Electroweak Symmetry Breaking with Holomorphic Supersymmetric Nambu–Jona-Lasinio Model

— **Talk at PHENO 2010**

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Nambu–Jona-Lasinio Model :-

- dynamical symmetry breaking
- four-fermion interaction

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$$

$$\longrightarrow \mathcal{L}_\psi - (\mu\phi^\dagger + g\psi_+\psi_-)(\mu\phi + g\bar{\psi}_+\bar{\psi}_-)$$

$$= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- - \mu^2\phi^\dagger\phi - \mu g(\phi^\dagger\bar{\psi}_+\bar{\psi}_- + \phi\psi_+\psi_-)$$

- auxiliary scalar field ϕ (no kinetic term)
- EL-eq for $\phi^\dagger$ gives ϕ as composite

$$\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$$

- $\langle\phi\rangle \neq 0 \Longrightarrow$ symmetry breaking and fermion mass

$\rightarrow$ **low energy effective field theory :-**

- 1-loop effective potential for ϕ gives gap equation

  $$\langle\phi\rangle \neq 0 \text{ solution for } \frac{g^2\Lambda^2}{8\pi^2} > 1$$

- Dirac fermion mass $m = \mu g \langle\phi\rangle$ for $\psi_+ - \psi_-$

  $$g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \qquad \Longrightarrow \qquad g^2 \left\langle \bar{\psi}_+ \bar{\psi}_- \right\rangle \psi_+ \psi_-$$

- kinetic term for ϕ through wave-function renormalization

  fermion-loop propagator with Yukawa vertices ($m \ll \Lambda$)

  $$Z = \frac{N_c \mu^2 g^2}{16\pi^2} \left[\ln \frac{\Lambda^2}{M^2} + O(1) \right]$$

- Higgs with mass $2m$ and a Goldstone boson

$\phi \longrightarrow \phi/\sqrt{Z}$:-

- $$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + \partial_\mu\phi^\dagger\partial^\mu\phi - \tilde{\mu}^2\phi^*\phi - \frac{\tilde{\lambda}}{2}(\phi^\dagger\phi)^2 - \tilde{y}\phi\psi_+\psi_- + h.c.$$

— $\tilde{y} = \frac{\mu g}{\sqrt{Z}} = \frac{4\pi}{\sqrt{N_c}}\frac{1}{\sqrt{\ln(\Lambda^2/M^2)}}$

— $\tilde{\mu}^2 = \left[\frac{8\pi^2}{N_c g^2} - (\Lambda^2 - M^2)\right]\frac{2}{\ln(\Lambda^2/M^2)}$

— $\tilde{\lambda} = \frac{32\pi^2}{N_c \ln(\Lambda^2/M^2)}$

- condition for $\langle\phi\rangle \neq 0$ gives gap equation result

Supersymmetrizing the NJL Model (Naively):-

- $i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \bar{\Phi}_+\Phi_+$
- $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \quad \longrightarrow \quad \int d^4\theta \;\; g^2\bar{\Phi}_+\bar{\Phi}_-\Phi_+\Phi_-$
- $-\mu g\,\phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g\,\Phi\Phi_+\Phi_-$
- $-\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2}\Phi\Phi$

BUT :-

- $\phi = -g/\mu\,\bar{\psi}_+\bar{\psi}_-$ implies

$$\mu^2\phi^*\phi = -\mu g\,\phi\psi_+\psi_- = g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \qquad \text{(no SUSY !)}$$

- no nontrivial vacuum without SUSY breaking

The Supersymmetric NJL Model :-

- $i\bar{\psi}_+ \sigma^\mu \partial_\mu \psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \bar{\Phi}_+ \Phi_+ \;\; (1 - m^2 \theta^2 \bar{\theta}^2)$
- $g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \quad \longrightarrow \quad \int d^4\theta \;\; g^2 \bar{\Phi}_+ \bar{\Phi}_- \Phi_+ \Phi_-$
- $-\mu g \, \phi \psi_+ \psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g \, \Phi_2 \Phi_+ \Phi_-$
- $-\mu^2 \phi^* \phi \quad \longrightarrow \quad \int d^2\theta \;\; \mu \, \Phi_1 \Phi_2 \; + \; \int d^4\theta \;\; \bar{\Phi}_1 \Phi_1$

BUT :-

- EL-eq for Φ_2 gives $\Phi_1 = -g \, \Phi_+ \Phi_-$ implies

$$\int d^4\theta \;\; \bar{\Phi}_1 \Phi_1 = \int d^4\theta \;\; g^2 \bar{\Phi}_+ \bar{\Phi}_- \Phi_+ \Phi_-$$

- Φ_2 not the composite Φ_1 plays the Higgs superfield $\quad \langle \Phi_1 \rangle = 0$

An Alternative Supersymmetrization ?

- $i\bar{\psi}_+ \sigma^\mu \partial_\mu \psi_+ \quad \longrightarrow \quad \int d^4\theta \;\; \bar{\Phi}_+ \Phi_+ \;\; (1 - m^2 \theta^2 \bar{\theta}^2)$
- $-\mu g \, \phi \psi_+ \psi_- \quad \longrightarrow \quad \int d^2\theta \;\; \mu g \, \Phi_0 \Phi_+ \Phi_-$
- $-\mu^2 \phi^* \phi \quad \longrightarrow \quad \int d^2\theta \;\; \frac{\mu}{2} \Phi_0 \Phi_0$

$$\Longrightarrow \qquad \mathcal{L} = \int d^4\theta \left[(\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_-)(1 - m^2 \theta^2 \bar{\theta}^2) \right] + \int d^2\theta \left[\frac{\mu}{2} \Phi_0^2 + \sqrt{\mu G} \Phi_0 \Phi_+ \Phi_- \right] + h.c.$$

- consider superpotential $W = \frac{G}{2} \Phi_+ \Phi_- \Phi_+ \Phi_-$

$$\longrightarrow W - \frac{1}{2} (\sqrt{\mu} \Phi_0 + \sqrt{G} \Phi_+ \Phi_-)(\sqrt{\mu} \Phi_0 + \sqrt{G} \Phi_+ \Phi_-)$$

With Holomorphic Four-Chiral Superfield Interaction :-

- $W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$ contains no $g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_-$

- EL-eq for auxiliary superfield Φ_0 gives $\Phi_0 = -\sqrt{G/\mu}\,\Phi_+\Phi_-$

  implies $\quad \frac{\mu}{2}\Phi_0^2 = -\frac{\sqrt{\mu G}}{2}\Phi_0\Phi_+\Phi_- = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$

- $\langle\Phi_0\rangle \implies \frac{G}{2}\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$ Dirac mass for Φ_+–Φ_-

- kinetic term for Φ_0 from wave-function renormalization

  through Φ_+–Φ_- loop with Yukawa vertices

→ low energy effective field theory :-

- (gauged-)kinetic term $\int d^4\theta \;\; Z_0 \, \bar{\Phi}_0 \, e^{2V_\Phi} \, \Phi_0 \;\; \left[1 + (2m^2 + A^2)\theta^2\bar{\theta}^2\right]$

  where $Z_0 = \frac{N_c \mu G}{16\pi^2}\left[\ln\frac{\Lambda^2}{M^2} + O(1)\right]$

— $\tilde{m}_0^2 = -(2m^2 + A^2)$, tachyonic soft mass (*cf.* radiative EWSB)

— $\tilde{y} = \frac{\sqrt{\mu G}}{\sqrt{Z_0}} = \frac{4\pi}{\sqrt{N_c}} \frac{1}{\sqrt{\ln\left(\Lambda^2/M^2\right)}}$

— $\tilde{\mu} = \frac{\mu}{Z_0} = \frac{16\pi^2}{N_c G} \frac{1}{\ln\left(\Lambda^2/M^2\right)}$

- $\frac{\mu}{2}\Phi^2$ term $\Longrightarrow$ Φ in real representation of symmetry

Condensate/Mass Generation — A Comparison :-

• NJL : $g^2\langle\bar\psi_+\bar\psi_-\rangle\ \psi_+\psi_- \quad\longrightarrow\quad -\mu g\,\langle\phi\rangle\ \psi_+\psi_-$

— symmetry breaking with bi-fermion condensate $\langle\phi\rangle$

• SNJL : $\int d^4\theta\ \ g^2\langle\bar\Phi_+\bar\Phi_-\rangle\ \Phi_+\Phi_-$

$$\longrightarrow\quad -g\left\langle F_1^\dagger\right\rangle\ [A_+F_- + A_-F_+ - \psi_+\psi_-] \qquad\qquad \left(F_1^\dagger = -\mu A_2\right)$$

— $\langle F_1\rangle = -g\,\langle A_+F_- + A_-F_+ - \psi_+\psi_-\rangle$, sbi-fermion condensate

• HSNJL : $\int d^2\theta\ \ -G\langle\Phi_+\Phi_-\rangle\ \Phi_+\Phi_-$

$$\longrightarrow\quad \sqrt{\mu G}\ \langle A_0\rangle\ [A_+F_- + A_-F_+ - \psi_+\psi_-]\ \ + \sqrt{\mu G}\,\langle F_0\rangle\ A_+A_-$$

— $\langle A_0\rangle = -\sqrt{G/\mu}\ \langle A_+A_-\rangle$, a bi-scalar condensate

Towards

EW Symmetry Breaking

NJL Model → SM :-

- four-fermion interaction $g^2\bar{Q}\bar{t^c}Qt^c$
- Higgs doublet as top-composite

$$\phi = -g/\mu(\bar{Q}\bar{t^c})$$

- top condensate breaks EW symmetry → fermion masses

— gives top quark mass at to infared quasi-fixed point

- high $m_t \sim 218$ GeV ($\Lambda \sim 10^{19}$ GeV) Bardeen, Hill, Lindner 90

  $m_t \sim 214 - 202$ GeV ($\Lambda \sim 10^{15} - 10^{19}$ GeV) Marciano 89,90

  $m_t \sim 253$ GeV Miransky, Tanabashi, Yamawaki 89; King & Mannan 90,91
- extensions, e.g. two-Higgs-doublet model

SNJL Models → MSSM (why SUSY ?):-

- SM → MSSM — hierarchy/fine-tuning problem

  scalar field is somewhat sick

- SM fermion spectrum sort of fixed (anomaly cancelation) e.g. OK 96

  scalar content — only part arbitrary (*cf.* gauge symmetry)

- SUSY — **technically** natural hierarchy

  scalar as (part of) **chiral** superfield (constrained as fermions)

  Vs Georgi's survival hypothesis

- **BUT μ-problem** — vectorlike pair of Higgs superfields

- SNJL models solve our problems

— and avoid fine-tuning of four-quark coupling(s)

Towards the MSSM :-

- consider $W = G\,\varepsilon_{\alpha\beta}\,\hat{Q}^{\alpha}\hat{U}^{c}\hat{Q}'^{\beta}\hat{D}^{c}\,(1+B\theta^2)$

$$W \longrightarrow W - \mu\,(\hat{H}_d - \lambda_u\hat{Q}\hat{U}^c)(\hat{H}_u - \lambda_d\hat{Q}'\hat{D}^c)(1+B\theta^2)$$
$$= (-\mu\hat{H}_d\hat{H}_u + y_u\hat{Q}\hat{H}_u\hat{U}^c + y_d\hat{H}_d\hat{Q}'\hat{D}^c)(1+B\theta^2)$$

- two composites — $\hat{H}_u = \frac{y_d}{\mu}\,\hat{Q}'\hat{D}^c$ and $\hat{H}_d = \frac{y_u}{\mu}\,\hat{Q}\hat{U}^c$

- low energy effective theory looks like MSSM ($A = B$)
- symmetric role for $\hat{H}_u$ and $\hat{H}_d$ $\left(\text{also}:\ \mu\lambda_\mathrm{u}\lambda_\mathrm{d} = \frac{\mathrm{y_u y_d}}{\mu} = \mathrm{G}\right)$

— numerical lifted through non-universal soft masses

— expect $\langle h_u\rangle \gtrsim \langle h_d\rangle$ (Vs UBB in D-flat)

Holomorphic Vs Old Model (for MSSM) :-

• bottom together with (vs only) top mass at quasi-fixed point

⋆ both (vs one) Higgs superfields as composites

• large (vs small) tanβ

• $A_t \simeq A_b \simeq B \quad$ (vs $A_t \simeq 0$)

• $m^2_{H_d} \simeq -(m^2_{\tilde{Q}} + m^2_{\tilde{b}} + |A_b|^2)$
plus (vs only) $m^2_{H_u} \simeq -(m^2_{\tilde{Q}} + m^2_{\tilde{t}} + |A_t|^2)$

⋆ full W $[= G_{ijkh}\, Q_i U^c_j Q_k D^c_h (1 + A\theta^2) + G^e_{ij}\, Q_3 U^c_3 L_i E^c_j (1 + A\theta^2)]$
— non-holomorphic case needs similar holomorphic terms
for Yukawa couplings of down-type quarks and charged leptons

• sbottom and stop condensates for u_i and $d_i + \ell_i$ masses
(vs top condensate and stop condensates for u_i and $d_i + \ell_i$ masses)

Numerical (RG analysis) Results :-

- earlier MSSM $t - b - \tau$ quasi-fixed point analysis (without background model) Froggatt *et.al* 93

$$m_t = 184.3 \pm 6.8 \text{ GeV}, \qquad m_h = 121.8 \pm 4.3 \text{ GeV}$$

- ? $m_t = 171.2 \pm 2.1$ GeV

- old SNJL: MSSM t quasi-fixed point analysis Carena *et.al* 92

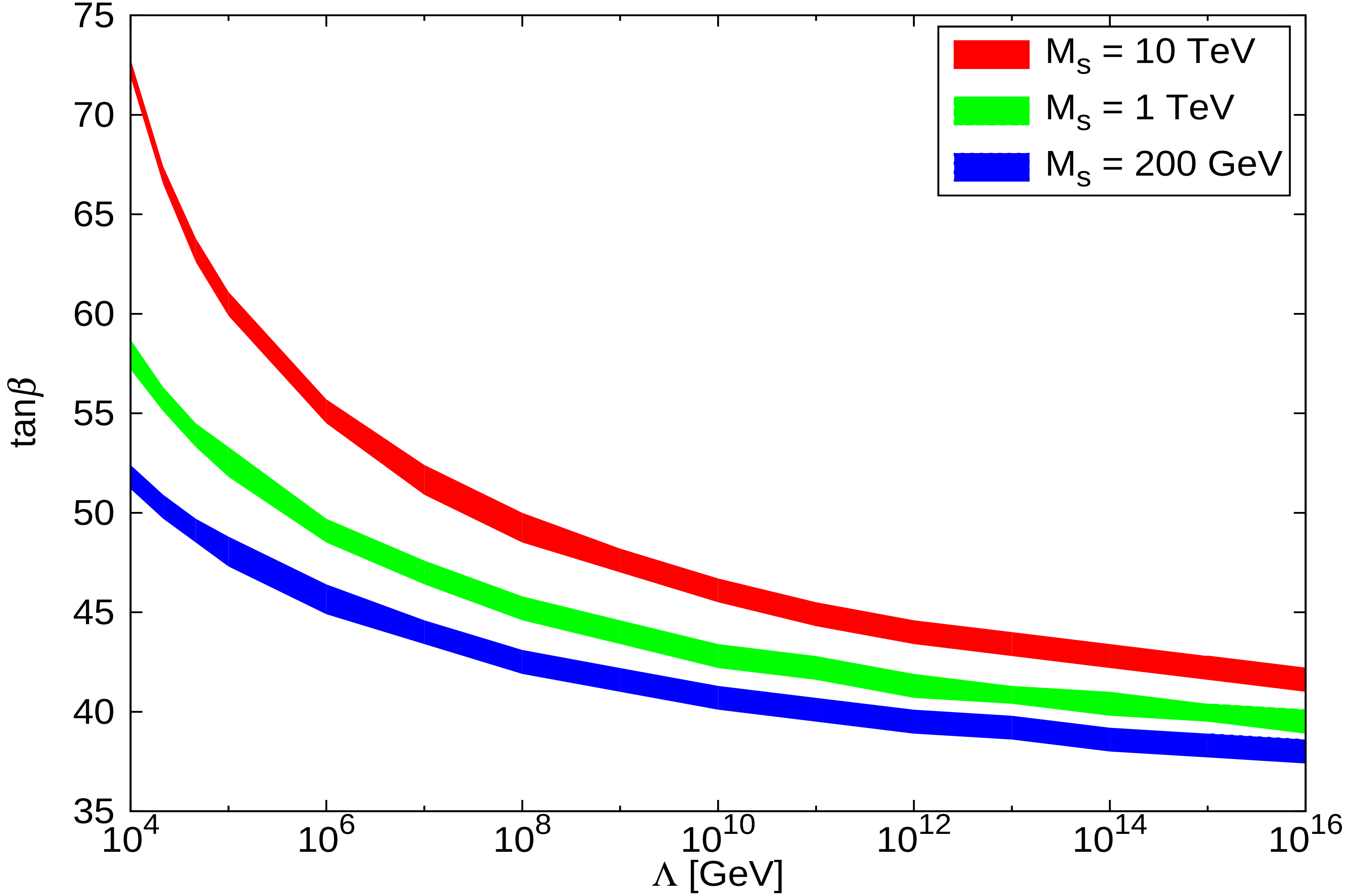
— high Λ and large tanβ lower m_t

- infared quasi-fixed point NOT necessary

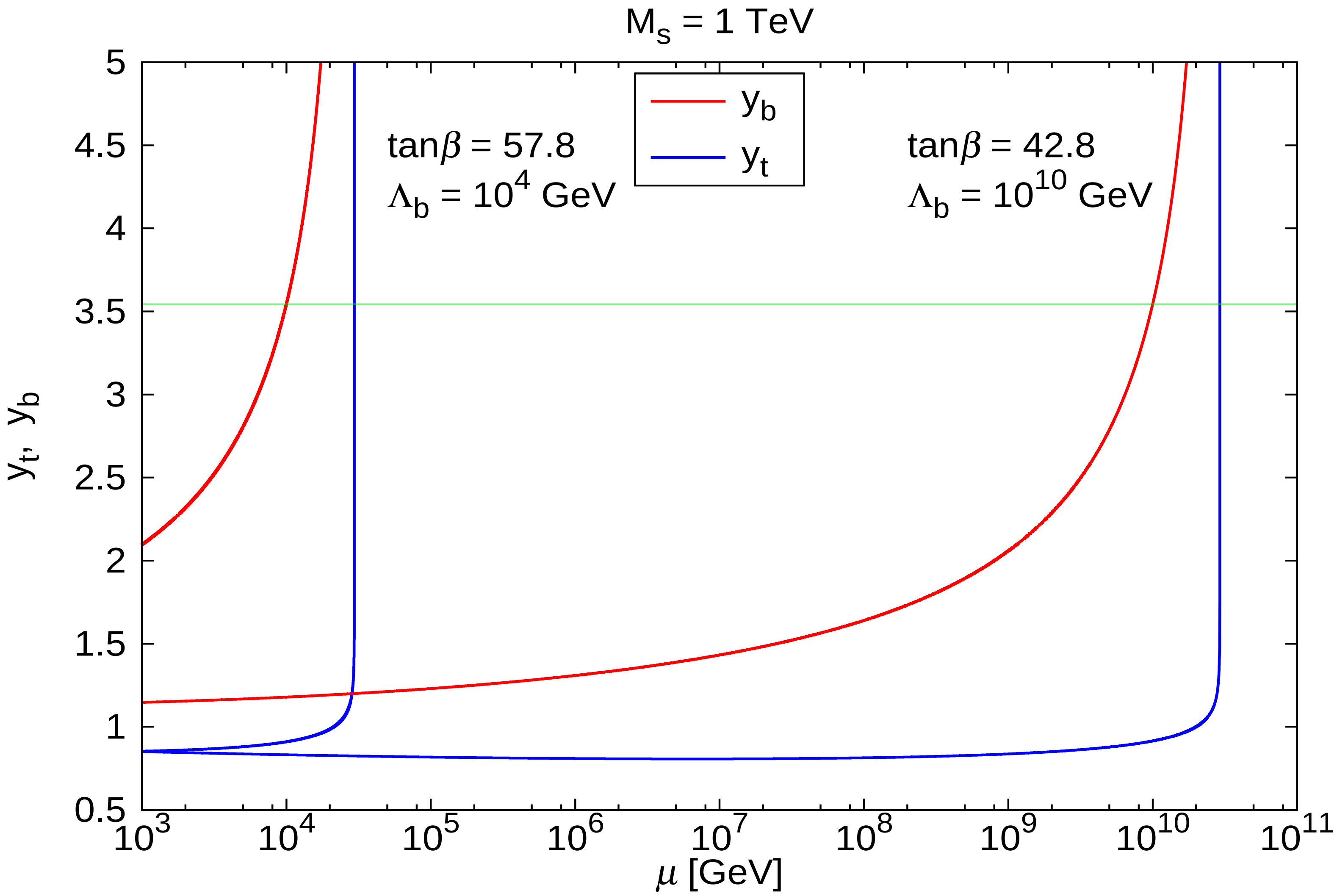
SNJL – y_t blows up at Λ

HSNJL – y_t and y_b blows up at $\sim \Lambda$

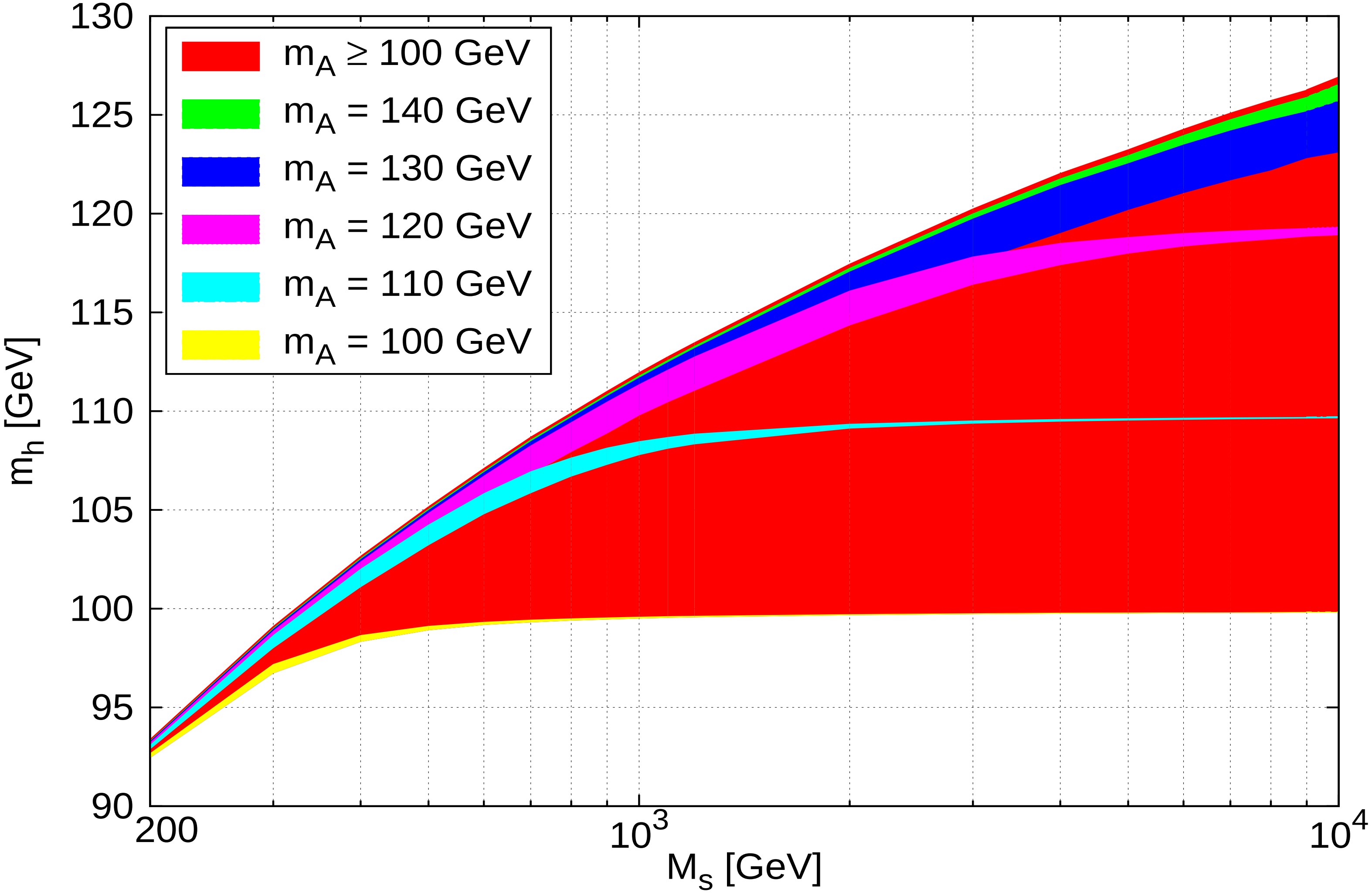
Our Solution :-



Illustrative y_t and y_b :-



Mass of the lightest Higgs boson :-



Final Remarks :-

- SNJL model with holomorphic term works
- may provide more interesting version of MSSM

— SUSY : scalar $\longrightarrow$ chiral superfield

— *problematic* MSSM superfield spectrum — vectorlike Higgs superfields, turn up as composites

— four-superfield (G) term from integrated out *heavy* Higgs superfields ?

— more natural B (and A) term, and all Yukawa coupling

- chiral symmetry explicitly broken

THANK YOU !