The Double Cover of A_5 and the Flavor Problem

Alexander Stuart

UW-Madison

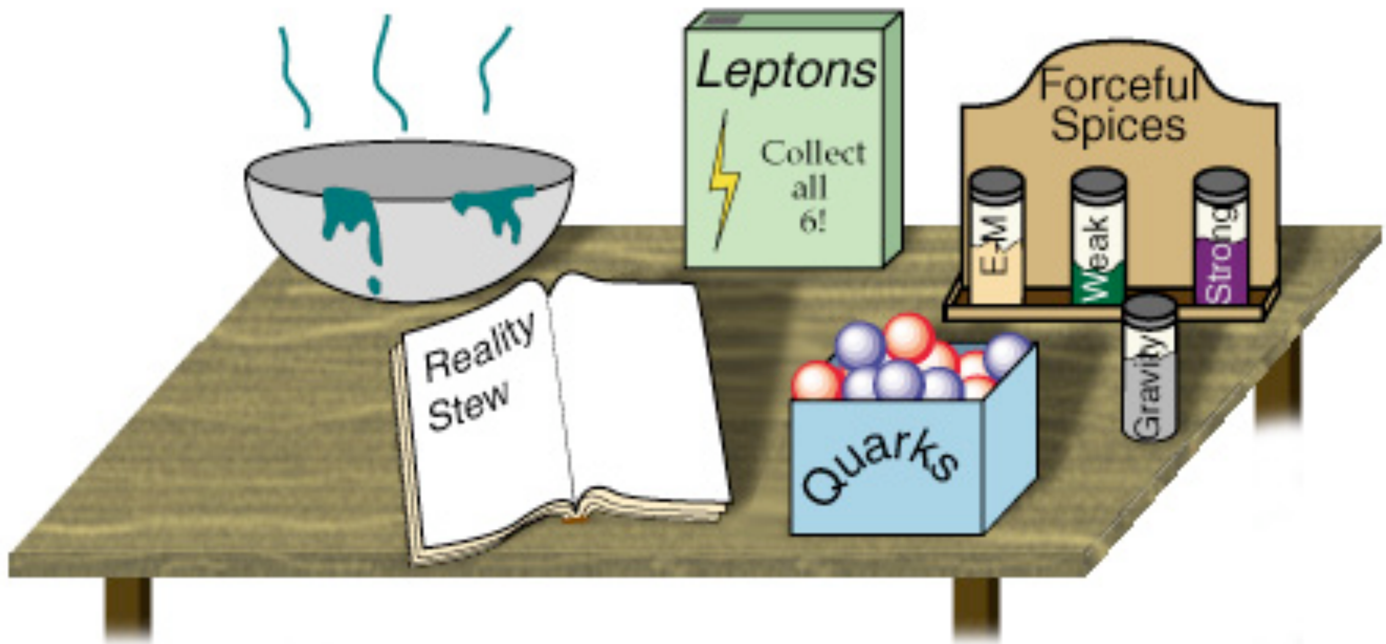
May 10th, 2010

Based on: L. Everett and A. Stuart, 0812.1057, and work in progress

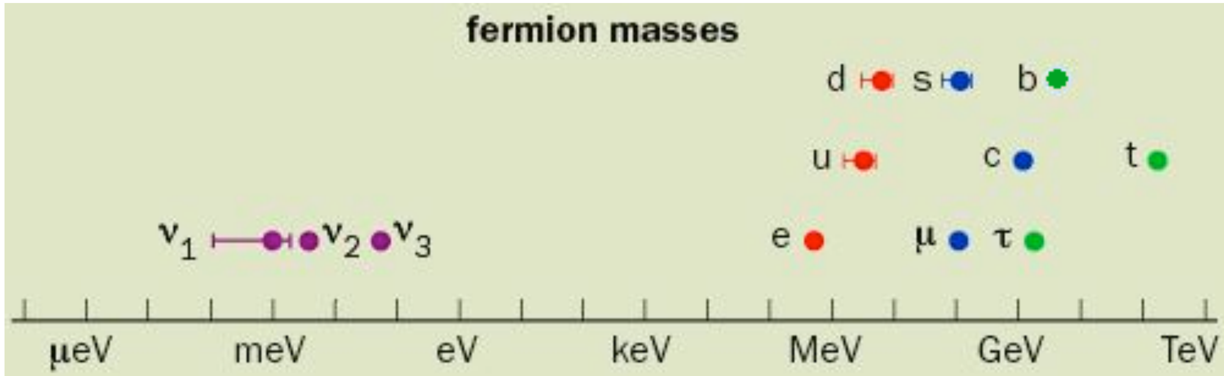
The Standard Model

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Triumph of modern science, but incomplete. Fails to predict measured masses and mixings of fermions (i.e. they are input parameters).
- What exactly do we "taste"?



What We "Taste"



Quark Mixing Angles

$$\mathcal{U}_{\rm CKM} = \mathcal{R}_1(\theta_{23}^{\rm CKM})\mathcal{R}_2(\theta_{13}^{\rm CKM}, \delta_{\rm CKM})\mathcal{R}_3(\theta_{12}^{\rm CKM})$$

$$\theta_{12}^{\rm CKM} = 13.0^\circ \pm 0.1^\circ$$
$$\theta_{23}^{\rm CKM} = 2.4^\circ \pm 0.1^\circ$$
$$\theta_{13}^{\rm CKM} = 0.2^\circ \pm 0.1^\circ$$
$$\delta_{\rm CKM} = 60^\circ \pm 14^\circ$$

Lepton Mixing Angles

$$\mathcal{U}_{\rm MNSP} = \mathcal{R}_1(\theta_\oplus)\mathcal{R}_2(\theta_{13}, \delta_{\rm MNSP})\mathcal{R}_3(\theta_\odot)\mathcal{P}$$

$$\theta_\odot = 33.9^\circ \, {}^{+2.4^\circ}_{-2.2^\circ}$$
$$\theta_\oplus = 45^\circ \, {}^{+10^\circ}_{-10^\circ}$$
$$\theta_{13} < 13^\circ$$

(presently no constraints on CP violation)

Adding Some "Ginger"

(i.e. a discrete flavor symmetry that is spontaneously broken to generate observed masses and mixings)

A_4/T is the most common discrete symmetry used (E. Ma, G. Altarelli et al.,…)

Icosahedral Symmetry

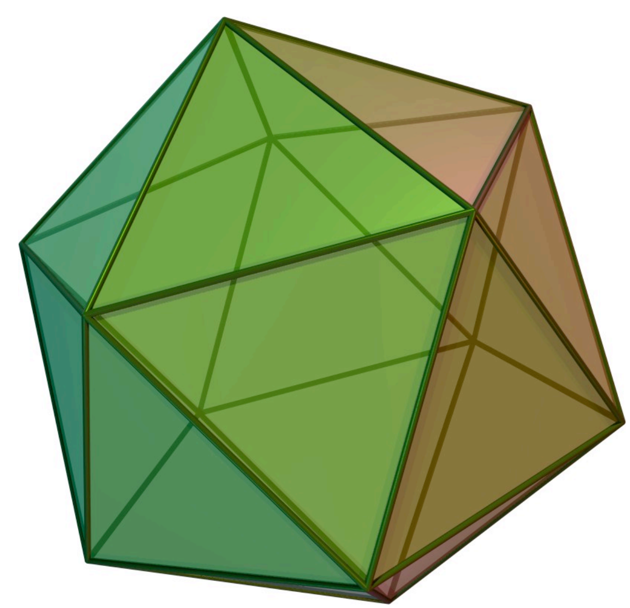
- All over nature
- Provides "natural" setting to look at $\theta_{sol} = ArcTan(\frac{1}{\phi}) = 31.7175°$
- Has been applied in this context in **arXiv:0812.1057** [hep-ph]
- Now let's apply it to the quarks.

What exactly is Icosahedral Symmetry?

The Icosahedral Group, I

- An icosahedron is the Platonic solid that consists of 20 equilateral triangles. → f = 20
- 20 triangles each have 3 sides → 60 edges but 2 triangles/edge → 30 edges → e=30
- 20 triangles each have 3 vertices → 60 vertices but 5 vertices/edge → v= 12
- Are we right? $\chi(g) = 2 - 2g = v - e + f$
- I consists of all rotations that take vertices to vertices i.e. $0, \pi, \frac{2\pi}{3}, \frac{2\pi}{5}, \frac{4\pi}{5}$

$$A_5 \cong I \subseteq SO(3)$$



http://upload.wikimedia.org/wikipedia/commons/e/eb/Icosahedron.jpg

Conjugacy Classes of I

Rotation by each angle forms its own conjugacy class.

Schoenflies Notation: C_n^k is a rotation by $\frac{2\pi k}{n}$

in front = # of elements in class

So for the icosahedral group we have:

$$e, 12C_5, 12C_5^2, 20C_3, 15C_2$$

Note:

$$1+12+12+15+20=60=1^2+3^2+3^2+4^2+5^2$$ **Two triplets.**

The Double Cover of I, I'

- To each element $g \in I$, associate another element $gR \ni R^2 = e$
- Therefore, $|I'| = 2|I| = 120$
- The characters (traces) of the new elements are related to the old by: $\chi(gR) = -\chi(g)$
- Furthermore, each conjugacy class gets a partner: $C_n^k R$
- One notable exception: $15C_2$
- We get 4 more (spinorial) irreps: $2^2 + 2^2 + 4^2 + 6^2 = 60$
- What about its character table?

$$I' \subseteq SU(2)$$

I' Character Table

$\mathcal{I}'$	**1**	**3**	**3'**	**4**	**5**	**2**	2'	4'	**6**
e	1	3	3	4	5	2	2	4	6
$12C_5$	1	ϕ	$1-\phi$	-1	0	ϕ	$1-\phi$	1	-1
$12C_5^2$	1	$1-\phi$	ϕ	-1	0	$\phi-1$	$-\phi$	-1	1
$20C_3$	1	0	0	1	-1	1	1	-1	0
$30C_2$	1	-1	-1	0	1	0	0	0	0
R	1	3	3	4	5	-2	-2	-4	-6
$12C_5R$	1	ϕ	$1-\phi$	-1	0	$-\phi$	$\phi-1$	-1	1
$12C_5^2R$	1	$1-\phi$	ϕ	-1	0	$1-\phi$	ϕ	1	-1
$20C_3R$	1	0	0	1	-1	-1	-1	1	0

…So now what?

Notable Kronecker Products of I'.

Use Character Table to easily obtain Kronecker Products (known)

$$3' \otimes 3' = 1 \oplus 3' \oplus 5 \quad 3 \otimes 3 = 1 \oplus 3 \oplus 5 \quad 3 \otimes 3' = 4 \oplus 5$$

$$2 \otimes 2 = 1 \oplus 3 \quad 2 \otimes 2' = 4 \quad 2' \otimes 2' = 1 \oplus 3'$$

$$4 \otimes 4 = 1 \oplus 3 \oplus 3' \oplus 4 \oplus 5$$

$$5 \otimes 5 = 1 \oplus 3 \oplus 3' \oplus 4 \oplus 4 \oplus 5 \oplus 5$$

These Kronecker Products will allow us to construct a simple Lagrangian (superpotential) that is invariant under the discrete symmetry that generates our observed quark masses and mixings.

All of this is abstract. We need actual representations.
Invariants not available in literature…. Work to be done.

Tensor Product Decomposition (I)

$$3 \otimes 3$$

$$3 = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$3 \otimes 3'$$

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$3 = (a_3 b_2 - b_2 a_3, a_1 b_3 - a_3 b_1, a_2 b_1 - a_1 b_2)^T$$

$$5 = \begin{pmatrix} a_2 b_2 - a_1 b_1 \\ a_2 b_1 + a_1 b_2 \\ a_3 b_2 + a_2 b_3 \\ a_1 b_3 + a_3 b_1 \\ -\frac{1}{\sqrt{3}}(a_1 b_1 + a_2 b_2 - 2 a_3 b_3) \end{pmatrix}$$

$$4 = \begin{pmatrix} \frac{1}{\phi} a_3 b_2 - \phi a_1 b_3 \\ \phi a_3 b_1 + \frac{1}{\phi} a_2 b_3 \\ -\frac{1}{\phi} a_1 b_1 + \phi a_2 b_2 \\ a_2 b_1 - a_1 b_2 + a_3 b_3 \end{pmatrix}$$

$$5 = \begin{pmatrix} \frac{1}{2}(\phi^2 a_2 b_1 + \frac{1}{\phi^2} a_1 b_2 - \sqrt{5} a_3 b_3) \\ -(\phi a_1 b_1 + \frac{1}{\phi} a_2 b_2) \\ \frac{1}{\phi} a_3 b_1 - \phi a_2 b_3 \\ \phi a_3 b_2 + \frac{1}{\phi} a_1 b_3 \\ \frac{\sqrt{3}}{2}(\frac{1}{\phi} a_2 b_1 + \phi a_1 b_2 + a_3 b_3) \end{pmatrix}$$

Tensor Product Decomposition (II)

$$2 \otimes 2 = 1 \oplus 3 \qquad 2' \otimes 2' = 1 \oplus 3' \qquad 2 \otimes 2' = 4$$

$$1 = a_2 b_1 - a_1 b_2$$

$$3 = \begin{pmatrix} -ia_1b_1 + ia_2b_2 \\ a_1b_1 + a_2b_2 \\ ia_2b_1 + ia_1b_2 \end{pmatrix}$$

$$1 = a_2 b_1 - a_1 b_2$$

$$3' = \begin{pmatrix} -ia_1b_1 - ia_2b_2 \\ -a_1b_1 + a_2b_2 \\ a_2b_1 + a_1b_2 \end{pmatrix}$$

$$4 = \begin{pmatrix} -a_2b_1 - a_1b_2 \\ ia_2b_1 - ia_1b_2 \\ -ia_1b_1 - ia_2b_2 \\ -a_1b_1 + a_2b_2 \end{pmatrix}$$

Now we have the explicit invariants… Onto the best part…

How to Build an T' Flavor Model

Seek scenario with diagonal CKM matrix (at leading order) and a hierarchical mass pattern. This "eliminates" embedding in a 3 and 3', and tells us to embed in 2's.

Our approach: Let 3rd generation quarks be singlets, and embed the 1st and 2nd generation quarks in doublets.

(Standard approach in A'_4/T' models: A. Aranda, et al., F. Feruglio et al., …)

$$U_{L,R} \to 2 \qquad\qquad 2 \otimes 2 = 1 \oplus 3$$

$$D_{L,R} \to 2$$

Forbid tree-level mass term (of 1st and 2nd generations) with an additional Z_3 symmetry

Introduce flavon fields to generate mass at higher level through spontaneous symmetry breaking

Let's Start "Cooking"

Field	$\overline{t}_L$	t_R	$\overline{b}_L$	b_R	$\overline{U}_L$	U_R	$\overline{D}_L$	D_R	H_u	H_d	ρ_u	ρ_d	ξ_u	ξ_d	ρ_u^0	ρ_d^0	ξ_u^0	ξ_d^0
I'	1	1	1	1	2	2	2	2	1	1	1	1	3	3	1	1	3	3
Z_3	0	0	0	0	1	1	2	2	0	0	1	2	1	2	1	2	1	2
$U(1)_R$	1	1	1	1	1	1	1	1	0	0	0	0	0	0	2	2	2	2

With these charge assignments, our effective mass terms to 'leading order' become:

$$w_q = \overline{t}_L t_R + \overline{b}_L b_R + \overline{U}_L U_R \frac{\rho_u}{\Lambda} + \overline{U}_L U_R \frac{\xi_u}{\Lambda} + \overline{D}_L D_R \frac{\rho_d}{\Lambda} + \overline{D}_L D_R \frac{\xi_d}{\Lambda}$$

Breaking the Flavor

$$\langle \xi_d \rangle = \begin{pmatrix} 0 \\ 0 \\ v_d \end{pmatrix} \quad \langle \rho_d \rangle = \rho_d \qquad \langle \xi_u \rangle = \begin{pmatrix} 0 \\ 0 \\ v_u \end{pmatrix} \quad \langle \rho_u \rangle = \rho_u$$

Spontaneously Breaking the flavor symmetry, as above, yields the following mass matrices:

$$m_{u,d}^2 \approx \begin{pmatrix} (v_{u,d} - i\rho_{u,d}^2)(v_{u,d}^* + i\rho_{u,d}^{2*}) & 0 & 0 \\ 0 & (v_{u,d} + i\rho_{u,d}^2)(v_{u,d}^* - i\rho_{u,d}^{2*}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Freedom to assume singlets are real and triplets are purely imaginary

$$m_{u,d} = \left| v_{u,d} - \rho_{u,d} \right| \qquad m_{c,s} = \left| v_{u,d} + \rho_{u,d} \right|$$

$$U_{CKM} = 1_{3x3}$$

Outlook and Conclusion

- Summary:
  - The Double Icosahedral Group provides a rich setting for investigating the flavor puzzle.
  - Our work provides a toolbox for using Icosahedral Symmetry as a flavor symmetry group.
- Where to next?
  - Flavon Potential and generate Cabibbo Angle.
  - Apply in Grand Unified Theory contexts and/or look into higher dimensional irreducible representation.
  - Write Papers.