The supersymmetric little hierarchy problem and possible solutions

Stephen P. Martin

Northern Illinois University

PHENO 2010

May 11, 2010

Based in part on `0910.2732`, and work to appear with James Younkin.

Supersymmetry is Too Big To Fail:

- A solution to the big hierarchy problem of $M_{\rm Planck}/M_W$
- A dark matter candidate
- Easily satisfies oblique precision electroweak constraints
- Gauge coupling unification

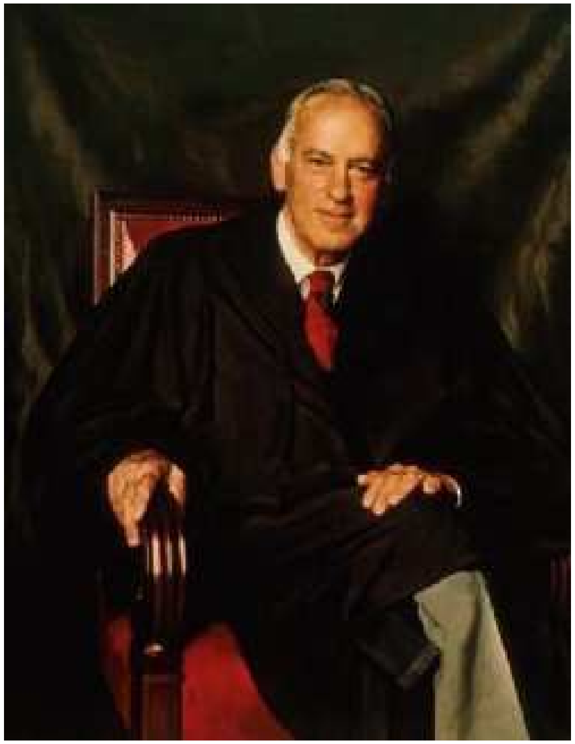
However, the non-discovery of the lightest Higgs boson h^0 at LEP2 is cause for doubt.

The <u>supersymmetric little hierarchy problem</u> is the rational fear that some percent-level fine-tuning is needed to explain how h^0 evades the LEP2 bounds ($M_h > 114$ GeV in most supersymmetric models).

What is fine tuning?

“I shall not today attempt further to define [it]... and perhaps I could never succeed in intelligibly doing so. **But I know it when I see it...**”

U.S. Supreme Court Justice Potter Stewart concurrence in *Jacobellis v. Ohio* (1964).



Like pornography, fine-tuning is impossible to define.

There is no such thing as an objective measure on the parameter space of SUSY, or any other theory. Only one set of parameters, at most, is correct!

But, like Potter Stewart, we usually know it when we see it.
So, even lacking the possibility of a real definition, let us proceed.

SUSY prediction for lightest Higgs mass:

$$M_h^2 \ = \ m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^4 \sin^2\beta \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Top squarks are spin-0 partners of top quark: $\tilde{t}_1$, $\tilde{t}_2$.

$\tan\beta = v_u/v_d$ = ratio of Higgs VEVs.

To evade discovery at LEP2, need $\sin\beta \approx 1$ and (naively)

$$\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \ \gtrsim \ 700 \text{ GeV}.$$

The logarithm apparently must be $\gtrsim \ 3$.

Meanwhile, the condition for Electroweak Symmetry Breaking is:

$$m_Z^2 \;=\; -2\left(|\mu|^2 + m_{H_u}^2\right) + \text{small loop corrections} \;+\; \mathcal{O}(1/\tan^2\beta).$$

Here $|\mu|^2$ is a SUSY-preserving Higgs squared mass,
$m_{H_u}^2$ is a (negative) SUSY-violating Higgs scalar squared mass.

The problem: typical models for SUSY breaking imply that $-m_{H_u}^2$ is comparable to $m_{\tilde{t}_1} m_{\tilde{t}_2} \;\gtrsim\; (700\ \mathrm{GeV})^2$. If so, then required cancellation is of order 1%, or worse.

Things may not be so bad, for at least four reasons:

- The previous formula for M_h is too simplistic.
  Top-squark mixing can raise M_h dramatically.
- The previous formula for M_h changes in extensions of the minimal SUSY model.
- It isn't obvious how $-m^2_{H_u}$ is related to $m_{\tilde{t}_1} m_{\tilde{t}_2}$.
  They are related by SUSY breaking, but in different ways in different models.
- Maybe h^0 cleverly hid from LEP2, and M_{h^0} really is significantly less than 114 GeV. (See e.g. Gunion and Dermisek.)

Much work on the SUSY Little Hierarchy Problem has been done in the last decade.

I will not attempt a proper survey today.

But, just in the PHENO10 parallel sessions, work directly motivated or informed by it includes J. Zurita, A. de la Puente, J.P. Olson, P. Draper, R. Dermisek, J. Younkin.

Possible Solution #1:

Large stop mixing.

Include effects of a stop mixing angle with (cosine, sine) $= c_{\tilde{t}},\ s_{\tilde{t}}$:

$$M_h^2 = m_Z^2 + \frac{3y_t^2}{4\pi^2} m_t^2 \Bigg[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{c_{\tilde{t}}^2 s_{\tilde{t}}^2}{m_t^2}(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)\ln\left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right) + \frac{c_{\tilde{t}}^4 s_{\tilde{t}}^4}{m_t^4}\left\{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2 - \frac{1}{2}(m_{\tilde{t}_2}^4 - m_{\tilde{t}_1}^4)\ln\left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right)\right\}\Bigg].$$

The Blue term is positive definite.

The Red term is negative definite.

Maximizing with respect to the stop mixing angle, one can show:

$$M_h^2 < m_Z^2 + \frac{3y_t^2}{4\pi^2} m_t^2 \Big[\ln\big(m_{\tilde{t}_2}^2/m_t^2\big) + 3 \Big].$$

The upper bound is the “maximal mixing” scenario.

Unfortunately, in many specific classes of models of SUSY breaking, the mixing angle is not large enough.

Possible Solution #2:

Non-universal gaugino masses.

(More generally, abandon mSUGRA.)

In mSUGRA, there are only 5 new parameters at $M_U = 2 \times 10^{16}$ GeV:

$$\begin{aligned} M_{1/2} &= \text{universal gaugino mass} \\ m_0 &= \text{universal scalar mass} \\ A_0 &= \text{universal (scalar)}^3 \text{ coupling} \\ \tan\beta &= \langle H_u \rangle / \langle H_d \rangle \\ \text{Arg}(\mu) & \end{aligned}$$

What if we allow the bino, wino, and gluino masses M_1, M_2, M_3 to be distinct at M_U ?

The large value of μ in mSUGRA is mostly the gluino's fault.

(G. Kane and S. King, `hep-ph/9810374`)

$$-m_{H_u}^2 = 1.92 M_3^2 + 0.16 M_2 M_3 - 0.21 M_2^2$$
$$+ \text{many terms with tiny coefficients}$$

The parameters on the right side are at M_U, left side is at the TeV scale after RG running.

If one takes a smaller gluino mass at M_U, say $M_3/M_2 \sim 1/3$, then $-m_{H_u}^2$ will be much smaller.

As a result, $|\mu|^2$ will also be very small, solving the little hierarchy problem.

An example:

$\langle F \rangle =$ order parameter that breaks SUSY.

Suppose $\langle F \rangle$ transforms in a linear combination of the singlet and the adjoint of the $SU(5)$ that contains $SU(3)_c \times SU(2)_L \times U(1)_Y$.

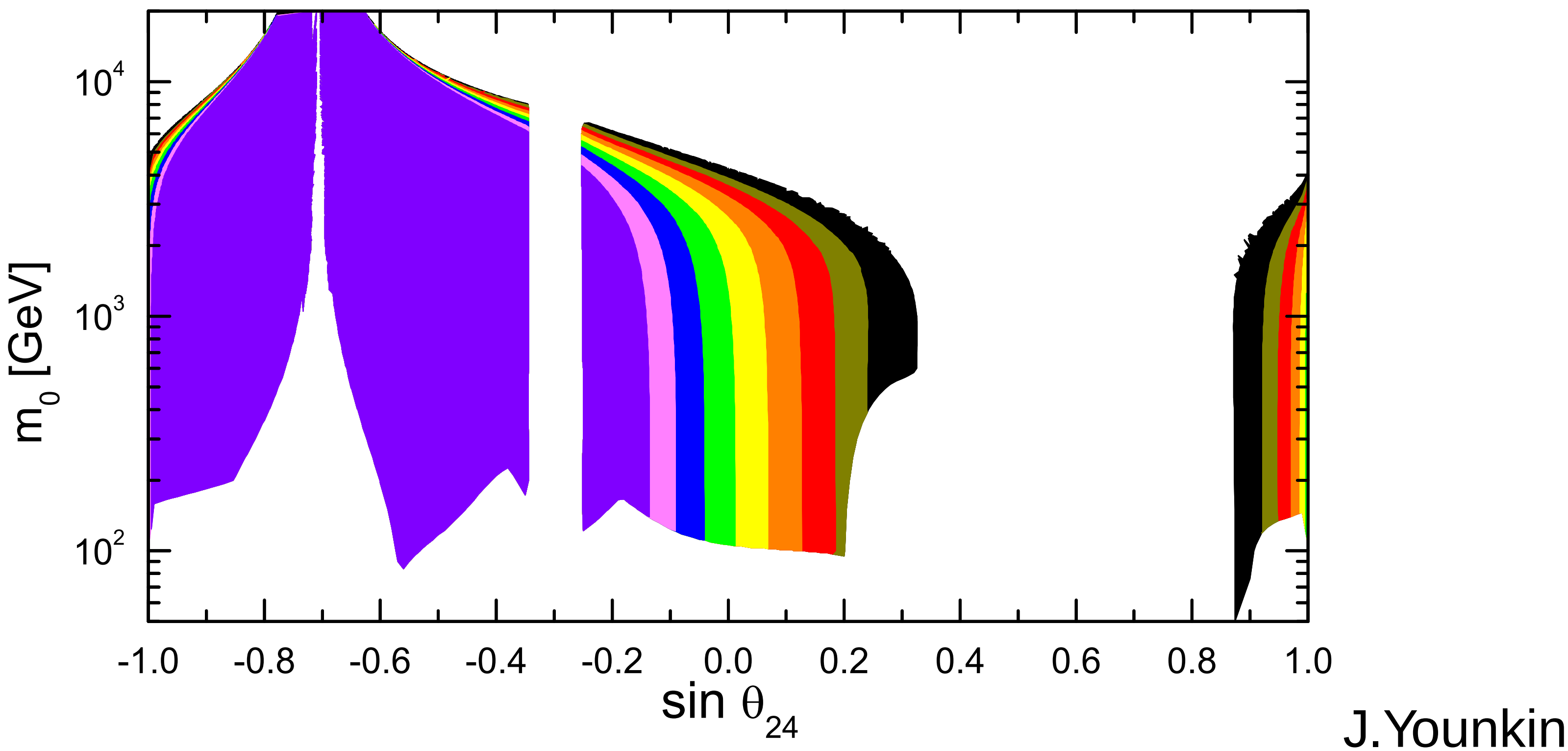
Then:

$$\begin{aligned} M_1 &= m_{1/2}(\cos\theta_{24} + \sin\theta_{24}) \\ M_2 &= m_{1/2}(\cos\theta_{24} + 3\sin\theta_{24}) \\ M_3 &= m_{1/2}(\cos\theta_{24} - 2\sin\theta_{24}) \end{aligned}$$

Note $\sin\theta_{24} = 0$ is usual mSUGRA.

$\sin\theta_{24} \gtrsim 0.2 \longrightarrow$ small M_3/M_2, “Compressed SUSY”, solution to little hierarchy problem.

Map of μ for varying $\sin\theta_{24}$, m_0; fixed $M_1 = 500$ GeV, $\tan\beta = 10$.



Black: $\mu < 300$ GeV, Brown: $300 < \mu < 400$,

Red: $400 < \mu < 500$, etc.

For much more, see Younkin's talk in SUSY3 parallel session.

Possible solution # 3:

Extend minimal SUSY.

The reason why $M_h^2 \sim m_Z^2$ at tree-level is because the Higgs quartic coupling is small: $(g^2 + g'^2)/8$.
Many candidate models work by adding to this coupling.

For example, the Next-to-Minimal Supersymmetric Standard Model extends the minimal model with a singlet S. The superpotential interaction is

$$W = \lambda S H_u H_d,$$

and leads to

$$\Delta M_h^2 = m_Z^2 \cos^2(2\beta) + \lambda^2 v^2 \sin^2(2\beta).$$

Possible Solution #4:

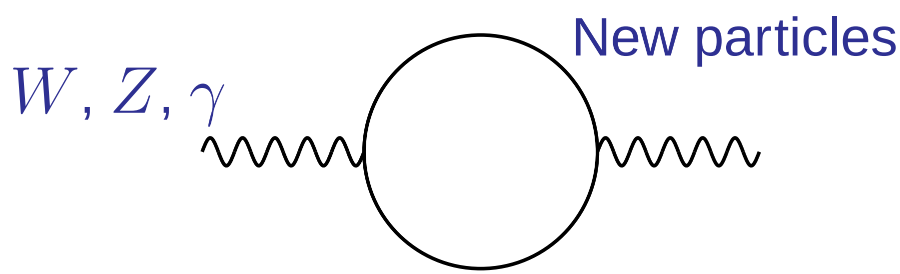
Extend minimal SUSY more radically.

I'll spend the remainder of my time on this.

Extend MSSM with new vectorlike matter =

fields in <u>real</u> representation of unbroken gauge group.

In general, new physics is highly constrained by precision electroweak observables (Peskin-Takeuchi S,T parameters):

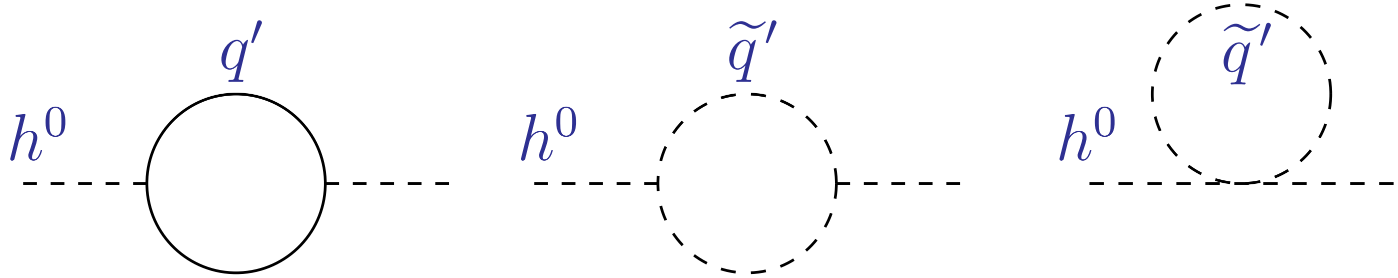


For vectorlike matter, contributions to S, T decouble like $1/M^2$.

In minimal SUSY, the new fermions (gauginos and Higgsinos) are vectorlike. Why not add more of them?

If the vectorlike fermions also have large Yukawa couplings, in addition to their bare masses, then they will contribute to M_h^2.

For example, new vectorlike quarks contribute to M_h^2 through these diagrams:



These contributions do **not** decouple for large $M_{q'}$, as long as there is a hierarchy of squark to quark masses, $M_{\tilde{q}'}/M_{q'}$.

Generic structure of new extra vectorlike matter superfields:

$$\begin{aligned} \Phi, \overline{\Phi} &= SU(2)_L \text{ doublets (vectorlike)} \\ \phi, \overline{\phi} &= SU(2)_L \text{ singlets (vectorlike)} \end{aligned}$$

Superpotential:

$$W = M_\Phi \Phi \overline{\Phi} + M_\phi \phi \overline{\phi} + k H_u \Phi \overline{\phi}$$

Yukawa coupling = k, and $\Delta m^2_{h^0} \propto k^4 v^2$.

So want k as large as possible = IR quasi-fixed point of renormalization group equations.

Important earlier work on this subject:

Moroi and Okada 1992,

Babu, Gogoladze, and Kolda 2004,

Babu, Gogoladze, Rehman, Shafi 2008.

But, corrections to the Peskin-Takeuchi T parameter were overestimated by a factor of about 4. So much less constrained than previously thought! (SPM, 2009)

Want to maintain or improve successes of minimal SUSY:

- Perturbative gauge coupling unification
- No unconfined fractional charges
- Avoid fine tuning: new particles not too heavy.

Building block superfields, under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$
\begin{aligned}
Q, \overline{Q}: &\quad (\mathbf{3}, \mathbf{2}, \frac{1}{6}),\ (\overline{\mathbf{3}}, \mathbf{2}, -\frac{1}{6}) \\
U, \overline{U}: &\quad (\mathbf{3}, \mathbf{1}, \frac{2}{3}),\ (\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}) \\
D, \overline{D}: &\quad (\mathbf{3}, \mathbf{1}, -\frac{1}{3}),\ (\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) \\
L, \overline{L}: &\quad (\mathbf{1}, \mathbf{2}, -\frac{1}{2}),\ (\mathbf{1}, \mathbf{2}, \frac{1}{2}) \\
E, \overline{E}: &\quad (\mathbf{1}, \mathbf{1}, -1),\ (\mathbf{1}, \mathbf{1}, 1) \\
N: &\quad (\mathbf{1}, \mathbf{1}, 0) \quad \text{(singlet)} \\
T: &\quad (\mathbf{1}, \mathbf{3}, 0) \quad \text{(electroweak triplet)} \\
O: &\quad (\mathbf{8}, \mathbf{1}, 0) \quad \text{(color octet)})
\end{aligned}
$$

All others give unconfined fractional charges, or will ruin perturbative unification.

Models with perturbative unification:

(LND)^n : $(L, \overline{L}, D, \overline{D}, N, \overline{N}) \times n$ $[\mathbf{5} + \overline{\mathbf{5}}$ of $SU(5), \ \ n = 1, 2, 3]$

QUE : $Q, \overline{Q}, U, \overline{U}, E, \overline{E}$ $[\mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)]$

QDEE : $Q, \overline{Q}, D, \overline{D}, E, \overline{E}, E, \overline{E}$

OTLEE : $O, T, L, \overline{L}, E, \overline{E}, E, \overline{E}$ [adjoint of $SU(3)_c \times SU(3)_L \times SU(3)_R$]

$\ldots$

(There are 5 more.)

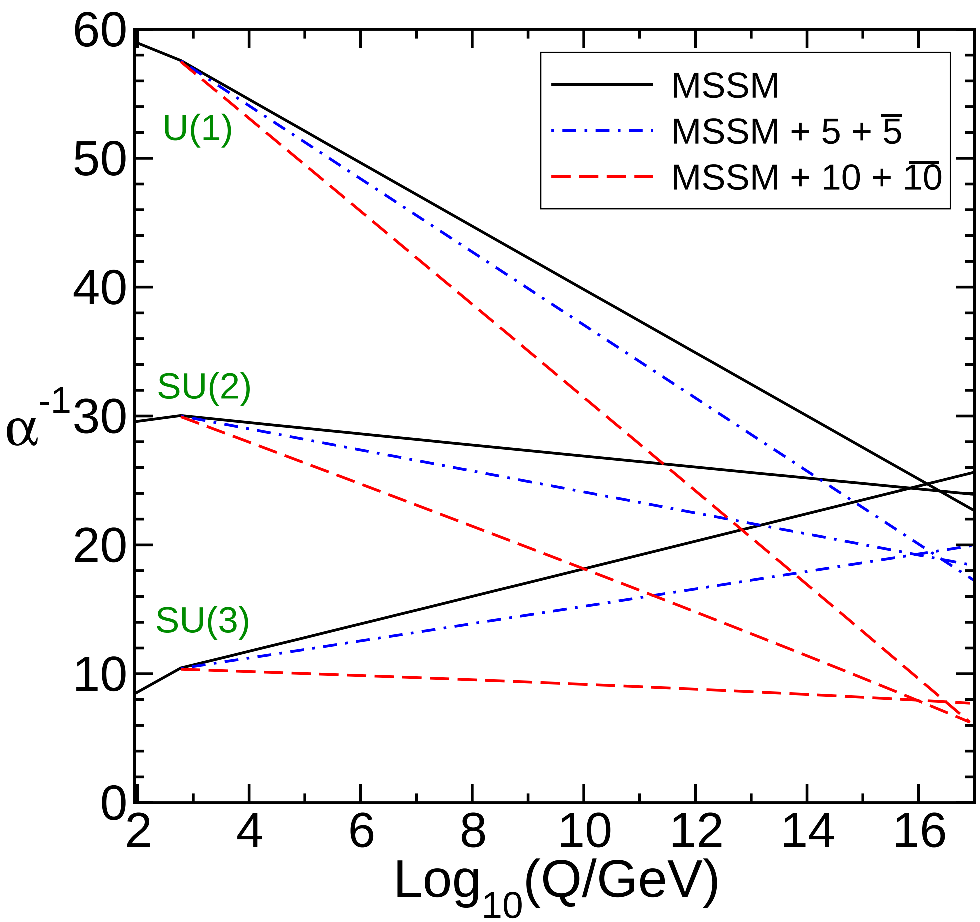
The OTLEE model has a qualitatively different feature than first three:

$\mathcal{L} = k H_u T L =$ (Higgs doublet)(triplet)(doublet) Yukawa coupling

Not discussed today; see forthcoming paper for details.

Gauge couplings still unify above 10^{16} GeV, but at stronger coupling.

Three-loop running:



Black = MSSM

Blue = LND Model

Red = QUE Model

(QDEE, OTLEE similar)

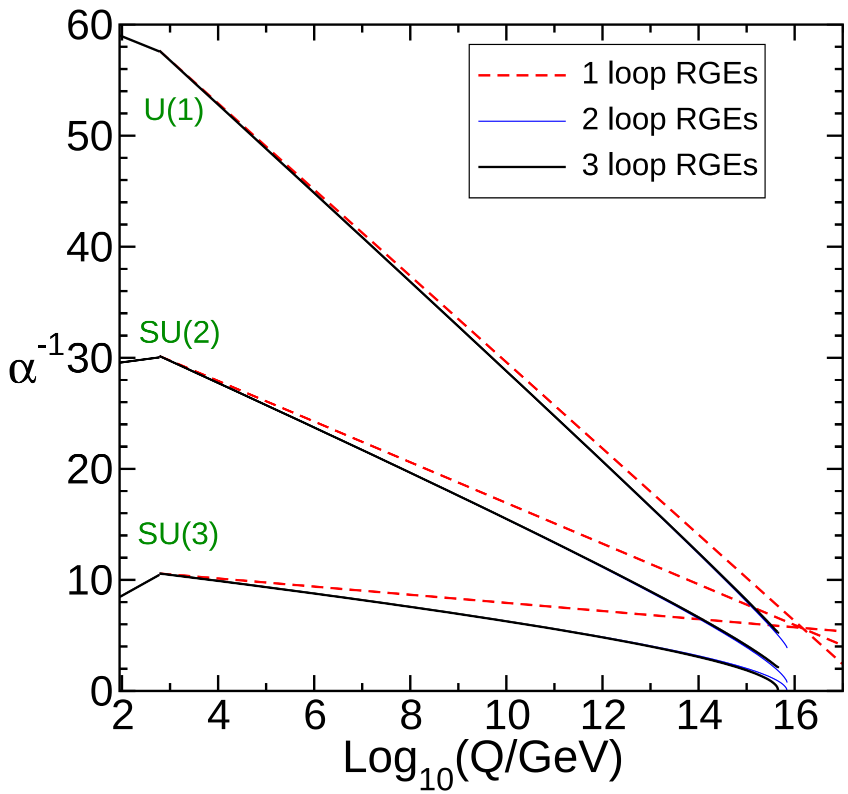
All new particle thresholds taken at $Q = 600$ GeV.

Extra fields contribute equally to the three beta functions at 1 loop.

An aside: why not a complete 4th vector-like family?

Explored by BGRS2008, and more recently in an interesting paper by Graham, Ismail, Saraswat and Rajendran `0910.2732` based on a 1-loop analysis.

However, taking into account higher-loop effects, perturbative unification fails (unless new particle masses $\gtrsim$ 2.5 TeV):



QUE Model:

$$W = M_Q Q\overline{Q} + M_U U\overline{U} + kH_u Q\overline{U} + M_E E\overline{E}$$

New fermions: $t', \;\; t'', \;\; b', \;\; \tau'$

New scalars: $\tilde{t}'_{1,2,3,4}, \;\; \tilde{b}'_{1,2}, \;\; \tilde{\tau}'_{1,2}$

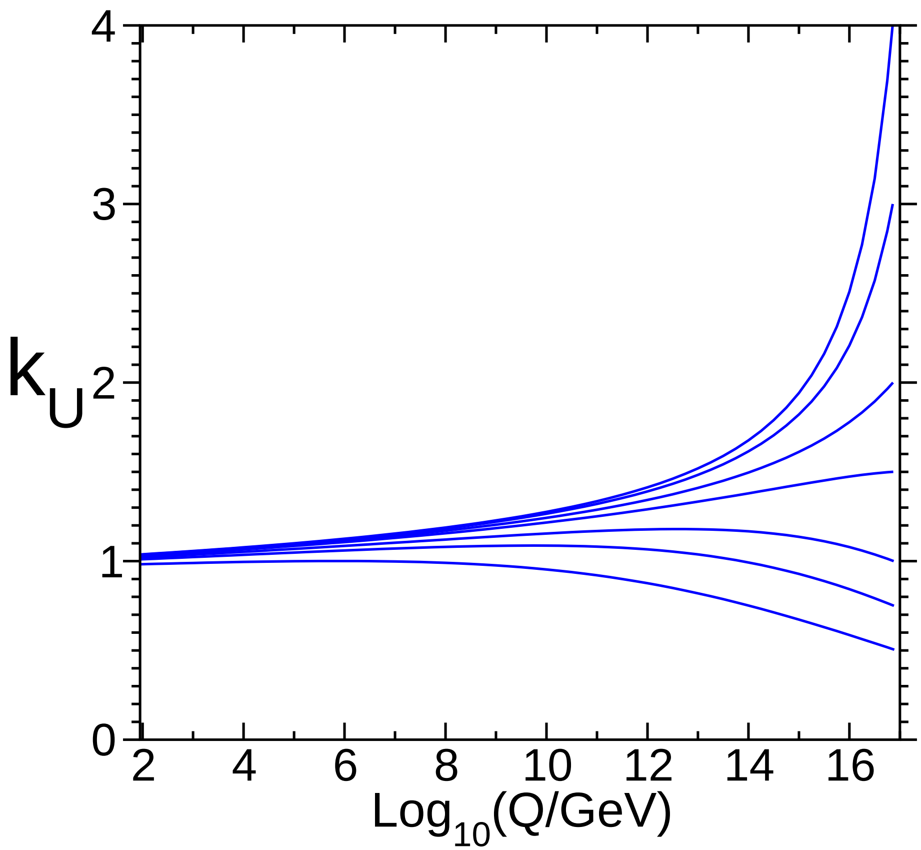
The Yukawa coupling k has an IR quasi-fixed point at $k \approx 1.05$.

Soft susy-breaking Lagrangian:

$$-\mathcal{L}_{\rm soft} = a_k H_u Q\overline{U} + m_Q^2 |Q|^2 + \ldots$$

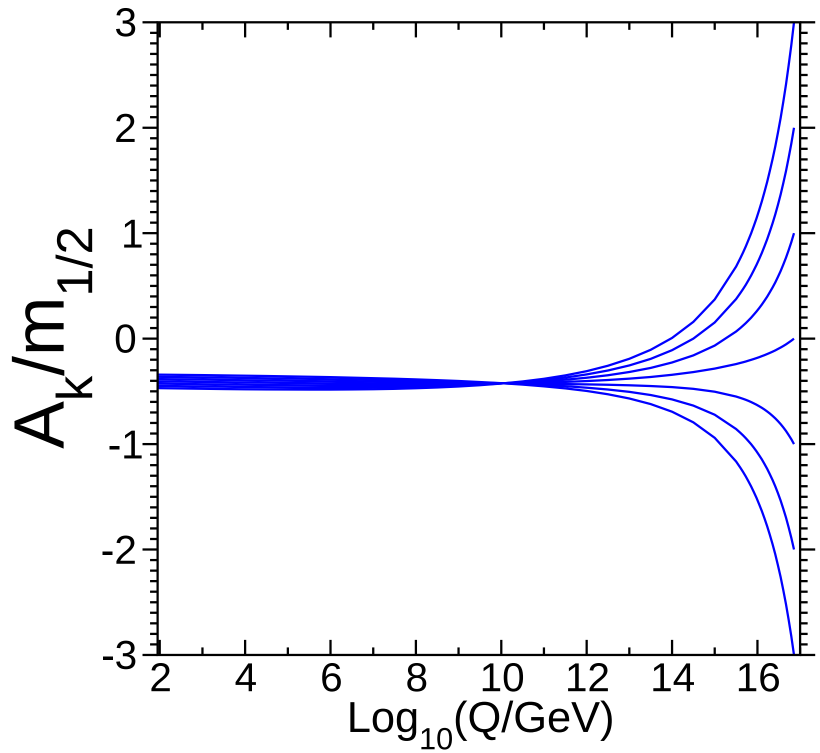
The corrections to Δm_h^2 depend strongly on a_k, which also has a strongly attractive fixed point.

Infrared-stable fixed point at $k = 1.05$ in the QUE model:



This large value is natural in the sense that many inputs at GUT scale end up there. The QDEE model behaves very similarly.

Near fixed point for k, there is also a strong focusing behavior for $A_k = a_k/k$:

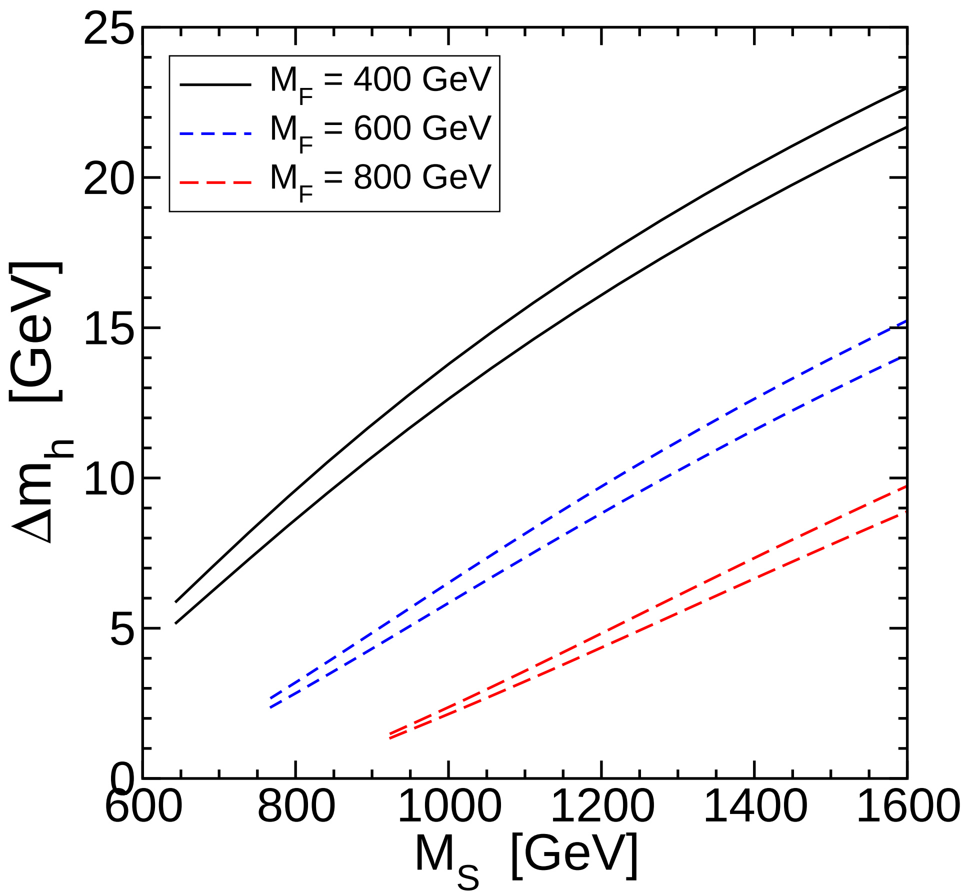


For almost every high-scale boundary condition,

$$-0.5 \ \lesssim \ A_k/m_{1/2} \ \lesssim \ -0.3.$$

This is much closer to the “No Mixing” scenario than to “Maximal Mixing”.

Higgs mass corrections near the fixed point with $k = 1.05$ in the QUE model, as a function of average scalar mass M_S:

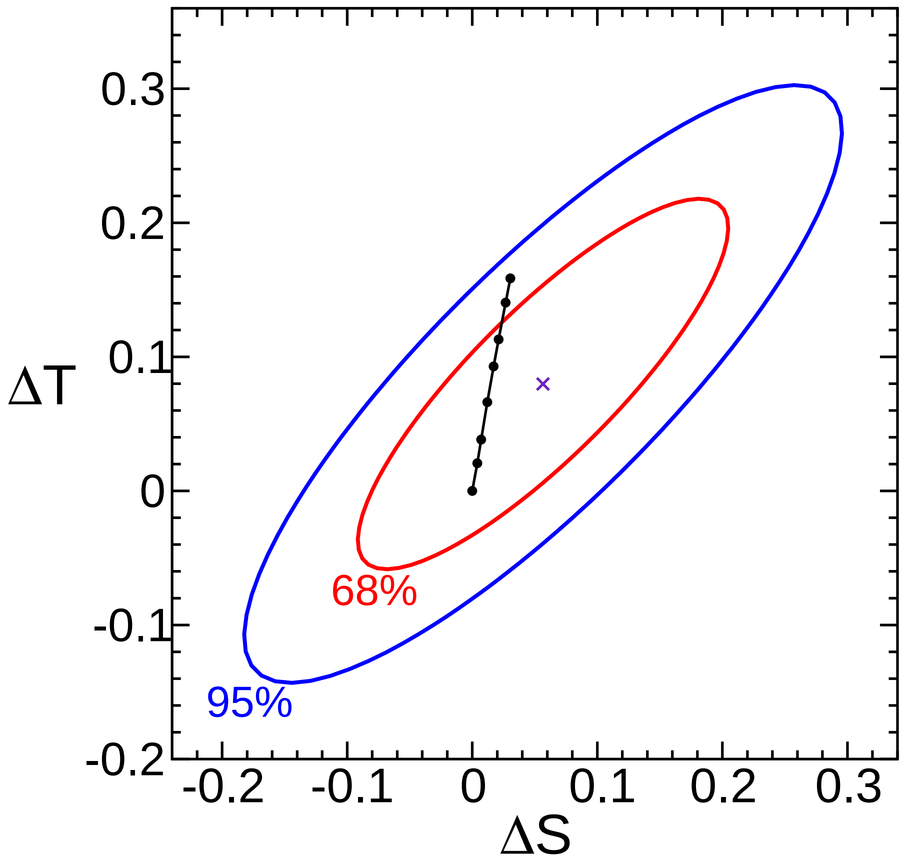


Upper lines: $A_k = -0.5 m_{1/2}$

Lower lines: $A_k = -0.3 m_{1/2}$

The most dramatic dependence is on M_S/M_F.

ΔS, ΔT for typical QUE model with varying $M \equiv M_Q = M_U$ and $m_{1/2} = 600$ GeV.



$\Delta S = \Delta T = 0$ defined here by Standard Model with $m_t = 173.1$ GeV, $M_h = 115$ GeV.

$\times$ = best fit to Z-pole data.

Black dots are $m_{t_1'} = 275, 300, 350, 400, 500, 700, 1000$ GeV and ∞.

General comments on collider phenomenology:

- Largest production cross-section involves the lightest new quark: always t' for QUE Model and b' for QDEE Model.
- New extra particles and sparticles probably won't appear in cascade decay of MSSM superpartners (notably the gluino), due to kinematic prohibition or suppression.
- Lightest new fermions can only decay by mixing with Standard Model fermions. If this is very small, the lightest new fermions could be long-lived on collider scales, yielding charged massive particles or displaced vertices.
- Mixing with Standard Model fermions is highly constrained (no GIM mechanism) except for the third family, so decays to t, b are most likely case.

Limits from Tevatron (CDF)

- $m_{t'} > 335$ GeV if BR($t' \to Wq$) is 100%.

  Based on lepton + jets + $E_T^{\rm miss}$ search with 4.6 fb^{-1}.

  CDF note 10110. (Slight excess. Expected limit was $m_{t'} > 372$ GeV.)

- $m_{b'} > 338$ GeV if BR($b' \to Wt$) is 100%.

  Based on same-charge dilepton search with 2.7 fb^{-1}.

- $m_{b'} > 268$ GeV if BR($b' \to Zb$) is 100%.

  Based on 1.06 fb^{-1}.

- $m_{b'} > 295$ GeV if BR($b' \to Wt, Zb, hb$) = $(0.5, 0.25, 0.25)$.

  Based on dilepton search with 1.2 fb^{-1}.

- $m_{q'} \gtrsim 350$ GeV if q' long-lived

  Based on time-of-flight measurement with 1.06 fb^{-1}.

How does the t' decay in QUE Model?

Depends on the form of the mixing term between the extra quarks and the Standard Model ones (assumed to be t, b). Possible terms are:

- $\mathcal{L} = H_d Q \bar{b}$

  Implies charged-current (“W-philic”) decays, with

  $BR(t' \to bW,\, tZ,\, th) = (1, 0, 0)$.

- $\mathcal{L} = H_u Q \bar{t}$

  Implies dominantly neutral current (“W-phobic”) decays, with

  $BR(t' \to bW,\, tZ,\, th) = (0, 0.5, 0.5)$ in the high mass limit.

- $\mathcal{L} = H_u \begin{pmatrix} t \\ b \end{pmatrix} \overline{U}$

  Implies “democratic” decays, with

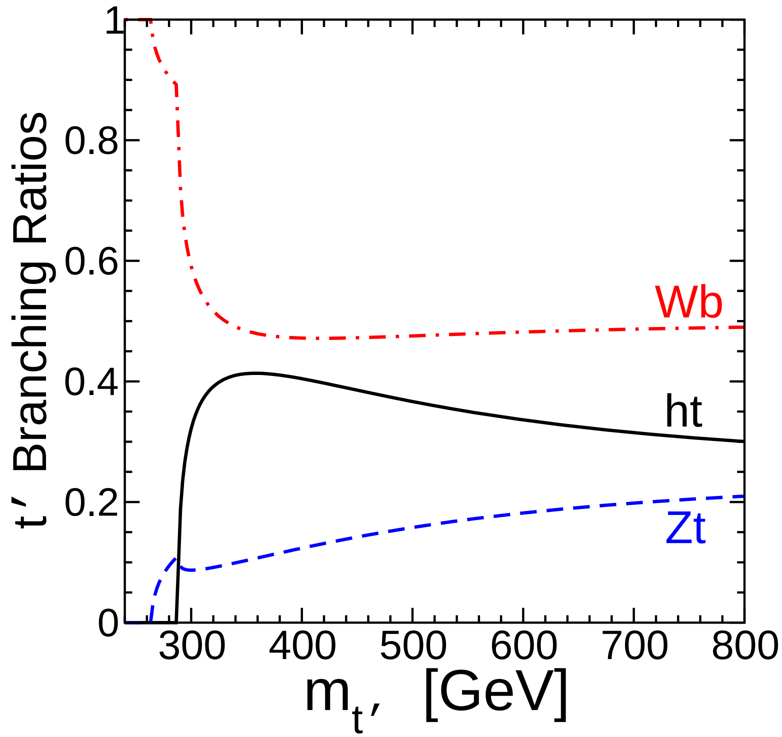
  $BR(t' \to bW,\, tZ,\, th) = (0.5, 0.25, 0.25)$ in the high mass limit.

Linear combinations of these are also possible.

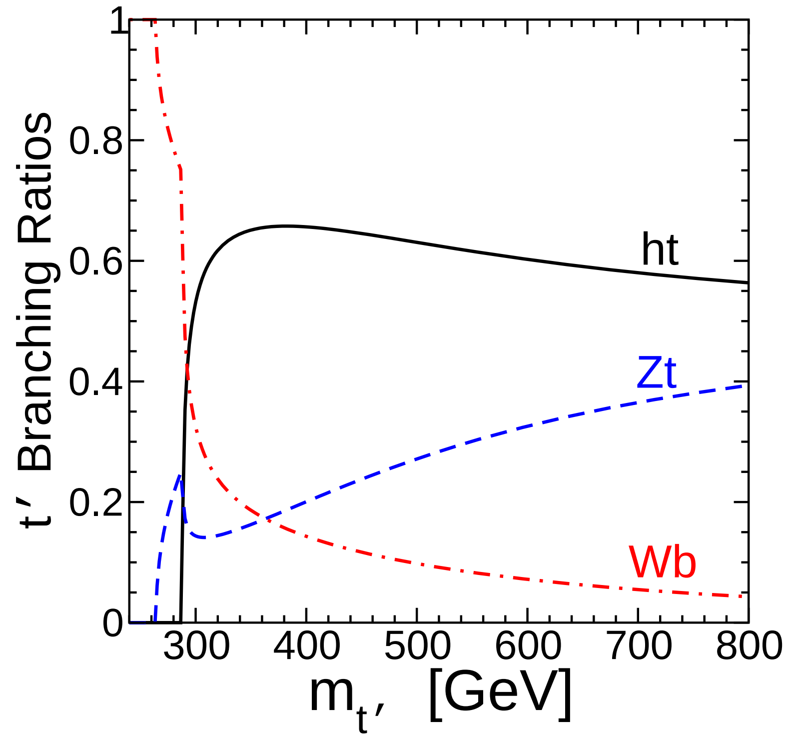
Mass effects are important in “W-phobic” and “democratic” cases.

Branching ratios for t' in QUE model:

“democratic”



“W-phobic”



Note CDF search assumes large BR$(t' \to Wq)$, but this is not inevitable.

Can Tevatron put bounds on $t' \longrightarrow tZ$ and/or $t' \longrightarrow th$?

(I strongly suspect so.)

LHC signals depend on the mixing of the new quarks with the Standard Model ones. For example, if the W decays dominate:

In the QUE Model, the t' signature is the same as for ordinary t, but with a larger mass:

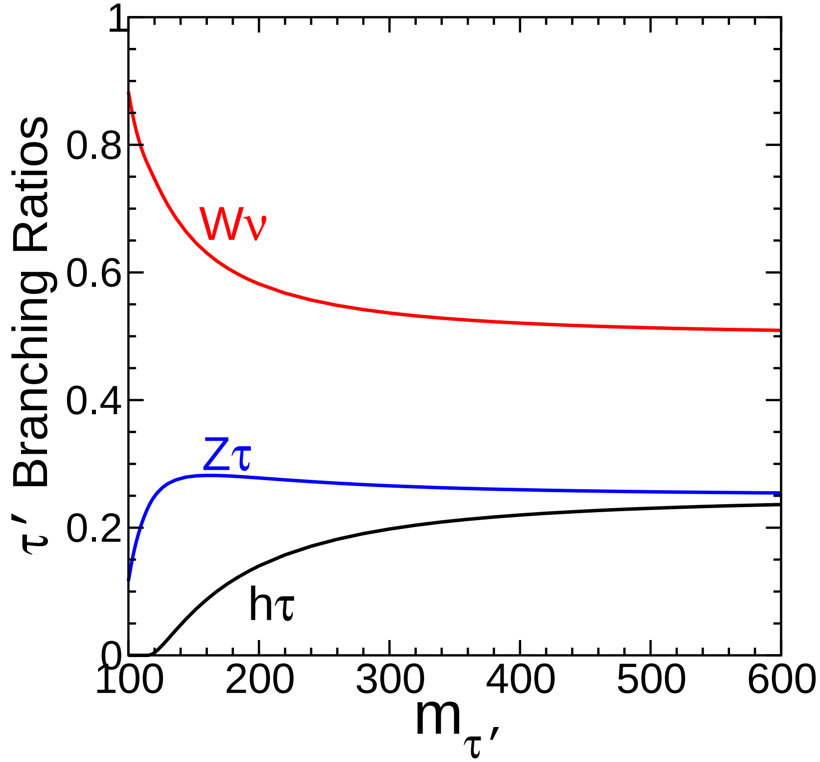
$$pp \to t'\overline{t'} \to W^+ b W^- \overline{b}.$$

In the QDEE Model, there could be a same-sign dilepton signal from

$$pp \to b'\overline{b'} \to W^- t W^+ \overline{t} \to W^+ W^+ W^- W^- b\overline{b} \to \ell^+ \ell^+ bbjjjj + E_T^{\rm miss}$$

Many more possibilities!

In both QUE and QDEE Models, gauge coupling unification demands a τ' whose branching ratios depend only on its mass:



For large $m_{\tau'}$, the Goldstone equivalence theorem implies

$$\text{BR}(W\nu) : \text{BR}(Z\tau) : \text{BR}(h\tau) = 2 : 1 : 1.$$

Can Tevatron place any bound on such a τ' ?

Outlook

The Supersymmetric Little Hierarchy Problem is a minor crisis.

Physics progress thrives on crisis!

We should regard it as an opportunity, to learn and make bold predictions. The models I've talked about are a small fraction of possible solutions. Explore your own ideas!