sqrt{2}}E^0 \end{pmatrix}_L, \qquad \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix}. \tag{2}$$

The lepton triplet must be a Majorana combination. It should be noted that the antisymmetric tensor $i\sigma_2$ follows from the antisymmetric property of the charge conjugation.

Since the gauge charges of the leptons are already assigned by anomaly constraints, the gauge charges of the Higgses are assigned by the combinations with leptons in the Yukawa potential under the $SU(2)_L \times U(1)_Y \times U(1)_X$ gauge invariance. If we introduce the assignment of $U(1)_X$ charges for the Higgses, the singlet η must have a $U(1)_X$ charge of 2 from the y_1-term since E has -1; the doublet ϕ may have a charge of 2 from the y_2-term and 0 from the y_4-term; and the triplet χ may have a charge of -2 from the y_3-term and 0 from the y_5-term. The other gauge charges may be assigned in an analogous way, and the assigned charges of the Higgses are listed at Table II. Notice that the Higgs doublet and triplet may have two distinctive $U(1)_X$ charges.

The Yukawa potential with distinct charges takes the form

$$y_1\,\mathrm{Tr}(\overline{E_L^c}E_L)\eta_{(2)} + y_2\overline{L_L}\phi_{(2)}\ell_R + y_3\overline{L_L^c}i\sigma_2\chi_{(-2)}L_L + y_4\overline{L_L^c}i\sigma_2 E_L\phi_{(0)} + y_5\,\mathrm{Tr}(\overline{E_L}\chi_{(0)})\ell_R + \mathrm{H.c.}, \tag{3}$$

where the indices in the lower brackets of the Higgses denote $U(1)_X$ charges of the Higgses.

A discrete Z_2 symmetry could be imposed to explain a certain phenomenology, dark matter. If E is odd and all other particles are even under Z_2 symmetry, this would prevent the exotic leptons from coupling with the ordinary leptons, and the neutral lepton E^0 becomes stable and thus could be a dark matter candidate. The Yukawa potential with Z_2 symmetry is given by

$$y_1\,\mathrm{Tr}(\overline{E_L^c}E_L)\eta_{(2)} + y_2\overline{L_L}\phi_{(2)}\ell_R + y_3\overline{L_L^c}i\sigma_2\chi_{(-2)}L_L + \mathrm{H.c.} \tag{4}$$

The Yukawa potential for the quark sector may be built in an analogous way.

$$y_6\overline{Q_L}\tilde{\phi}_{(0)}u_R + y_7\overline{Q_L}\phi_{(0)}d_R + \mathrm{H.c.}, \tag{5}$$

where $Q = (u, d)^T$ is the quark doublet and $\tilde{\phi}_{(0)} = i\sigma_2\phi^*_{(0)}$. The hypercharge combinations in the potential prohibit the couplings between quarks and Higgs triplets. Since quarks receive masses only from $\phi_{(0)}$, there is no tree-level flavor-changing neutral currents. Note that leptons and quarks interact with two distinct Higgs doublets, which is different from the standard two Higgs doublet model (2HDM) where one Higgs couples to the up-type quarks and the other couples to the charged leptons and down-type quarks.

The size of the couplings and vacuum expectation values (VEVs) may be approximately constrained with the known experimental measurements. The VEV of $\phi_{(0)}$, $\langle\phi_{(0)}\rangle$, must be of the order of 100 GeV to meet the top quark mass, and $\langle\phi_{(2)}\rangle$ must be $1 \sim 100$ GeV to satisfy the τ-lepton mass and the known SM VEV.[4] The Higgs triplet VEV $\langle\chi\rangle$ must be very small compared to the Higgs doublet VEV to avoid large corrections to the ρ parameter [5]. The neutrino mass may be generated at the tree level in this model. The mass matrix of neutral leptons is

$$\mathcal{M}_{\nu E} = \begin{pmatrix} m_\nu & y_4\langle\phi_{(0)}\rangle \\ y_4\langle\phi_{(0)}\rangle & M_E \end{pmatrix},$$

where $m_\nu \equiv y_3\langle\chi_{(-2)}\rangle$ and $M_E \equiv y_1\langle\eta\rangle$. The nature of the neutrino is not known; however, we have approximately predicted the size of the neutrino mass. We take the exotic

[4] $\langle\phi\rangle = \sqrt{\langle\phi_{(0)}\rangle^2 + \langle\phi_{(2)}\rangle^2} \simeq 174$ GeV.

lepton E at the weak scale, so the coupling y_4 must be very small[5] since $\langle \phi_{(0)} \rangle$ is of the order of 100 GeV. The seesaw-like mechanism is applicable to generate the neutrino mass. If Z_2 symmetry is imposed, $y_4 = 0$. The neutrino mass may be taken as m_ν, where y_3 and/or $\langle \chi_{(-2)} \rangle$ are sized for the neutrino mass. For either case, we predict Majorana-type neutrinos in this model. Since $\langle \chi \rangle$ and the coupling y_4 are small, we can consider that the massive leptons are in the mass eigenstates for the Yukawa potential of (3).

The Higgs potential is also amenable to the gauge invariance with the extra $U(1)$.

$$V \supset V_{2\mathrm{HDM}} + \{\mu_1 \phi_{(0)}^T \chi_{(0)}^\dagger \phi_{(0)} + \mu_2 \phi_{(2)}^\dagger \chi_{(-2)}^\dagger \phi_{(0)} + \mathrm{H.c.}\} + \{\lambda_1 \phi_{(2)}^\dagger \phi_{(0)} \mathrm{Tr}(\chi_{(-2)}^\dagger \chi_{(0)}) + \lambda_2 \phi_{(2)}^\dagger \sigma^a \phi_{(0)} \times \mathrm{Tr}(\chi_{(-2)}^\dagger \sigma^a \chi_{(0)}) + \mathrm{H.c.}\}, \tag{6}$$

where $V_{2\mathrm{HDM}}$ stands for the Higgs potential involving only Higgs doublets, and the functional form is the same as the 2HDM with Z_2 symmetry. In addition to the two complex trilinear couplings, two complex quartic couplings are possible, those involving the CP-violation phenomenology. The phenomenology with two complex trilinear couplings can be found in Ref. [6],[6] and the complex quartic couplings are related to the electric dipole moment of fermions which will be discussed in this paper. The other interaction terms are trivial and almost irrelevant to the phenomenology.

III. Z' DISCOVERY LIMIT

The interactions of the Z' boson with the fermions are described by

$$\sum_f z'_f g_{Z'} Z'_\mu \bar{f} \gamma^\mu f, \tag{7}$$

where $f = E_L, Q_L, L_L, u_R, d_R, e_R$ are the lepton and quark fields and z'_f is the gauge charge corresponding to the fermion.

The leptonic decays $Z' \to \ell^+\ell^-$ (e^+e^- and $\mu^+\mu^-$) provide the most distinctive signature for observing the Z' signal at hadron colliders. The cross section of the $p\bar{p}$ collision in the $\ell^+\ell^-$ channel can be calculated at the narrow width Z' pole in the center-of-momentum (CM) frame. The hadronic cross section is given by

$$\sigma(Z') = K \sum_{q,\bar{q}} \int_0^1 dx_1 dx_2 (f_q^p(x_1) f_{\bar{q}}^{\bar{p}}(x_2) + f_{\bar{q}}^p(x_1) f_q^{\bar{p}}(x_2)) \hat{\sigma}(Z'), \tag{8}$$

where $\hat{s} = x_1 x_2 s$ is the partonic fraction of s, $f(x)$'s are the partonic distribution functions (PDFs), and the sum is performed over all the light quarks. K is the QCD correction factor (~ 1.3) [7], which accounts for higher order QCD corrections. The partonic cross section $\hat{\sigma}(Z')$ is calculated in a sum over the spins of the final states and an average over the spins and colors of the initial states:

$$\hat{\sigma}(Z') = \frac{\pi z_f'^2 g_{Z'}^2}{48} \delta(\hat{s} - M_{Z'}^2). \tag{9}$$

Equations (8) and (9) lead to the hadronic cross section in the $\ell^+\ell^-$ channel.

$$\sigma(Z') \cdot \mathrm{Br}_{\ell^+\ell^-} = K \frac{\pi z_f'^2 g_{Z'}^2}{48 s} \sum_{q,\bar{q}} \int_{m_{Z'}^2/s}^1 \frac{dx}{x} \times \left(f_q^p(x) f_{\bar{q}}^{\bar{p}}\left(\frac{M_{Z'}^2}{xs}\right) + f_{\bar{q}}^p(x) f_q^{\bar{p}}\left(\frac{M_{Z'}^2}{xs}\right)\right) \cdot \mathrm{Br}_{\ell^+\ell^-}, \tag{10}$$

where $\mathrm{Br}_{\ell^+\ell^-}$ is the branching ratio of Z' to $\ell^+\ell^-$. We may take $z_f'^2 \simeq 1$, since precision measurements of Z-pole observables predict the small Z-Z' mixing angle ($\le 10^{-3}$) [5].

For pp collision at the LHC, the proton PDF takes the place of the antiproton PDF. Figure 1 shows the predicted cross sections with the present experimental sensitivity at the Tevatron run II[7] and the projected experimental sensitivity at the LHC [10]. The actual experimental analysis shows an experimental line with a more complicated structure than the horizontal line in the figure. For a nonzero background,[8] $N_{Z'} = 3$ events are excluded at the Tevatron. The Z' discovery limits are 300 GeV, 870 GeV for $g_{Z'} = 0.1, 0.7$ at the Tevatron, and the LHC may probe Z' up to 3.1 TeV, 5.7 TeV for $g_{Z'} = 0.1, 0.7$. Since the $U(1)_X$ gauge charge of the Higgs singlet η is 2, $M_{Z'} \simeq 2 g_{Z'} \langle \eta \rangle$. We predict the lower limit of the extra $U(1)$ symmetry breaking to be around 200–800 GeV at the Tevatron (CDF detector).

[5]According to the famous canonical seesaw mechanism, the order unity coupling is assumed, with the scale of new physics of 10^{13} GeV. However, we relax the constraint, as the coupling y_4 could approximately be of the order of the electron Yukawa coupling ($\sim 10^{-6}$).

[6]They assigned the Higgs triplets of the order of 10^{13} GeV to explain neutrino masses and baryogenesis via leptogenesis. However, the Higgs triplets are assumed to have masses of the order of the weak scale to explain the interesting phenomenology in our scenario.

[7]The CDF Collaboration [8] has set a better luminosity for $\sigma(Z') \cdot \mathrm{Br}_{\ell^+\ell^-}$ than the D0 Collaboration[9] for some reason, so this is considered for the CDF collider.

[8]The nonzero background is roughly taken from Ref. [8], in which all the expected backgrounds are considered. The most significant source of background in this channel is the SM Drell-Yan process via Z/γ^* as reported in Ref. [8].

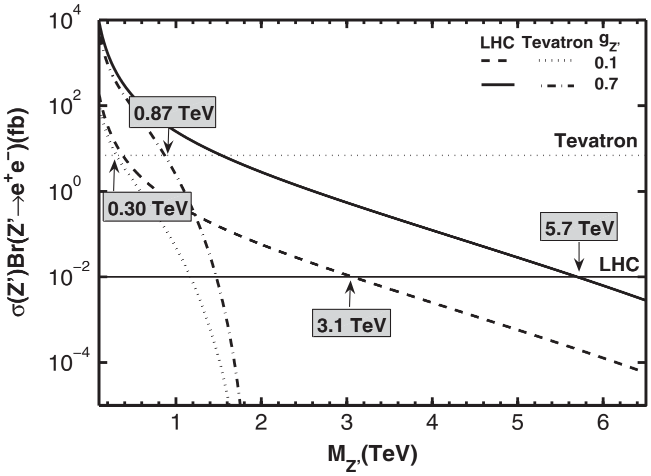

FIG. 1. The Z' discovery limit at the Tevatron ($\sqrt{s} = 1.96$ TeV and $L = 1.3\ \mathrm{fb}^{-1}$) and the LHC ($\sqrt{s} = 14$ TeV and $L = 100\ \mathrm{fb}^{-1}$). The horizontal lines indicate the experimental sensitivities, and the bold lines are predictions of the cross section. The predictions are for the coupling, $g_{Z'} = 0.1$ and 0.7 (SM coupling). MRST LO PDFs [22] are used. The intersections of the curves determine the lower mass limits.

IV. MUON ANOMALOUS MAGNETIC MOMENT

The deviation of the current experimental value from the SM prediction is approximately 3.0σ, and the numerical deviation is $\Delta a_\mu = 27.5(8.4) \times 10^{-10}$ [11] or $27.7(9.3) \times 10^{-10}$ [12]. The experimental value is the measurement of the Brookhaven National Laboratory experiment [13]. We investigate one- and two-loop contributions.

The diagrams of Fig. 2 display one-loop contributions involving the new particles, E, χ, and Z'. The relevant interaction Lagrangian for diagrams (a) and (b) of Fig. 2 comes from the y_5-term of the Yukawa potential of (3). The states $\chi_{(0)}$, $\chi_{(-2)}$ may be rotated into the mass eigenstates χ_ℓ, χ_h, where χ_ℓ and χ_h are the light and heavy mass eigenstates. The rotational angle is determined in the Higgs potential. However, the couplings with the Higgs triplet are free parameters, so we redefine the new couplings in the mass eigenstates. The relevant Lagrangian for the light scalar state χ_ℓ is given by

$$-y(\overline{E^0_L}\chi^+_\ell \mu_R + \overline{E^-_L}\chi^0_\ell \mu_R - \overline{E^+_L}\chi^{++}_\ell \mu_R + \mathrm{H.c.}), \quad (11)$$

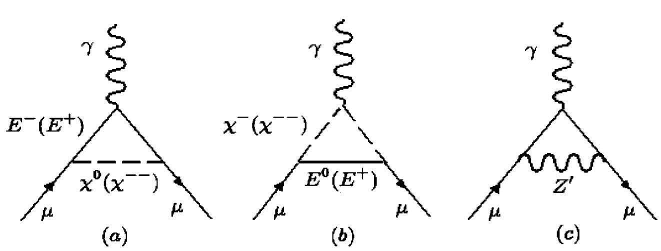

FIG. 2. The one-loop contributions to a_μ involving the extra particles, E, χ, and Z'.

where y is the Yukawa coupling in the mass eigenstates of χ. The Lagrangian for the heavy mass eigenstates can be given in the same fashion. The y_4-term with which the Higgs doublet is involved is neglected due to the small coupling constrained by the neutrino mass.

The contribution of Fig. 2(a) is negligible, since the particles E^+, E^- on the line which are hooked up by the photon have opposite electric charges. We calculated the contribution of Fig. 2(b), and it is given by

$$\Delta a_\mu^{(\mathrm{one})} = \frac{3y^2}{8\pi^2}\left(\frac{m_\mu}{M_E}\right) f\left(\frac{M_\chi^2}{M_E^2}\right), \quad (12)$$

where the prefactor of 3 comes from the electric charges of $\chi^\pm$, $\chi^{\pm\pm}$.

The corresponding one-loop function is

$$f(z) = \int_0^1 dx \frac{(1-x)x}{zx + 1 - x} = \frac{1 - z^2 + 2z\ln z}{2(1-z)^3} \quad (13)$$

which has asymptotic behaviors,

$$f(z) \to \begin{cases} \frac{1}{6} & \text{as } z = 1 \\ \frac{1}{2z} - \frac{\ln z}{z^2} & \text{for } z \gg 1 \\ \frac{1}{2} + z\ln z & \text{for } z \ll 1. \end{cases} \quad (14)$$

We neglect the contribution from the other scalars (called the heavy scalars), since those scalars are split into light and heavy mass eigenstates, in general, and the one-loop function behaves as $f(z) \to 0$ as $z \to \infty$. Furthermore, the large splitting is necessary to generate the sizable electric dipole moment that will be discussed in the next section. The M_χ or M_{χ_ℓ} implies the mass of the light scalar in this paper.

Figure 3 shows the predictions of the anomalous magnetic moment for 0.1 TeV $< M_E, M_\chi <$ 1 TeV. The range of deviations from the SM is presented in the dark "allowed" band [11]. The predictions that lie in the allowed band indicate that the Yukawa coupling y should be around

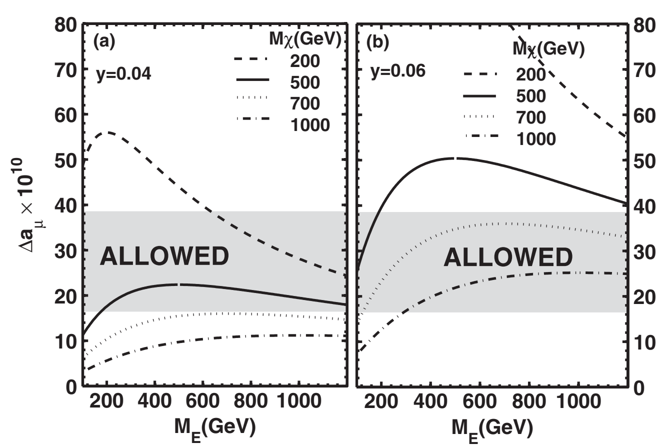

FIG. 3. Δa_μ as a function of the exotic lepton mass M_E for various values of M_χ at the one-loop level.

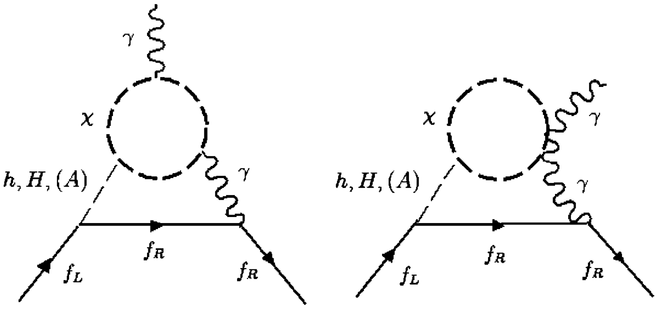

FIG. 4. Two-loop contributions to a_μ (d_e) (mirror graphs are not displayed).

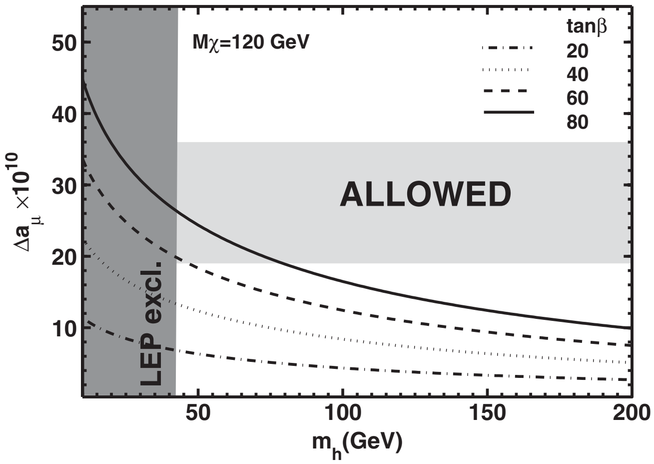

FIG. 5. Δa_μ as a function of the light Higgs boson mass m_h at the two-loop level for various values of $\tan\beta$.

0.05. Since $\Delta a_\mu^{(\mathrm{one})} \sim y^2/M_E$, the Yukawa coupling y is very sensitive to the deviation Δa_μ. Besides the above region, a possible scenario is $M_E \approx M_\chi > 1$ TeV for the Yukawa coupling $y > 0.06$. The contribution by the Z' gauge boson of Fig. 2(c) is negligible, since $\Delta a_\mu \sim m_\mu^2/M_{Z'}^2$.

If Z_2 symmetry is imposed, there is no one-loop contribution to explain the deviation. We consider the two-loop contribution via the Barr-Zee type of mechanism [14], which is depicted in Fig. 4. The relevant Lagrangian to induce the Barr-Zee two-loop contribution is given by

$$-\frac{\sqrt{2}m_\mu r_{\mathcal{H}}}{v}\bar{\mu}\mathcal{H}\mu - \frac{\lambda_+ v}{\sqrt{2}}\mathcal{H}(\chi_\ell\chi_\ell + \chi_h\chi_h),$$

where $\mathcal{H} = h$ or H, $v = \sqrt{2}\langle\phi\rangle$, and λ_+ is the coupling in the mass eigenstates of χ for $\mathcal{H}$. The rotational angles[9] $r_h = -\sin\beta_h/\cos\beta$ and $r_H = \cos\beta_h/\cos\beta$ to the muon are the same as in the standard 2HDM, since the scalar $\phi_{(2)}$, which is consistent with the scalar to couple to the charged leptons in the 2HDM, couples to the muon. There is no contribution from the CP-odd Higgs (pseudoscalar) A because the interaction with the CP-odd Higgs violates CP symmetry, so the effect of the CP-odd Higgs involves the electric dipole moment.

The contribution of two loops is given by

$$\Delta a_\mu^{(\mathrm{two})} \simeq -\sum_{\mathcal{H},\chi}\frac{\alpha m_\mu^2}{16\pi^3}\frac{Q_\chi^2 r_{\mathcal{H}}\lambda_+}{m_{\mathcal{H}}^2}\left[F\left(\frac{M_{\chi_\ell}^2}{m_{\mathcal{H}}^2}\right) + F\left(\frac{M_{\chi_h}^2}{m_{\mathcal{H}}^2}\right)\right]. \tag{15}$$

Note that $\sum Q_\chi^2 = 5$ due to singly and doubly charged scalars in the inner loop. The two-loop function is

$$F(z) = \int_0^1 dx\frac{x(1-x)}{z - x(1-x)}\ln\left[\frac{x(1-x)}{z}\right] \tag{16}$$

which has asymptotic behaviors,

$$F(z) \to \begin{cases} -0.344 & \text{as } z = 1 \\ -\frac{1}{6z}\ln z - \frac{5}{18z} & \text{for } z \gg 1 \\ (2 + \ln z) & \text{for } z \ll 1. \end{cases} \tag{17}$$

The Barr-Zee two-loop contributions, according to Eq. (15), are suppressed by the muon mass and the loop factor, and thus the large $r_{\mathcal{H}}$ and the small $m_{\mathcal{H}}$ are necessary. The lower limit of the light Higgs boson mass is around 44 GeV for $r_h \simeq \tan\beta$ from the CERN LEP [15], but the light Higgs boson keeps the same lower limit of the SM Higgs boson, 113.5 GeV for $r_H \simeq \tan\beta$. The case for $r_h \simeq \tan\beta$ is taken. We can approach these analyses in the 2HDM since the VEVs of the Higgs triplets have the small size. Besides, the doubly charged scalar χ^{++} in the inner loop gives the main contribution to the deviation due to its double electric charge. The lower limit of the doubly charged scalar, around 120 GeV from the Tevatron [16] and the LEP [17], is considered.

Figure 5 shows the predictions for the Barr-Zee two-loop contribution, $\Delta a_\mu^{(\mathrm{two})}$, as a function of the light Higgs boson mass m_h. To predict the two-loop contribution $\Delta a_\mu^{(\mathrm{two})}$, we assume a coupling λ_+ of the same size as the SM Higgs quartic coupling for the SM Higgs of 120 GeV. The predictions barely reside in the allowed region.

V. ELECTRON ELECTRIC DIPOLE MOMENT

The EDM of fermions predicted by the standard model is extremely small compared to the present experimental bounds. Another mechanism beyond the SM has been required to induce the sizable EDM. There are also explicit CP-violation interactions related to the Barr-Zee two-loop mechanism [18–20] for the EDM in this model. Since the interaction must involve the CP violation, it is comprised of only the CP-odd Higgs (pseudoscalar) A. The irreducible CP phase appears in the diagonalization[10] of the mass

[9]Conventionally, the rotational angle between the neutral Higgses in the 2HDM is denoted by the symbol α. But in this paper, the symbol α is used for the electric fine-structure constant, so we use the symbol β_h for the rotational angle.

[10]The detailed process for diagonalization of the mass matrix by the unitary transformation can be found in Ref. [20].

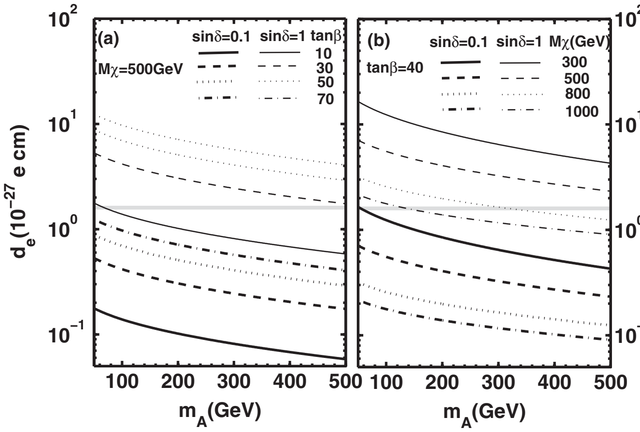

FIG. 6. Numerical estimates of the EDMs as a function of the CP-odd (or pseudoscalar) Higgs boson mass for various values of $\tan\beta$ and M_χ. Also shown are estimates for the CP phase, $0.1 \leq \sin\delta \leq 1$ $[6° \leq \arg(A) \leq 90°]$. The horizontal line indicates the current 90% C.L. experimental bound [21].

matrix for the Higgs triplets in the Higgs potential of (6). If we introduce the new phenomenological couplings, the relevant interaction Lagrangian is given by

$$\frac{\sqrt{2}m_\mu r_A}{v}\bar{e}i\gamma^5 Ae - \frac{\lambda_- v}{\sqrt{2}}A(\chi_\ell\chi_\ell - \chi_h\chi_h),$$

where $r_A = \tan\beta$ is the rotational angle and $\lambda_- = \lambda\sin\delta$, with $\sin\delta$ being the CP-violation effect which comes from combinations of the complex quartic couplings in the potential of (6). The Barr-Zee diagrams were well calculated in many papers to induce the sizable electric dipole moment, and the result is identical to the Barr-Zee two-loop contribution of the anomalous magnetic moment, except for CP-violation effects. The electron electric dipole moment results in

$$\left(\frac{d_e}{e}\right)^\gamma = -\sum_\chi \frac{\alpha m_e}{32\pi^3}\frac{Q_\chi^2 r_A\lambda_-}{m_A^2}\left[F\left(\frac{M_{\chi_\ell}^2}{m_A^2}\right) - F\left(\frac{M_{\chi_h}^2}{m_A^2}\right)\right], \tag{18}$$

where the two-loop function is given in Eq. (16); also note that $\sum Q_\chi^2 = 5$ due to singly and doubly charged scalars from the Higgs triplets. The electron EDM results in the difference between two contributions from the light and heavy scalars, χ_ℓ and χ_h. The contribution from the heavy scalar is neglected, since the two-loop function behaves like $F(z) \to 0$ as $z \to \infty$.

In order to predict the electron electric dipole moment numerically, we also assume the coupling λ of the same size as the SM Higgs quartic coupling for the SM Higgs of 120 GeV. Figure 6 shows the predictions of the electron electric dipole moment as a function of the CP-odd Higgs (or pseudoscalar) mass with the current 90% C.L. experimental bound [21]. The sizable contributions are expected for the moderate size of the CP phase, $0.1 \leq \sin\delta \leq 1$ $[6° \leq \arg(A) \leq 90°]$.

VI. CONCLUSIONS

The model (Lagrangian) with a peculiar extra $U(1)$, that Barr and Dosner suggested, has clearly been presented. The gauge charges of the extra $U(1)$ give a strong constraint to build the Lagrangians. Z' discovery limits are estimated and predicted at the Tevatron and the LHC. The discovery limit at the Tevatron (CDF detector) gives the lower limit of the extra $U(1)$ symmetry breaking scale, approximately 200–800 GeV. The muon anomalous magnetic moment could be explained at the one-loop level for a Yukawa coupling around 0.05. If we allow masses of the new particles to be more than 1 TeV, the larger Yukawa coupling is possible. However, smaller Yukawa couplings are prohibited by the discovery limits of new particles at the Tevatron and the LEP. The muon anomalous magnetic moment could also be explained at the two-loop level, but the region of parameters is very narrow. There are explicit CP-violation interactions in this model. A sizable electron electric dipole moment is expected for a moderately sized CP phase, $0.1 \leq \sin\delta \leq 1$ $[6° \leq \arg(A) \leq 90°]$ via the Barr-Zee mechanism.

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