Lepton and Quark Flavor Violation in a Minimal $SO(10)\times A_4$ SUSY GUT

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Outline

- Motivation
- A brief description of $SO(10)\times A_4$ SUSY GUT Model
- Lepton and quark flavor mixings
    - Introduction
    - A-terms mass insertion
    - The sfermion mass insertion
    - Universality hypothesis at the Planck scale.
- Conclusion

Motivation

- To investigate the flavor mixing of quarks and charged leptons in the minimal $SO(10)\times A_4$ SUSY GUT.

- Consequently, to obtain the constraints on the soft terms which might gives us a hint about how supersymmetry is broken.

A brief description of $SO(10)\times A_4$ SUSY GUT Model

Features of SO(10):

- $(Q_i, L_i, u_i^c, d_i^c, e_i^c) + \nu^c \rightarrow 16$
- See-Saw Mechanism is automatic
- $M_u \propto M_d \rightarrow V_{CKM} = diag\ \{1,1,1\}$

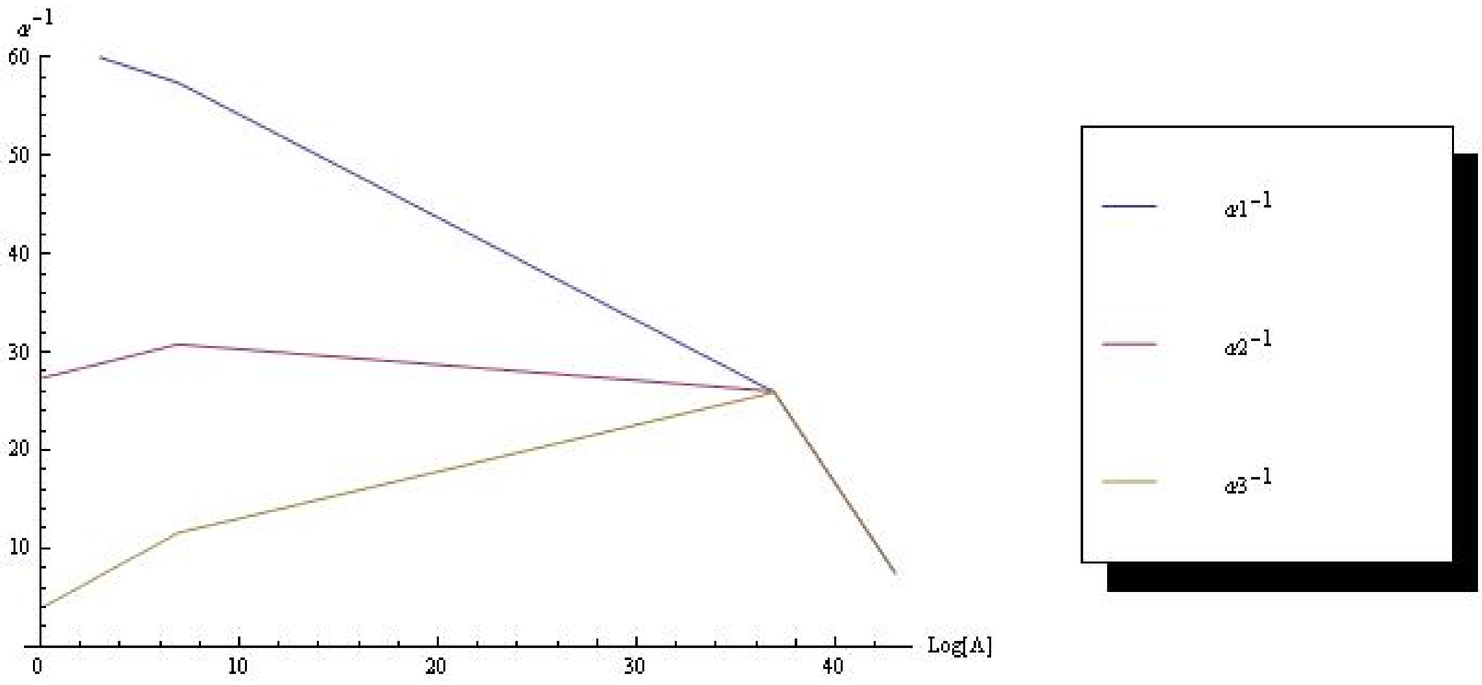
Features of A4:

- A4 is the smallest discrete group that has a three dimensional irreducible representation [E. Ma, G. Rajasekaran, 2001]
- A4 flavor symmetry very easily gives tri-bi-maximal mixing matrix [P.F Harrison et al, 2002]

SO(10)	16_i	$16_1, \overline{16}_1$	$16_2, \overline{16}_2$	$16_3, \overline{16}_3$	1_i^c
A4	3	1	1	1	3
$Z_2\times Z_4\times Z_2$	$+,+,+$	$+,-,+$	$-,+,+$	$+,+,-$	$+,+,+$
SO(10)	10_i	$10'_i$	$10''_i$	$10'''_i$	1_i
A4	3	1	1	1	3
$Z_2\times Z_4\times Z_2$	$+,i,+$	$+,-i,+$	$+,i,-$	$+,-i,-$	$+,-i,+$

SO(10)	10_H	45_H	16_H	$\overline{16}_H$	1_{Hi}	$1'_{Hi}$	$1''_{Hi}$	$1'''_{Hi}$
A4	1	1	1	1	3	3	3	3
$Z_2\times Z_4\times Z_2$	$-,+,-$	$+,-,-$	$+,-i,+$	$+,-i,+$	$+,-,+$	$-,+,+$	$+,+,-$	$+,-i,+$

A brief description of $SO(10) \times A_4$ SUSY GUT Model



$$\alpha^{-1} = \frac{-b_G}{2\pi} \mathrm{Log}[\frac{Q}{Q_G}] + \alpha_G^{-1}$$

$$b_G = S(R) - 3C(G)$$

The unified gauge coupling is perturbative $b_G < 26$

The model gives $b_G = 19$

$\Rightarrow$ $\alpha(M_P) = 0.1$

A brief description of $\mathrm{SO}(10)\times \mathrm{A}_4$ SUSY GUT Model

$$M_F^o = C_F \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & Q_F\ s \\ 0 & Q_{F^c}\ s & (Q_{F^c} + Q_F)\ c \end{bmatrix}$$

Where, $Q_F = 2 I_{3R} + \frac{6}{5}\delta\left(\frac{Y}{2}\right),$

- ☆ $s \to 0$ Hierarchy between the second and third generation
- ☆ $Q_D + Q_{D^c} = Q_L + Q_{L^c} \Rightarrow m_b^0 = m_\tau^0$
- ☆ $\frac{m_\mu^0}{m_s^0} = \frac{Q_{L^+} Q_{L^-}}{Q_{d^c} Q_d} = -3\frac{1+\frac{6}{5}\delta}{1+\frac{2}{5}\delta} \overset{?}{=} \pm 3$ ⟹ $\delta \to 0$, $\delta \to -1.25$
- ☆ $\delta \to -1.25$ $\quad \frac{m_c^0 / m_t^0}{m_s^0 / m_b^0} = 0$

  $\delta \to 0$ $\quad \frac{m_c^0 / m_t^0}{m_s^0 / m_b^0} = 1$ ⟹ Excluded by experiment

A brief description of $SO(10) \times A_4$ SUSY GUT Model

By including the couplings with the vector 10-plet fermions, the model leads to the doubly lopsided structure.

$$M_L \approx M_D^T \approx m_d^0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_2 & \delta_1 & 1 \end{bmatrix}$$

A brief description of $SO(10) \times A_4$ SUSY GUT Model

	Model predictions	Experiment	Pull
$m_e(m_e)$	0.511×10^{-3}	0.511×10^{-3}	...
$m_\mu(m_\mu)$	105.6×10^{-3}	105.6×10^{-3}	...
$m_\tau(m_\tau)$	1.776	1.776	...
$\overline{m}_{ud}$	4.32×10^{-3}	$(3.85 \pm 0.52)\times10^{-3}$	0.9
$m_c(m_c)$	1.4	$1.27^{+0.07}_{-0.11}$	1.85
$m_t(m_t)$	172.5	171.3 ± 2.3	0.52
$\frac{m_s}{\overline{m}_{ud}}$	25.36	27.3 ± 1.5	1.29
$m_s(2Gev)$	109.6×10^{-3}	$105^{+25}_{-35} \times 10^{-3}$	0.184
$m_b(m_b)$	4.31	$4.2^{+0.17}_{-0.07}$	0.58
V_{us}	0.2264	0.2255 ± 0.0019	0.473
V_{cb}	39.2×10^{-3}	$(41.2\pm1.1)\times10^{-3}$	1.82
V_{ub}	4.00×10^{-3}	$(3.93\pm0.36)\times10^{-3}$	0.194
η	0.3569	$0.349^{+0.015}_{-0.017}$	0.526
$\sin\theta_{12}^{sol}$	0.551	0.566 ± 0.018	0.83
$\sin\theta_{23}^{atm}$	0.776	0.707 ± 0.108	0.63
$\sin\theta_{13}$	0.154	< 0.22	-

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Lepton and Quark flavor mixings

Introduction

- Sources of quark and lepton flavor violation

  - The sfermion mass insertions $\delta^{ij}_{LL\;,RR}$

  - The chirality-flipping mass insertion (A-terms) $\delta^{ij}_{LR\;,RL}$

Both sources arise from the mixing of the light states with the heavy ones.

- There are two transformations

  - Transformation needed to block diagonalize the femion mass matrix

  - Transformation needed to diagonalize the 3 by 3 light femion mass matrix

Lepton and quark flavor mixings

Introduction

In order to block diagonalize the fermion mass matrix, we define the light and heavy states as follows:

$$L_2 = N_1(\psi_2 - s_\theta T_2 \chi_2)$$

$$L_3 = \frac{N_1}{R_q}\left(-\psi_3 + \frac{c_\theta s_\theta T_2^2}{N_1^2}\psi_2 + T_1\chi_1 - QT_3T_1\chi_3 + \frac{c_\theta T_2}{N_1^2}\chi_2\right)$$

$$H_1 = \frac{1}{K_q}(QT_3\chi_1 + \chi_3)$$

$$H_2 = \frac{1}{N_2}(\chi_2 + T_2(s_\theta\psi_2 + c_\theta\psi_3))$$

$$H_3 = \frac{T_1K_q}{N_2R_q}(-N_1^2\psi_3 + c_\theta s_\theta T_2^2\psi_2 + c_\theta T_2\chi_2) + \frac{N_2}{K_qR_q}(QT_3\chi_3 - \chi_1)$$

Where,

$$N_1 = \sqrt{1+s_\theta^2T_2^2}, \quad N_2 = \sqrt{1+T_2^2}$$

$$R_q = \sqrt{1+T_2^2+T_1^2(1+s_\theta^2T_2^2)(1+Q^2T_3^2)},$$

$$K_q = \sqrt{1+Q^2T_3^2},$$

$$T_1 = \frac{\langle S\rangle}{M_1}, \quad T_2 = \frac{\langle \varepsilon\rangle}{M_2}, \quad T_3 = \frac{\langle 45_H\rangle}{M_3}.$$

$s_\theta \approx 0.04,\ s_\theta T_2 \ll 1$ ⇒ The first and second generations are almost degenerate

$\frac{c_\theta T_2}{N_1^2},\ Q\,T_3\,T_1 \approx 1$ ⇒ The third generation splits and Causes flavor violation

Lepton and Quark flavor mixings

A-terms mass insertion

First Transformation gives

$$\tilde{M}^2_{RL} = \tilde{a}_1 M_{1f} + \tilde{a}_2 M_{2f}$$

Second diagonalization $V_R^* M_f V_L = M_f^{diag}$

$$V_R^* \tilde{M}^2_{RL} V_L = \tilde{a}_1 M_f^{diag.} + \tilde{a}_{21} V_R^* M_{2f} V_L . \qquad \text{Where,} \quad \tilde{a}_{21} = \tilde{a}_2 - \tilde{a}_1$$

$$\delta^l_{12} = (0.075, 0.01)\text{GeV}\frac{\tilde{a}_{21}}{\tilde{m}^2} \leq 10^{-5}, \ \delta^d_{12} = (0.34, 0.69)\text{GeV}\frac{\tilde{a}_{31}}{\tilde{m}^2} \leq 9\times 10^{-5}$$

$$\delta^l_{13} = (0.035, 0.07)\text{GeV}\frac{\tilde{a}_{13}}{\tilde{m}^2} \leq 0.04, \delta^d_{13} = (0.079, 0.35)\text{GeV}\frac{\tilde{a}_{31}}{\tilde{m}^2} \leq 1.7\times 10^{-2}$$

$$\delta^l_{23} = (0.056, 0.88)\text{GeV}\frac{\tilde{a}_{13}}{\tilde{m}^2} \leq 0.03, \delta^d_{23} = (0.055, 0.8)\text{GeV}\frac{\tilde{a}_{31}}{\tilde{m}^2} \leq (6, 4.5)\times 10^{-3}$$

$\tilde{a}_{21} \approx 100\,\text{GeV}$, and $\tilde{m} \approx 500\,\text{GeV}$ ⇨ The Correct FCNC Suppression

Lepton and Quark flavor mixings

The sfermion mass insertion

A4 discrete symmetry is broken at GUT scale. From top Yukawa coupling

$$1.5 \le T_2, T_3 \le 2.4 \quad T_1 \approx 1$$

The quadratic mass soft terms are give by:

$$L_{\text{soft}} = \overline{m}^2 \psi_i^* \psi_i + \overline{M}_\alpha^2 \chi_\alpha^* \chi_\alpha$$

In terms of the new orthogonal eigenstates, The soft mass matrix for light state is modified as follows:

$$\begin{pmatrix} \overline{m}^2 & 0 & 0 \\ 0 & \overline{m}^2 & 0 \\ 0 & 0 & \overline{m}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \overline{m}^2 & 0 & 0 \\ 0 & \overline{m}^2 & 0 \\ 0 & 0 & \overline{m}^2 \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \alpha \\ \mathbf{0} & \alpha & \delta \end{pmatrix}$$

Where,

$$\delta = \frac{\overline{m}^2 + \overline{M}_1^2 T_1^2 + \overline{M}_2^2 T_2^2 + \overline{M}_3^2 T_1^2 T_3^2 Q^2}{R_q^2} - \overline{m}^2, \quad \alpha = \frac{(\overline{m}^2 - \overline{M}_2^2) s_\theta T_2^2}{R_q}$$

Lepton and Quark flavor mixings

The sfermion mass insertion

$$T_1 = 1, \quad T_2 = 2, \quad T_3 = 2.3, s_\theta = 0.042, \quad M_1 = M_2 = M_3$$

$$\delta = 0.87(\overline{M}^2 - \overline{m}^2), \quad \alpha = -0.059(\overline{M}^2 - \overline{m}^2)$$

Second diagonalization $V_R^* M_f V_L = M_f^{diag}$

$$(\delta_{13}^l)_{LL} = 0.027\ \sigma \le 0.15, \delta_{12}^d = (2.3\times 10^{-4}, 0.13)\ \sigma \le (1.4, 90)10^{-4},$$

$$(\delta_{23}^l)_{LL} = 0.36\ \sigma \le 0.12, \delta_{13}^d = (0.013, 0.077)\ \sigma \le (0.09, 0.07),$$

$$(\delta_{12}^l)_{RR} = 7\times 10^{-4}\ \sigma \le 0.09, \delta_{23}^d = (0.06, 0.36)\ \sigma \le (0.16, 0.22).$$

$$\boldsymbol{(\delta_{12}^l)_{LL} = 0.051\sigma \le 6\times 10^{-4}}$$

The above stringent upper bounds comes from $\Gamma(\mu \to e\gamma)$

Where, $\sigma = \dfrac{\tilde{m}^2 - \tilde{M}^2}{\tilde{m}^2} \approx 0.01$

So, it is required high degree of degeneracy to suppress $\Gamma(\mu \to e\gamma)$

Note: The above numerical upper bound is obtained from arXive:hep-ph/0702144v2

Lepton and quark flavor mixings

Universality hypothesis at the Planck scale.

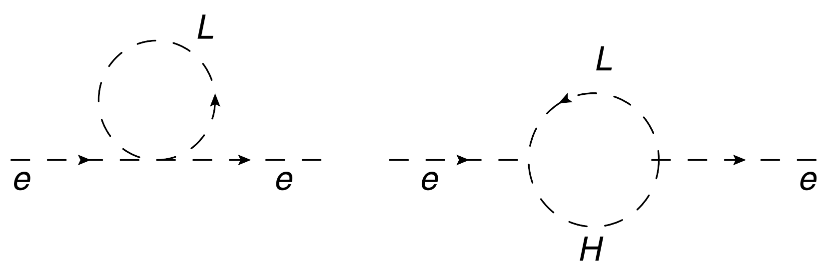
Can the universality hypothesis of masses at the Planck scale reproduce this high degeneracy?

At Planck scale $\tilde{M}_1^2 = \tilde{M}_2^2 = \tilde{M}_3^2 = \tilde{m}^2$

RGE $\Longrightarrow$ $\tilde{M}_2^2 = \tilde{M}_3^2, \quad \dfrac{\tilde{m}^2 - \tilde{M}_1^2}{\tilde{m}^2} \approx 1, \quad \dfrac{\tilde{m}^2 - \tilde{M}_2^2}{\tilde{m}^2} \approx 0.07$

Gauge Mediating Supersymetry Breaking (GMSB):

FV appears only if there is mixing between the messenger fields and ordinary fields :



$$\frac{\delta m^2}{m^2} \leq 1 \quad \Longrightarrow \quad |F_s|/M^2 < 0.03$$

In this Model, $10_i \Rightarrow \bar{5}_i + 5_i$. If $\dfrac{|F_s|}{M^2} \leq 0.03,\ Y_{exotic} < Y_{ordinary}$

then, the FCNC constraints are satisfied (Dine et al.,1996)

Conclusion

- The supersymmetric model of SO(10)XA4 with minimum Higgs representation makes the unified gauge coupling perturbative to the Planck scale, It successfully describes the fermion masses, CKM and neutrino mixings.

- The sources of the flavor mixings arise from the order one mixing of the third light generation and the heavy states. Consequently, the mass of the third generation splits.

- FCNC of $\mu \to e\gamma$ is only suppressed if we make the soft mass degeneracy arrangement of the light and heavy state. However, this arrangement is not consistent with universality hypothesis of PMSB.

- Therefore, the model adopts the GMSB. It contains the fields that serve as SUSY breaking messengers.