Model of leptons from $SO(3) \to A_4$

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Outline

- Apparent pattern in neutrino mixing can be explained using nonabelian discrete symmetry A_4
- Problem: where does discrete group A_4 come from?
- Idea: get A_4 by spontaneously breaking continuous group $SO(3)$ with scalar in **7** representation
- Can get correct mixing and mass spectrum, but fine-tuning remains

Neutrino Mixing Matrix

- 3 light neutrino states have flavor state-mass state mixing
- Described by 3×3 unitary matrix U

$$|U| \approx \begin{pmatrix} 0.823 & 0.554 & 0.126 \\ 0.480 & 0.558 & 0.677 \\ 0.305 & 0.618 & 0.725 \end{pmatrix}.$$

- Is there a pattern in $|U|$?

Discrete Symmetries and U_{HPS}

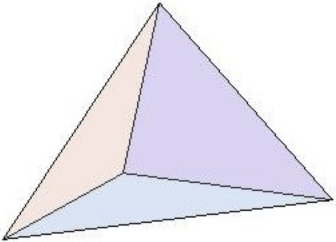
- Harrison, Perkins, and Scott pointed out that

$$U \approx U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Can we get such a pattern in U?
- One way: use non-abelian discrete symmetries
- One possible group: A_4 (Ma et. al., Altarelli et. al.)
- Industry of ν model building using A_4: many common features

A_4: Rotational Symmetries of the Tetrahedron

- Rotational symmetries of the tetrahedron
- 12 elements: Identity, 3 rotations by 180°, 4 rotations by 120°, 4 rotations by 240°
- Subgroup of $SO(3)$
- Representation: singlet, vector, and two "weird" complex 1 dimensional representations



From Continuous to Discrete

- One issue with A_4 models: why A_4?
- Generally no motivation from UV physics
- Try to get A_4 out of a more familiar continuous symmetry group
- A_4 contained in $SO(3)$, so try to start with it
- In order to get the results of A_4 models:
  - Spontaneously break $SO(3) \to A_4$
  - Put matter in appropriate representations

Breaking $SO(3)$ to A_4

- Single triplet scalar won't do it
- But higher representation scalars have non-trivial potentials
- Can get vacua with unbroken non-abelian discrete symmetries
- Motivate a choice of representation: look for a representation of $SO(3)$ that contains a singlet of A_4
- The first representation that does is the **7** (spin 3) representation
- Minimize the potential for **7**: over a large portion of parameter space, get $SO(3) \to A_4$

Getting Matter Content

- A_4 models need non-trivial ("weird" complex) representations of right-handed charged leptons
- But all representations of $SO(3)$ are "normal" and real
- Consequence: RH μ and τ part of the same $SO(3)$ multiplet
- An extra scalar with particular VEV is needed to get the right masses

Spectrum and Mixing Matrix

- Mass matrices have same form as in A_4 models
- With a little effort, can get low-energy spectrum for SM leptons (more on this soon)
- U_{HPS} is reproduced as the neutrino mixing matrix
- Lepton mass measurements constrain scales of the model
- Many scales: $\Lambda \gg v_T \gg v \sim v' \sim v_5 \gg M \gg v_H$
  - v not more than a factor of about 100 below Λ
  - $v' \gg M$ to get right neutrino mass splittings

Fine-tuning

- Right-handed μ, τ come as part of the same multiplet
- Non-trivial to find a way to split the mass of μ and τ
- $m_\mu / m_\tau \sim 1/16$ from measurements
- In our model: unrelated scales must cancel to within $1/16$
- Also: need an arbitrary phase to be near-maximal
- There is fine-tuning in this model

Conclusions

- Models with an A_4 discrete symmetry can explain apparent pattern of neutrino mixing
- The A_4 symmetry and matter content can be obtained by SSB of $SO(3)$, giving $U = U_{\mathrm{HPS}}$
- Issues with the model:
  - Vacuum alignment: why to scalars get VEVs with the right pattern?
  - Anomalies: new fermions can generate anomalies in gauge groups
  - Fine-tuning: cancelation between scales to get lepton masses

Backup: Extrema of the potential

$$V = -\frac{\mu^2}{2} T^{abc} T^{abc} + \frac{\lambda}{4} (T^{abc} T^{abc})^2 + c\, T^{abc} T^{bcd} T^{def} T^{efa}.$$

Three cases:

- For $c > 0$, minimum has A_4 symmetry
- For $-\lambda/2 < c < 0$, minimum has D_3 symmetry
- For $c < -\lambda/2$, potential is unstable

Two cases where discrete symmetries arise: breaking a continuous gauge symmetry to a discrete subgroup is generic!

Backup: Model with SSB

- Model with symmetries $SU(2)_L \times U(1)_Y \times SO(3)_F \times Z_2$

Field	$SU(2)_L$	$U(1)_Y$	$SO(3)_F$	Z_2
ψ_ℓ	**2**	$-1/2$	**3**	$-$
ψ_f	**1**	-1	**3**	$-$
ψ_e	**1**	-1	**1**	$+$
ψ_m	**1**	-1	**5**	$+$
ψ_n	**1**	0	**3**	$-$
H	**2**	$1/2$	**1**	$+$
ϕ	**1**	0	**3**	$-$
ϕ'	**1**	0	**3**	$+$
ϕ_5	**1**	0	**5**	$-$
T	**1**	0	**7**	$-$

Field	$SO(3)$ SB
H	None
ϕ	Z_3
ϕ'	Z_2
ϕ_5	Z_3
T	A_4

Backup: Degenerate μ and τ?

- ϕ couples equally to Ψ_μ and $\Psi_\tau \implies$ degenerate muon and tau
- Add another scalar ϕ_5 that transforms as a 5
- New coupling y_m^5 between $\psi_\ell \phi_5 \psi_m$
- However, mass splitting depends on phase difference between couplings to ψ_m!

$$m_\tau - m_\mu \sim v v_5 \cos[\arg(y_m y_m^{5*})]$$