$SU(9)$ Family Unification

S. Nandi

Oklahoma State University
and
Oklahoma Center for High Energy Physics

(in collaboration with J. Dent and T. W. Kephart)
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Goals

- To present a $SU(9)$ model of family unification with 3 light chiral families.
- To provide a natural hierarchy for the charged fermion masses and mixings.
- To link the scale of the tiny neutrino masses with the hierarchy of the quark masses and mixings.

Outline of Talk

- Introduction
- Model and the Formalism
- Phenomenological Implications
- Conclusions and Outlook

Introduction

- 3 light families of quarks and leptons
- 5 orders of magnitude hierarchy among the charged fermion masses
- 2 orders of magnitude hierarchy among the quark mixing angles
- Neutrino masses are 7 orders of magnitude smaller that the lightest charged fermion masses

Our approach

- Use $SU(5)$ as one family GUT
- Enlarge the symmetry to $SU(N)$, $N > 5$, to include more families
- Use only anti-symmetric representations of $SU(9)$
  $\Rightarrow$ only $\mathbf{5}$, $\bar{\mathbf{5}}$, $\mathbf{10}$, $\bar{\mathbf{10}}$, $\mathbf{1}$
- Choose a set of anti-symmetric representations such that
  $\Rightarrow 3(\bar{\mathbf{5}} + \mathbf{10}) + n_1(\mathbf{5} + \bar{\mathbf{5}}) + n_2(\mathbf{10} + \bar{\mathbf{10}}) + n_3(\mathbf{1})$
- $(\mathbf{5} + \bar{\mathbf{5}})$, $(\mathbf{10} + \bar{\mathbf{10}})$, and $\mathbf{1}$ in the GUT scale
  $\Rightarrow$ only $3(\bar{\mathbf{5}} + \mathbf{10})$ light families of fermions

Introduction

Questions:

- How to generate fermion masses and mixing hierarchy?
- How to suppress FCNC processes?
- How to explain $m_\nu \ll m_q, m_l$?
  or how to generate $m_{N_R}/M_{GUT} = 10^{-2}$?

Introduction

Our philosophy

- Use additional discrete symmetries to arrange the hierarchy of the Yukawa couplingss
- Only top quark have dim 4 Yukawa coupling
- Yukawa couplings of the lighter quarks will appear successively as higher dimensional operators
- Link the neutrino mass scale to the suppression of these Yukawa couplings

Model & Formalism

Our concrete model

- Gauge Symmetry: $SU(9)$
- Use additional discrete symmetries, Z_2, Z_2', Z''_2, and Z_3 to constrain the Yukawa couplings suitably
- Only top quark has dim 4 Yukawa coupling
- Yukawa couplings of all other quarks are successively higher dimensional (hence suppressed), and are constrained by the above symmetries
- Supression factor, $\varepsilon = V_S/V_{GUT}$ where $V_S =$ VeV of an $SU(5)$ singlet Higgs field, and $V_{GUT} =$ GUT scale VeV.

$$\mathcal{F} = \overline{\mathbf{126}} + \mathbf{84} + 2(\mathbf{36}) + 14(\bar{\mathbf{9}})$$

Note that this assignment is anomaly free.

$$\begin{aligned}
\overline{\mathbf{126}} &\to \mathbf{5} + 4(\mathbf{10}) + 6(\overline{\mathbf{10}}) + 4(\bar{\mathbf{5}}) + \mathbf{1} \\
\mathbf{84} &\to \overline{\mathbf{10}} + 4(\mathbf{10}) + 6(\mathbf{5}) + 4(\mathbf{1}) \\
\mathbf{36} &\to \mathbf{10} + 4(\mathbf{5}) + 6(\mathbf{1}) \\
\bar{\mathbf{9}} &\to \bar{\mathbf{5}} + 4(\mathbf{1})
\end{aligned}$$

Hence the complete set of fermions in the model is

$$\mathcal{F} = 3(\mathbf{10} + \bar{\mathbf{5}})_F + 15(\mathbf{5} + \bar{\mathbf{5}})_F + 7(\mathbf{10} + \overline{\mathbf{10}})_F + 73(\mathbf{1})_F.$$

$\Rightarrow$ 3 light families of fermions.

Assignment of three chiral families:

$$\begin{aligned}
\text{3rd family}: \quad & (\overline{\mathbf{126_3}})_F \to t_L, t_R, b_L \qquad && \bar{\mathbf{9}}_3 \to b_R \\
\text{2nd family}: \quad & \mathbf{84}_2 \to c_L, c_R, s_L && \bar{\mathbf{9}}_2 \to s_R \\
\text{1st family}: \quad & \mathbf{36}_1 \to u_L, u_R, d_L && \bar{\mathbf{9}}_1 \to d_R
\end{aligned}$$

In addition, the Higgs represetations that we shall use are $\mathbf{36}_H$, $\mathbf{36'}_H$, $\mathbf{36''}_H$, $\mathbf{9}_H$, and $\mathbf{315}_H$.

Charges under the discrete symmetries:

	Z_2	Z_2'	Z_2''	Z_3
$(126_3)_F$	-1			
$(84_2)_F$	-1			
$(36_1)_F$	-1			
$(9_3)_F$				α
$(9_2)_F$		-1		
$(9_1)_F$		-1	-1	α
36_H				
$36'_H$	-1			
$36''_H$				α
9_H		-1	-1	
315_H				

Allowed Yukawa interactions:

$$
\begin{aligned}
D=4: & \begin{cases} tt: & (\overline{126}_F)^2\ 315_H \end{cases} \\
D=5: & \begin{cases} cc: & (84_2)_F\ (84_2)_F\ 36_H\ 315_H \\ ct=tc: & (126_3)_F\ (84_2)_F\ [36_H\ 315_H + (315)^2_H] \\ bb: & (\overline{126}_3)_F\ (\bar{9}_3)_F\ 36'_H\ 36''_H \end{cases} \\
D=6: & \begin{cases} tu=ut: & (\overline{126}_3)_F\ (36_1)_F\ [(36_H)^2 + (36'_H)^2]\ (\overline{36}_H) \\ cu=uc: & (84_2)_F\ (36_1)_F\ 36_H\ [(9_H)^2 + (315_H)^2] \\ bs: & (\overline{126}_3)_F\ (\bar{9}_2)_F\ 36_H\ 36'_H\ 9_H \\ sb: & (\bar{9}_3)_F\ (84_2)_F\ (\overline{36}''_H)^2\ 36'_H \\ ss: & (84_2)_F\ (\bar{9}_2)_F\ \overline{36}'_H\ 9_H\ 315_H \\ bd: & (\overline{126}_3)_F\ (\bar{9}_1)_F\ (36''_H)^2\ 9_H \end{cases}
\end{aligned}
$$

Allowed Yukawa interactions (continued):

$$D=7:\begin{cases} uu: & (36_1)_F\ (36_1)_F\ (9_H)^2\ 315_H\ 36_H \\ dd: & (36_1)_F\ (\bar{9}_1)_F\ 36'_H\ (\overline{36}''_H)^2\ 9_H \\ db: & (36_1)_F\ (\bar{9}_3)_F\ 36'_H\ (\overline{36}''_H)^2\ 315_H \\ sd: & (84_2)_F\ (\bar{9}_1)_F\ 36_H\ 36'_H\ 36''_H\ 9_H \end{cases}$$

Up and down quark mass matrices

$$M_u = \begin{pmatrix} h^u_{11}\varepsilon^3 & h^u_{12}\varepsilon^2 & h^u_{13}\varepsilon^2 \\ h^u_{21}\varepsilon^2 & h^u_{22}\varepsilon & h^u_{23}\varepsilon \\ h^u_{31}\varepsilon^2 & h^u_{32}\varepsilon & h^u_{33} \end{pmatrix} v.$$

$$M_d = \begin{pmatrix} h^d_{11}\varepsilon^3 & h^d_{12}\varepsilon^3 & h^d_{13}\varepsilon^3 \\ h^d_{21}\varepsilon^3 & h^d_{22}\varepsilon^2 & h^d_{23}\varepsilon^2 \\ h^d_{31}\varepsilon^2 & h^d_{32}\varepsilon^2 & h^d_{33}\varepsilon \end{pmatrix} v \ .$$

Phenomenological Implications

Quark and lepton masses, CKM mixing, FCNC and Higgs Decays

- Existence of 3 light families from a gauge $SU(9)$ family symmetry
- Parameters of the model: h^u_{ij}, h^d_{ij},$\varepsilon = V_S/V_{GUT}$
  With choices h^u_{ij}, $h^d_{ij} \sim \mathcal{O}(1)$, and $\varepsilon \sim 1/50$, good agreement with all the quark and charged lepton masses.
- No SM singlet Higgs at the EW scale (Our singlet Higgs are close to the GUT scale)
- Yukawa coupling matrices and the mass matrices are proportional $\Rightarrow$ No FCNC as in SM
- SM Higgs boson decays are identical to those in SM

Phenomenological Implications

Neutrino masses and mixings

- $\bar{\mathbf{9}}$ of $SU(9) \Rightarrow \bar{\mathbf{5}}_F + 4(\mathbf{1})_F$
- Singlet fermions are unavoidable
- Existence of RH neutrinos are required in our model
- this is similar to $SO(10)$ GUT, but we have a family unification gauge symmetry
- Mass scale for these RH neutrinos are linked to the suppression factor needed to explain charged fermion mass hierarchy and CKM mixings, $\varepsilon = V_S/V_{GUT}$
- $\Rightarrow M_R \sim 10^{14}$ GeV, as required to explain the observed light neutrino masses via see-saw

Conclusions

- Presented a family unification model with three light chiral families
- Based on $SU(9)$ gauge symmetry with additional discrete symmetries
- Generate the hierarchy of quark and charged lepton masses, and CKM mixings
- SM singlet neutrinos are unavoidable
- Links the scale of the RH singlet neutrinos $\sim 10^{14}$ GeV to quark mass hierarchy and mixings