Dark Matter Relative Velocity Effects in the Extragalactic Gamma-rays

The contribution of a p-wave annihilation component to the intensity of annihilation photons

Sheldon Campbell[1]

In collaboration with: B. Dutta[1] and E. Komatsu[2]

[1]Texas A&M University, College Station
[2]University of Texas, Austin

Pheno 2010 Symposium
May 10, 2010

Outline

- The calculation of extragalactic annihilation gamma-ray intensity to date.
  - Spherical halo model of dark matter distribution.
- Addition of dark matter relative velocities in the spherical model of halos.
- First application: p-wave component of dark matter annihilation.
- Examples of p-wave effects on intensity in real particle physics models.
- Some prospects for the extragalactic gamma-ray signal.

Outline

- The calculation of extragalactic annihilation gamma-ray intensity to date.
  - Spherical halo model of dark matter distribution.
- Addition of dark matter relative velocities in the spherical model of halos.
- First application: p-wave component of dark matter annihilation.
- Examples of p-wave effects on intensity in real particle physics models.
- Some prospects for the extragalactic gamma-ray signal.

Outline

- The calculation of extragalactic annihilation gamma-ray intensity to date.
  - Spherical halo model of dark matter distribution.
- Addition of dark matter relative velocities in the spherical model of halos.
- First application: p-wave component of dark matter annihilation.
- Examples of p-wave effects on intensity in real particle physics models.
- Some prospects for the extragalactic gamma-ray signal.

Outline

- The calculation of extragalactic annihilation gamma-ray intensity to date.
  - Spherical halo model of dark matter distribution.
- Addition of dark matter relative velocities in the spherical model of halos.
- First application: p-wave component of dark matter annihilation.
- Examples of p-wave effects on intensity in real particle physics models.
- Some prospects for the extragalactic gamma-ray signal.

Outline

- The calculation of extragalactic annihilation gamma-ray intensity to date.
  - Spherical halo model of dark matter distribution.
- Addition of dark matter relative velocities in the spherical model of halos.
- First application: p-wave component of dark matter annihilation.
- Examples of p-wave effects on intensity in real particle physics models.
- Some prospects for the extragalactic gamma-ray signal.

Mean Intensity of Extragalactic Annihilation Gamma-Rays

$$\langle I_\gamma \rangle (E_\gamma) = \int_0^\infty \frac{dz}{H(z)} \left\langle \delta_\rho^2 \right\rangle (z) \, W\Big((1+z) E_\gamma, z \Big)$$

Mean Intensity of Extragalactic Annihilation Gamma-Rays

$$\langle I_\gamma \rangle(E_\gamma) = \int_0^\infty \frac{dz}{H(z)} \left\langle \delta_\rho^2 \right\rangle(z)\, W\Big((1+z)E_\gamma, z\Big)$$

- **Line-of-sight integration**
  Hubble function $H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$

Mean Intensity of Extragalactic Annihilation Gamma-Rays

$$\langle I_\gamma \rangle(E_\gamma) = \int_0^\infty \frac{dz}{H(z)} \left\langle \delta_\rho^2 \right\rangle(z) \, W\Big((1+z)E_\gamma, z\Big)$$

- **Line-of-sight integration**
  Hubble function $H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$
- **Mean square mass overdensity**
  $\delta_\rho = \frac{\rho}{\overline{\rho}} - 1 \approx \frac{\rho}{\overline{\rho}}$ Densest regions most important.
  Best modeled with spherical halo model:
  - Mean halo distribution: Sheth-Tormen mass function $\frac{dn}{dM}(M, z)$
  - Universal halo density profile: $\rho_h(r|M, z)$
    Our examples will use a truncated NFW profile with Bullock et. al. concentration distribution.

  $\left\langle \delta_\rho^2 \right\rangle(z) = \int_{M_{\rm min}}^\infty dM \frac{dn}{dM}(M, z) \int d^3\mathbf{r} \frac{\rho_h^2(r|M,z)}{\overline{\rho}^2(z)}$

Mean Intensity of Extragalactic Annihilation Gamma-Rays

$$\langle I_\gamma \rangle(E_\gamma) = \int_0^\infty \frac{dz}{H(z)} \left\langle \delta_\rho^2 \right\rangle(z)\, W\Big((1+z)E_\gamma, z\Big)$$

- **Line-of-sight integration**
  Hubble function $H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$
- **Mean square mass overdensity**
  $\delta_\rho = \frac{\rho}{\bar{\rho}} - 1 \approx \frac{\rho}{\bar{\rho}}$ Densest regions most important.
  Best modeled with spherical halo model:
  - Mean halo distribution: Sheth-Tormen mass function $\frac{dn}{dM}(M,z)$
  - Universal halo density profile: $\rho_h(r|M,z)$
    Our examples will use a truncated NFW profile with Bullock et. al. concentration distribution.

  $$\left\langle \delta_\rho^2 \right\rangle(z) = \int_{M_{\rm min}}^\infty dM \frac{dn}{dM}(M,z) \int d^3\mathbf{r}\, \frac{\rho_h^2(r|M,z)}{\bar{\rho}^2(z)}$$
- **Window function**
  $W(E_\gamma, z) = \frac{(1+z)^3}{8\pi} (\sigma v) n_{\rm DM}^2 \frac{dN_\gamma(E_\gamma)}{dE_\gamma} e^{-\tau(E_\gamma, z)}$
  Involves: optical depth, annihilation spectrum, current background number density, and relative velocity weighted annihilation cross section.

Mean Intensity of Extragalactic Annihilation Gamma-Rays

$$\langle I_\gamma \rangle (E_\gamma) = \int_0^\infty \frac{dz}{H(z)} \left\langle \delta_\rho^2 \right\rangle (z)\, W\Big((1+z) E_\gamma, z \Big)$$

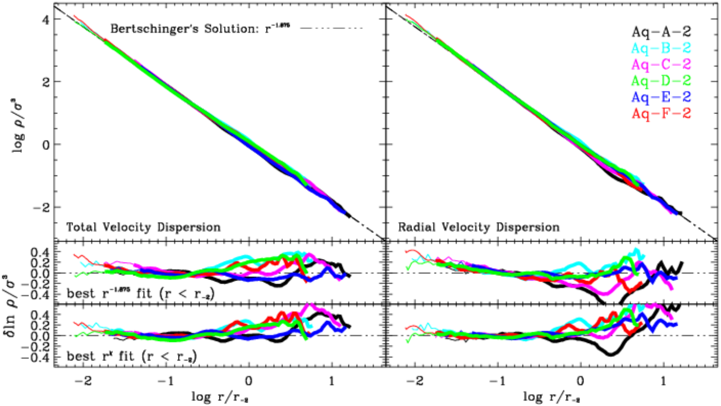
- **Line-of-sight integration**
  Hubble function $H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$
- **Mean square mass overdensity**
  $\delta_\rho = \frac{\rho}{\bar{\rho}} - 1 \approx \frac{\rho}{\bar{\rho}}$ Densest regions most important.
  Best modeled with spherical halo model:
  - Mean halo distribution: Sheth-Tormen mass function $\frac{dn}{dM}(M, z)$
  - Universal halo density profile: $\rho_h(r|M, z)$
    Our examples will use a truncated NFW profile with Bullock et. al. concentration distribution.

  $\left\langle \delta_\rho^2 \right\rangle (z) = \int_{M_{\rm min}}^\infty dM \frac{dn}{dM}(M, z) \int d^3\mathbf{r} \frac{\rho_h^2(r|M,z)}{\bar{\rho}^2(z)}$
- **Window function**
  $W(E_\gamma, z) = \frac{(1+z)^3}{8\pi} (\sigma v) n_{\rm DM}^2 \frac{dN_\gamma(E_\gamma)}{dE_\gamma} e^{-\tau(E_\gamma, z)}$
  Involves: optical depth, annihilation spectrum, current background number density, and relative velocity weighted annihilation cross section.

This is the s-wave approximation. $\sigma v =$constant
In general, σv is v-dependent. Need velocity distribution to investigate.

Universal Halo Velocity Dispersion $\sigma_{vh}(r)$

Navarro, et. al. (2008), Aquarius Project Simulations



$$\frac{\rho_h(r)}{\sigma^3_{vh}(r)} \propto r^{-\alpha}$$

Our examples use the critical solution of the radial Jeans equation to determine α and the proportionality constant (see Dehnen and McLaughlin 2005).

Distribution of 1-point relative velocities

Relation between mean relative velocity v and local velocity dispersion σ_v depends on the underlying velocity distribution. We assume:

$$v^2 = \lambda \sigma_v^2.$$

For Maxwell-Boltzmann distribution, $\lambda = 6$.

First application of this formalism:

Consider a model with a p-wave component to the annihilation cross-section:

$$\sigma v = a + b v^2$$

The p-Wave Contribution to the Intensity of Extragalactic Photons

$$\langle I_\gamma\rangle(E_\gamma) = \int_0^\infty \frac{dz}{H(z)}\left[\left\langle\delta_\rho^2\right\rangle(z) + \beta(z)\left\langle\delta_\rho^2\delta_{v^2}\right\rangle(z)\right] W_v\Big((1+z)E_\gamma, z\Big)$$

- Simplest to calculate in terms of "overdensity" $\delta_{v^2} = \frac{\sigma_v^2}{\overline{\sigma_v^2}} - 1$
- $\beta(z) \equiv \frac{1}{1+\frac{1}{\lambda\frac{b}{a}\overline{\sigma_v^2}(z)}}$
- Here, the cross section in W_v is now the mean value at redshift z $\overline{\sigma v}(z) = a + b\lambda\overline{\sigma_v^2}(z)$.
- In the spherical halo model:

$$\overline{\sigma_v^2}(z) = \int dM \frac{dn}{dM}(M,z) \int d^3\mathbf{r}\ \sigma_{vh}^2(r|M,z)$$

$$\left\langle\delta_\rho^2\delta_{v^2}\right\rangle(z) = \int dM \frac{dn}{dM}(M,z) \int d^3\mathbf{r} \frac{\rho_h^2(r|M,z)\sigma_{vh}^2(r|M,z)}{\overline{\rho}^2(z)\overline{\sigma_v^2}(z)}$$

Effects of the p-Wave

There are two main effects:

1. It adds a second term to the intensity calculation.
   - under what conditions is the second term important?
2. It raises the magnitude of the cross section, particularly at freezeout.
   - need a lower s-wave component to maintain correct relic density
   - this is the p-wave suppression of the intensity

First let's ignore the relic density constraint and consider the effect of the second term.

Expectation:

dark matter velocities today $\sim 10^{-3}$,

so bv^2 term in the cross section should become important when:

$$\frac{b}{a} \gtrsim 10^6, \quad \text{or} \quad \lambda \frac{b}{a} \gtrsim 10^7$$

Effects of the p-Wave

Contribution of the Second Intensity Term

The second term boosts the s-wave approximation $\langle I_\gamma \rangle_0$ by a factor of

$$\frac{\langle I_\gamma \rangle (E_\gamma)}{\langle I_\gamma \rangle_0 (E_\gamma)} = 1 + \lambda \frac{b}{a} \Delta_I(E_\gamma)$$

where

$$\Delta_I(E_\gamma) = \frac{\int \frac{dz}{H(z)} \overline{\sigma_v^2}(z) \left\langle \delta_\rho^2 (1 + \delta_{v^2}) \right\rangle (z) W((1+z)E_\gamma, z)}{\int \frac{dz}{H(z)} \left\langle \delta_\rho^2 \right\rangle (z) W((1+z)E_\gamma, z)}$$

depends only on

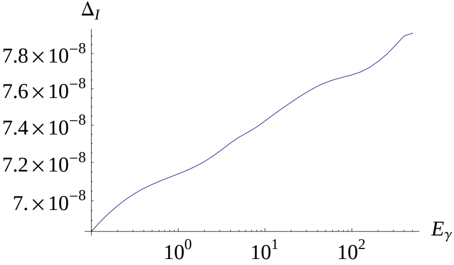
- annihilation spectrum,
- dark matter phase space distribution,
- opacity, neglected in this talk.

Expectation: Negligible for $\lambda \frac{b}{a} \lesssim 10^7 \Longrightarrow \Delta_I \sim 10^{-7}$.

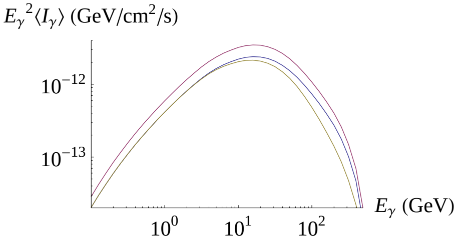
Effects of the p-Wave

Contribution of the Second Intensity Term

$$\frac{\langle I_\gamma \rangle}{\langle I_\gamma \rangle_0} = 1 + \lambda \frac{b}{a} \Delta_I$$



- $\Delta_I(E_\gamma)$ for co-annihilation region mSUGRA model
- $m_{\tilde{\chi}_1^0} = 550$ GeV, $\tan\beta = 50$, $A_0 = 0$, sign$(\mu) > 0$, $\frac{b}{a} = 5$
- Numerical integration within 1% precision.



Blue Curve: Intensity due to s-wave term only.

Gold Curve: Shows effect of the opacity–neglected for all other plots in this talk.

Red Curve: Intensity if by hand $\lambda\frac{b}{a} = 6\times10^6$. 45% increase at the peak.

Effects of the p-Wave

Maintaining the Relic Density in the Simple Thermal Relic Picture

- The cross section at freezeout needs to be a particular value, $\langle \sigma v \rangle_f$.
- Thermal square relative velocities at freezeout are $v_f^2 = \frac{6T_f}{m_{\rm DM}}$.
- S-wave approximation sets $a = \langle \sigma v \rangle_f$
- With p-wave, relic density constraint requires

$$a \approx \frac{\langle \sigma v \rangle_f}{1 + \left(\frac{b}{a}\right) \frac{6T_f}{m_{\rm DM}}}$$

- Two models equal but for the value of b have relative intensities

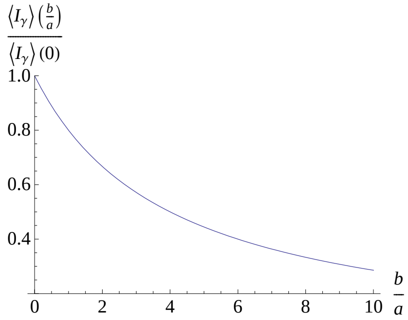
$$\frac{\langle I_\gamma \rangle \left(E_\gamma, \frac{b}{a}\right)}{\langle I_\gamma \rangle \left(E_\gamma, 0\right)} \approx \frac{1 + \left(\frac{b}{a}\right) \lambda \Delta_I(E_\gamma)}{1 + \left(\frac{b}{a}\right) \frac{6T_f}{m_{\rm DM}}}$$

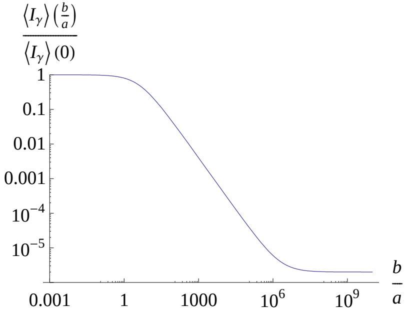
- Co-annihilations require even further suppression of a.

Effects of the p-Wave

The p-wave suppression effect

$$\frac{\langle I_\gamma \rangle \left(E_\gamma, \frac{b}{a}\right)}{\langle I_\gamma \rangle \left(E_\gamma, 0\right)} \approx \frac{1 + \left(\frac{b}{a}\right) \lambda \Delta_I(E_\gamma)}{1 + \left(\frac{b}{a}\right) \frac{6 T_f}{m_{\text{DM}}}}$$





Plots of the p-wave suppression for $6T_f/m_{DM} = 1/4$ and $\lambda\Delta_I = 5 \times 10^{-7}$.

$$\frac{b}{a} \ll \frac{m_{DM}}{6T_f} \sim 4 \implies \text{s-wave dominated}$$

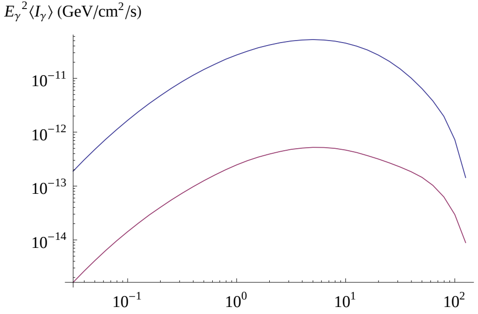
$$\frac{b}{a} \gg \frac{1}{\lambda\Delta_I} \sim 10^6 \implies \text{p-wave dominated}$$

p-Wave Annihilations in the MSSM

- Largest p-Wave components in parameter space with
  - lightest neutralino as pure bino as possible
  - large non-universal 3rd generation sfermion masses
  - here the s-wave is dominated by gauge boson production ($\gamma\gamma$, $Z\gamma$, gg)
- $\frac{b}{a} \lesssim 10^4 \Longrightarrow$ can ignore velocity distribution of dark matter, but must use correct value of $\sigma_0 v$ s-wave component.

p-Wave Annihilations in the MSSM

- Largest p-Wave components in parameter space with
  - lightest neutralino as pure bino as possible
  - large non-universal 3rd generation sfermion masses
  - here the s-wave is dominated by gauge boson production ($\gamma\gamma$, $Z\gamma$, gg)
- $\frac{b}{a} \lesssim 10^4 \Longrightarrow$ can ignore velocity distribution of dark matter, but must use correct value of $\sigma_0 v$ s-wave component.



mSUGRA models with
$A_0 = 0$, sign$(\mu) > 0$,
$\tan\beta = 10$, $m_{\tilde{\chi}_1^0} = 150$GeV

Top Curve: Focus Point/Hyperbolic Region
$\sigma_0 v = 2 \times 10^{-26}$ cm^3/s, $\frac{b}{a} = 1.5$

Bottom Curve: Co-annihilation Region
$\sigma_0 v = 2 \times 10^{-28}$ cm^3/s, $\frac{b}{a} = 378.8$

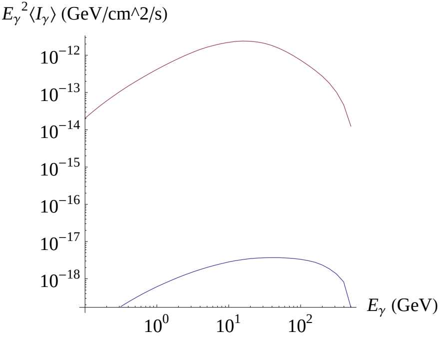
Particle physics model properties and relic density calculated using DarkSUSY 5.0.5.

Example of a p-Wave Dominated Scenario

- MSSM$\otimes U(1)_{B-L}$ with right-handed sneutrino LSP.
- annihilates dominantly via s-channel Z' to $f\,\overline{f}$
- $m_{\tilde{\nu}}$ near Z' resonance to satisfy relic density constraint

$$m_{\tilde{\nu}} = 550 \text{ GeV}, \; m_{Z'} = 1300 \text{ GeV}$$

- no tree level s-wave annihilation
- coupling suppression at 1-loop, estimate $\frac{b}{a} \sim 10^8$



Top Curve: mSUGRA co-annihilation region $\tan\beta = 50$, $m_{\tilde{\chi}_1^0} = 550$ GeV

Bottom Curve: Described $B-L$ model with $m_{\tilde{\nu}} = 550$ GeV. Annihilation spectrum generated by Pythia 8.135.

Note s-wave dominated $B-L$ models also exist via s-channel H'.

Conclusions and Future Directions

- Being able to model the dark matter velocity distribution in the spherical halo model allows us to begin considering velocity effects in the extragalactic annihilation signal.
- We have determined the effects of p-wave annihilation as a first application of these methods.
- There are many interesting velocity effects to investigate:
  - Sommerfeld effect–example of low velocity intensity enhancement,
  - resonances,
  - opening of new annihilation channels,
  - other evolutions of the annihilation spectrum with collision energy.
- How robust are the results to the astrophysical uncertainties?
- To what extent can astrophysical and particle physics information be disentangled?
- What are the prospects for experiments to measure a signal and extract information in the presence of astrophysical foregrounds and backgrounds?

Simplifications Made in the Sample Calculations

- Neglected small halos, setting $M_{\mathrm{min}} = 10^6 M_\odot$. If $M_{\mathrm{min}} = 10^{-6} M_\odot$, this increases the intensity by about an order of magnitude.
- Neglected halo substructures. This is expected to increase the mean intensity by a factor of 3–5.
- Neglected optical depth.
- Neglected anisotropy in the halo velocity dispersion profile. Simulations show that the dispersion is isotropic at the halo cores.

Non-thermal Scenarios

Re-heating due to the decay of another relic

- It is possible for a re-heating to have occured after dark matter freezeout, but before big bang nucleosynthesis.
  - For example, decay of string moduli, producing new dark matter.
- The dark matter annihilation cross section must be larger to bring the dark matter relic density back down, enhanced by a factor of freezeout temperature over re-heating temperature, as high as $10^3 - 10^4$.
- This enhancement in cross section will also enhance the gamma-ray intensity today.