Top Quark Mass Measurement in the ℓ + Jets Channel at CDF Using a Matrix Element Method with Quasi-Monte Carlo Integration

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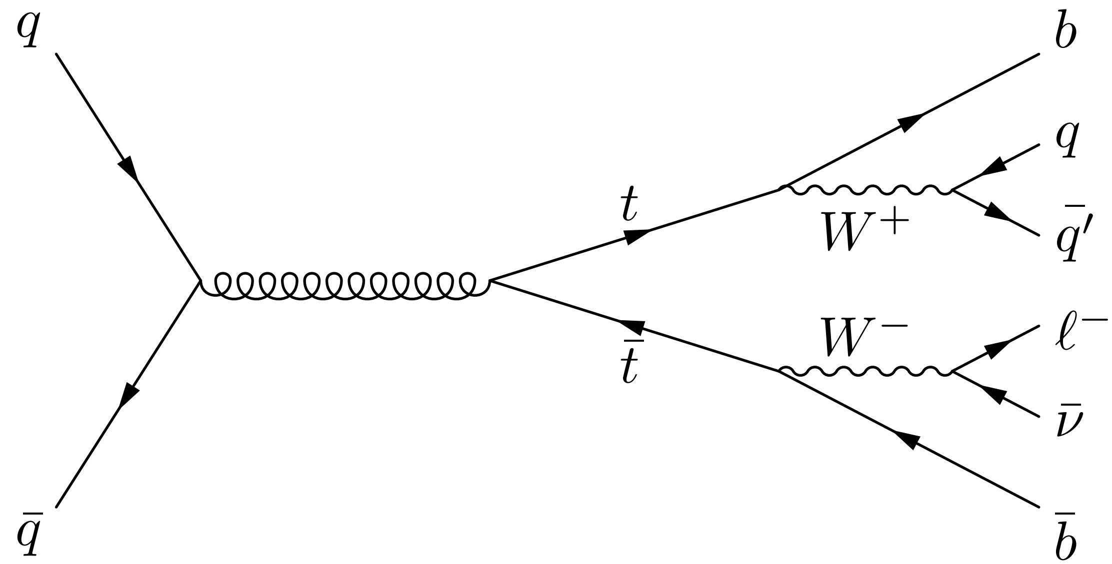
- The Matrix Element Technique
- Quasi-Monte Carlo Integration Method
- Calibration and Tests
- Data Result and Systematics

Matrix Element Method

Matrix element-based techniques have become increasingly popular as a way to perform high-precision measurements of the top mass.

Our measurement is performed in the "lepton + jets" channel, where one of the W quarks produced decays hadronically and the other leptonically: $t\bar{t} \to WWb\bar{b} \to b\bar{b}q\bar{q}'\ell\nu$

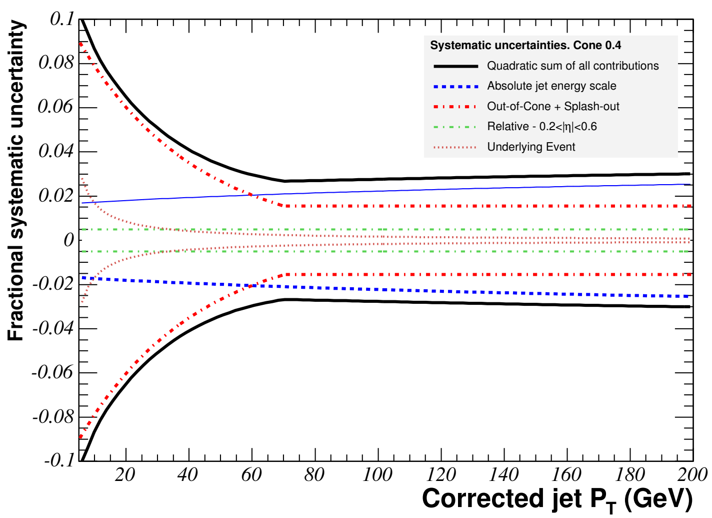


- Use the matrix element for $t\bar{t}$ production and decay to calculate a probability of seeing the observed quantities in our detector ($\vec{y}$).
- Integrate the matrix element over the unobserved variables corresponding to the parton-level quantities ($\vec{x}$).
- Combine these curves for different events to get a single final likelihood.

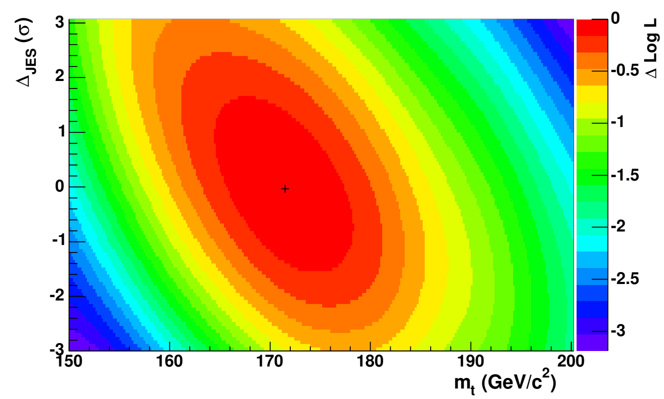
Matrix Element Method (cont'd)

- Of the $t\bar{t}$ decay products that we measure, the four jets are the most difficult to measure precisely. We expect the uncertainty in the jet energy calibration (bottom left) to be the single largest source of systematic uncertainty.

- A useful technique for dealing with this systematic is to build our likelihood as a function of two variables: $L = L(m_t, \Delta_{\text{JES}})$. This allows the information contained in the W decay to be incorporated into the event likelihood. Here, Δ_{JES} represents a shift of all jet energies by their relative uncertainties. For instance, $\Delta_{\text{JES}} = +1$ means that all jet energies are shifted 1 σ upwards.



Jet systematic uncertainty vs. p_T



Sample 2-D likelihood curve

Integration Formula

Likelihood for each event, assuming that the event is signal:

$$L(\vec{y} \mid m_t, \Delta_{\mathrm{JES}}) = \frac{1}{N(m_t)} \frac{1}{A(m_t, \Delta_{\mathrm{JES}})} \sum_{i=1}^{24} w_i L_i(\vec{y} \mid m_t, \Delta_{\mathrm{JES}})$$

- $\vec{y}$ are the momenta measured in the detector
- m_t is the pole mass of the top quark
- Δ_{JES} is the JES shift, in the units of standard JES systematic error
- $N(m_t)A(m_t, \Delta_{\mathrm{JES}})$ is the likelihood normalization factor: tree-level cross section times acceptance
- w_i are the 24 permutation weights, calculated using tagging probabilities
- L_i are likelihoods for individual permutations (assignments of jets to partons)

Integration Formula (Cont'd)

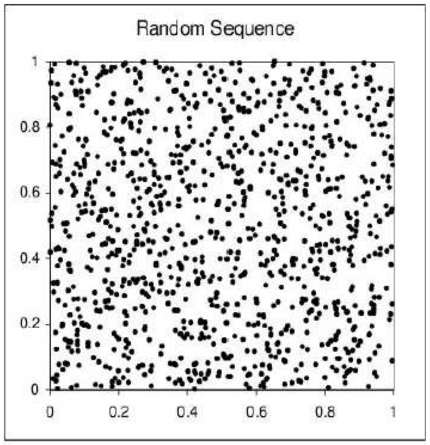
Likelihood for each permutation:

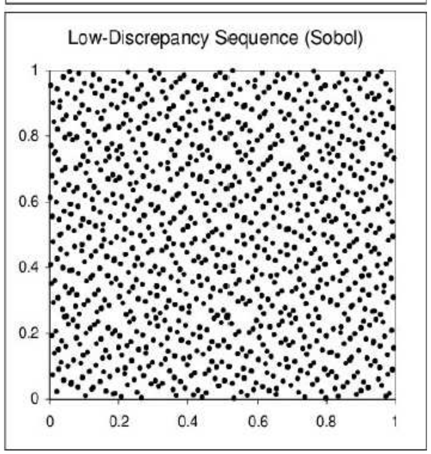
$$L_i(\vec{y} \mid m_t, \Delta_{\text{JES}}) = \int \frac{f(z_1) f(z_2)}{FF} \, \text{TF}(\vec{y} \mid \vec{x}, \Delta_{\text{JES}}) \, |M(m_t, \vec{x})|^2 \, d\Phi(\vec{x})$$

- $\vec{x}$ are the parton-level momenta
- $f(z)$ are the quark/gluon distribution functions, and FF is the flux factor
- $\text{TF}(\vec{y} \mid \vec{x}, \Delta_{\text{JES}})$ are the transfer functions connecting the parton-level and the detector-level quantities
- M is the matrix element for the $t\bar{t}$ production and decay. We use the Kleiss-Stirling matrix element, which includes both $q\bar{q}$ and gg, as well as full spin correlations.
- Φ is the parton-level phase space being integrated over

Quasi-Monte Carlo Integration

The full phase space $\Phi(\vec{x})$ has a total of 19 degrees of freedom, assuming the lepton momentum is perfectly measured. In the past, this was viewed as too many to integrate over, requiring us to make additional assumptions which simplified the integral but cost us some resolution.

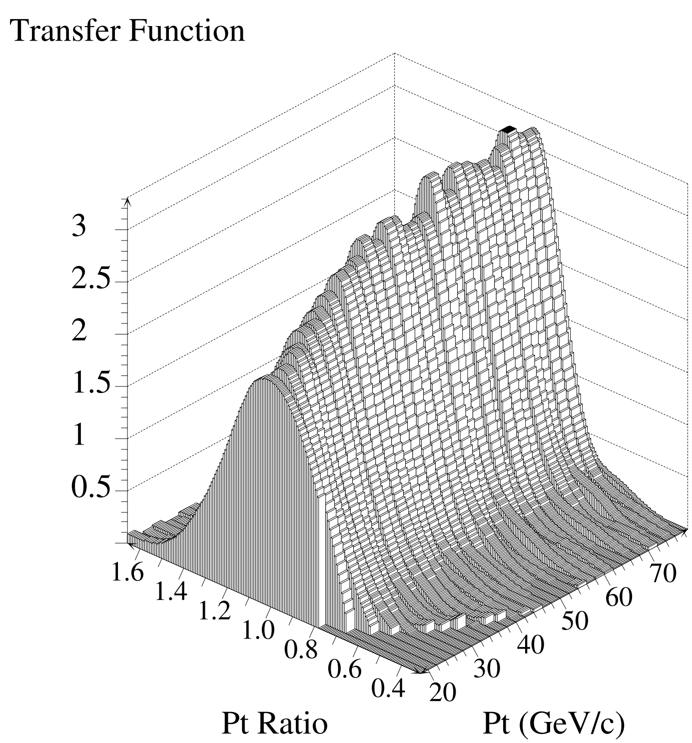


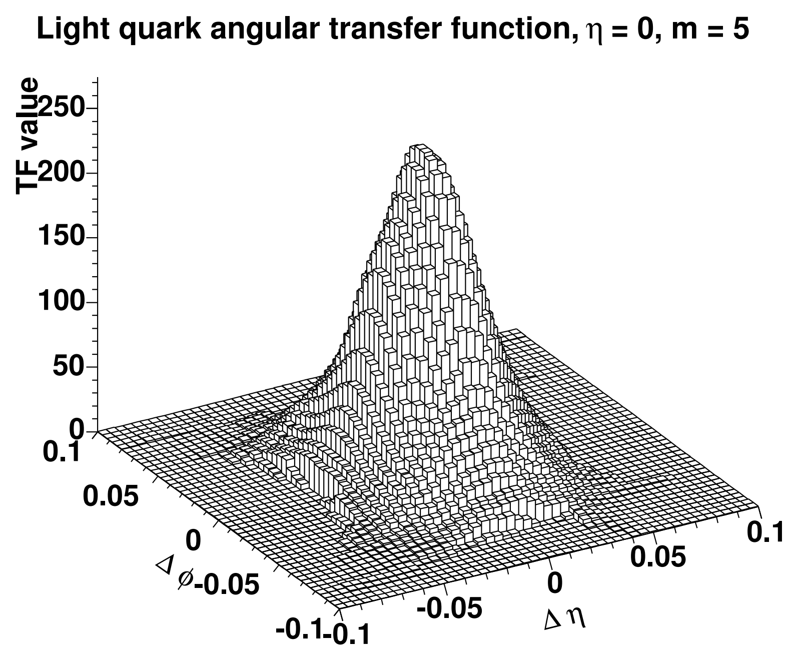


- Now, we use Quasi-Monte Carlo integration. This allows to integrate over all of the variables in the phase space in a practical amount of time without needing to make any additional assumptions.
- QMC integration uses a quasi-random sequence (we use a variant of the Sobol sequence, bottom left) instead of purely random points. See hep-ph/9601270 for more.
- For "well-behaved" functions, convergence rate is guaranteed to be at least as good as $O(\log(N)^d/N)$. Compare with $O(1/\sqrt{N})$ for standard MC integration.
- On average, it takes $\sim$ 80 minutes to integrate an event.

Transfer Functions

- The transfer functions $\mathrm{TF}(\vec{y}|\vec{x})$ are a crucial component of any matrix-element based analysis. They give the probability of seeing a reconstructed jet with momentum $\vec{y}$ given a parton with momentum $\vec{x}$. In our integration, we have separate transfer functions for the P_T (left) and angular (right) terms, built by maching jets to partons in Monte Carlo events.

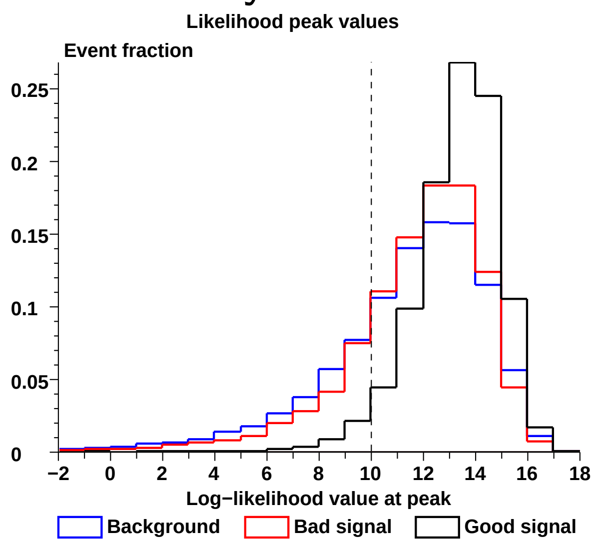




Background and Bad Signal

- We subtract off the expected background contribution, obtained from Monte Carlo, from our total likelihood to recover our likelihood for signal. We also have a class of events which we call "bad signal" — events which are $t\bar{t}$ but where the final observed objects are not directly from $t\bar{t}$ decay (extra jets from ISR/FSR, lost/merged jets, τ decay, misidentified dilepton/all-hadronic events, etc.) These make up $\sim$ 35% of our signal!

- To reduce the effect of these events on our likelihood, we adopt a cut on the peak value of the final log-likelihood curve of 10. This cut improves the expected resolution by more than 10%.



Efficiency of likelihood cut at 10:

Type of event	Total
Good signal	96.3% ± 0.2%
Bad signal	79.2% ± 0.4%
Background	72.7% ± 0.3%

Event Selection

Our selection requirements are as follows:

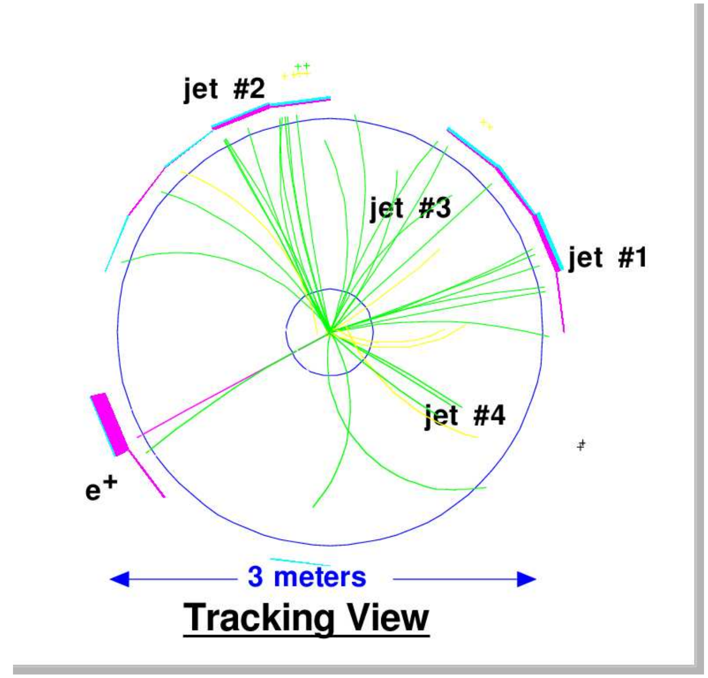
- Exactly one lepton with $E_T > 20$ GeV (electron) or $p_T > 20$ GeV$/c$ (muon), either in the central region ($|\eta| < 1$) on a high-p_T lepton trigger or a muon obtained on a missing E_T trigger, separated from all jets

- Exactly 4 tight jets with $E_T > 20$ GeV in the central region ($|\eta| < 2$)

- Missing $E_T > 20$ GeV (from ν)

- At least one jet tagged as being from a b quark

Total of 1070 events observed in 4.8 fb^{-1} of data at CDF



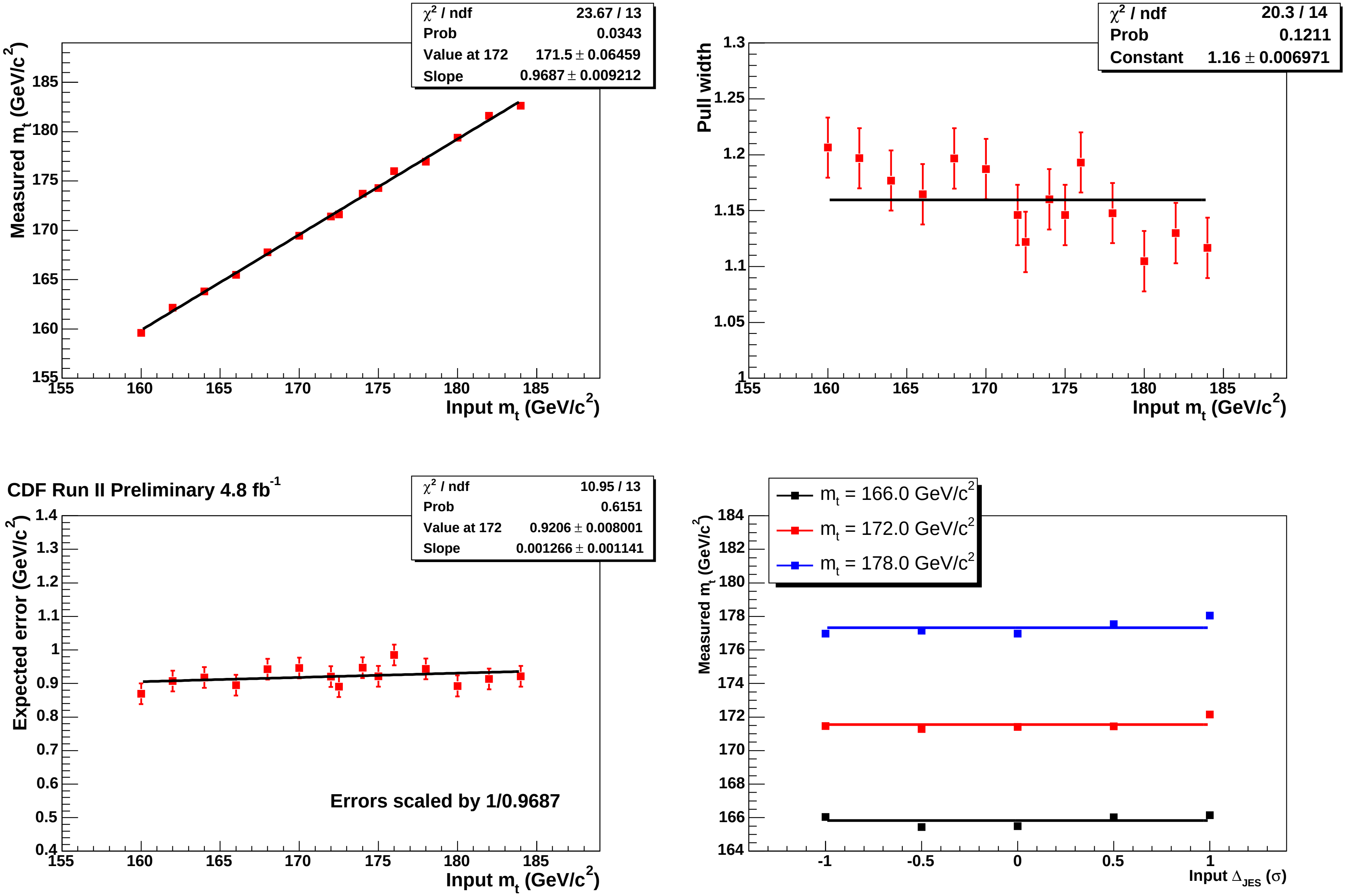
Sample $t\bar{t} \rightarrow b\bar{b}q\bar{q}'\ell\nu$ event in data

Principal backgrounds:

· W + heavy flavor ($b\bar{b}$, $c\bar{c}$, c) ($\sim$ 11.0% of total)
· W + mistag light jets ($\sim$ 3.9%)
· non-W QCD events ($\sim$ 4.5%)

Monte Carlo Results

We test and calibrate our method on PYTHIA Monte Carlo samples with a variety of true top mass and $\Delta_{\rm JES}$ values.

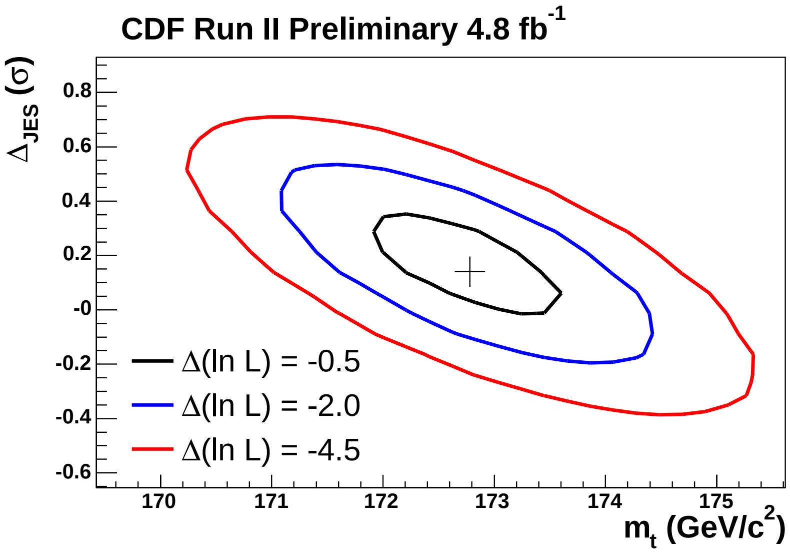


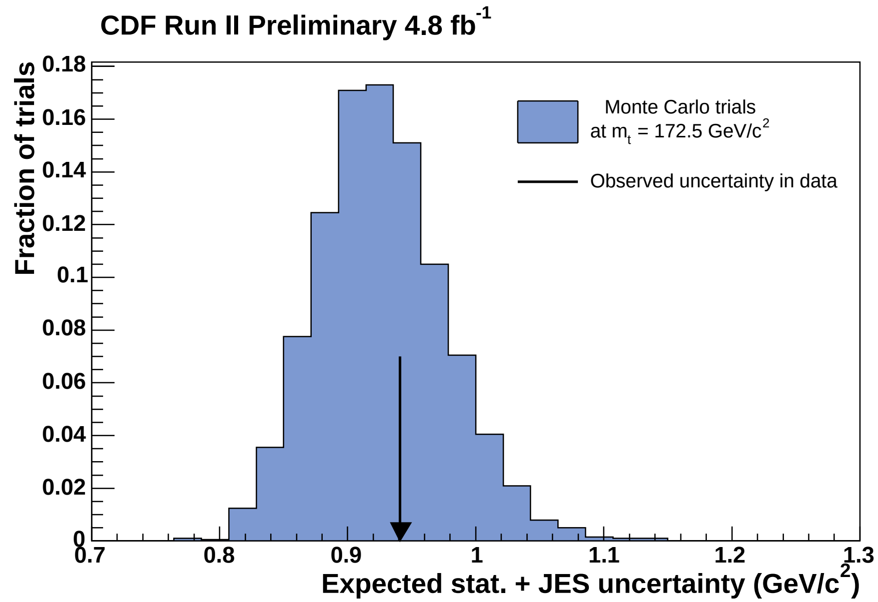
Systematic Error Summary

CDF Run II Preliminary, 4.8 fb^{-1}

Systematic source	Systematic uncertainty (GeV/c^2)
Calibration	0.11
MC generator	0.25
ISR and FSR	0.15
Residual JES	0.49
b-JES	0.26
Lepton P_T	0.14
Multiple hadron interactions	0.10
PDFs	0.14
Background modeling	0.33
Gluon fraction	0.03
Color reconnection	0.37
Total	0.84

Results with 4.8 fb^{-1}





With 918 out of 1070 events passing our likelihood cut, we obtain:

$$m_t = 172.8 \pm 0.7 \text{ (stat.)} \pm 0.6 \text{ (JES)} \pm 0.8 \text{ (syst.) GeV}/c^2$$

$$m_t = 172.8 \pm 1.3 \text{ (total) GeV}/c^2$$

$$\Delta_{\rm JES} = 0.14 \pm 0.20 \text{ (stat.) } \sigma$$

This is the best individual top mass measurement in the world!

Conclusion

$$m_t = 172.8 \pm 1.3 \text{ (total) GeV}/c^2$$

- Total precision of 0.73% on the m_t measurement — we've already exceeded the Tevatron Run II goal of 1% precision with a single measurement!

- As the measurement moves into the systematics-dominated realm, improvements to the analysis need to focus on these systematics.

- However, the Tevatron top mass measurement should continue to be a landmark for many years.

- PRL currently in preparation.

BACKUP SLIDES

Expected Background

W+light is estimated using the mistag rate. Relative contributions in W+HF are taken from MC and the overall normalization from data. Non-W estimate is obtained by fitting the missing E_T distribution with templates for signal and QCD.

CDF Run II Preliminary, 4.8 fb^{-1}

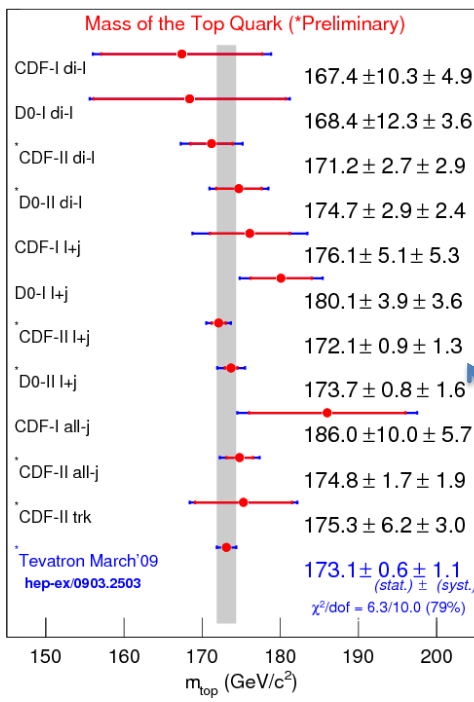
Event type	1 tag	$\geq$ 2 tags
non-W QCD	44.5 $\pm$ 38.6	3.8 $\pm$ 4.0
W+light mistag	40.7 $\pm$ 10.1	0.8 $\pm$ 0.3
diboson (WW, WZ, ZZ)	10.6 $\pm$ 1.1	1.0 $\pm$ 0.1
$Z \to \ell\ell$ + jets	8.5 $\pm$ 1.2	0.7 $\pm$ 0.1
$W + b\bar{b}$	54.6 $\pm$ 20.7	10.5 $\pm$ 3.5
$W + c\bar{c}$	33.5 $\pm$ 11.5	1.5 $\pm$ 0.5
$W + c$	16.5 $\pm$ 5.7	0.7 $\pm$ 0.3
Single top	8.7 $\pm$ 0.7	2.6 $\pm$ 0.2
Total background	217.6 $\pm$ 56.9	21.6 $\pm$ 7.8
Predicted top signal ($\sigma = 7.4$ pb)	644.2 $\pm$ 107.5	238.7 $\pm$ 36.8
Events observed	859	211

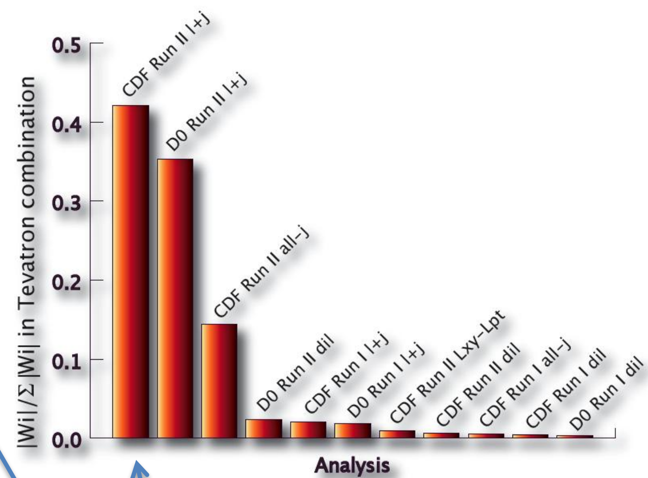
Loose Muons

Adding the new ("loose") muons actually doesn't hurt our S/B ratio – while they're not clean as the "tight" muons, they're still better than the electrons.

Lepton type	Expected Non-W QCD
Tight electrons (central high-E_T electron trigger)	9.26 $\pm$ 7.41%
Tight muons (central high-p_T muon trigger)	0.47 $\pm$ 0.42%
Loose muons (missing E_T trigger)	0.91 $\pm$ 0.73%
All leptons	4.51 $\pm$ 3.62%

March '09 Tevatron Combination





3.2 fb^{-1} version of this analysis

Integration Details

- We choose as our 19 variables the m_t^2 and m_W^2 on the hadronic and leptonic side, $\beta = \log \frac{\rho_q}{\rho_{\bar{q}}}$, the logarithm of the ratio of the 3-momentum magnitudes of the two partons from the hadronically decaying W, the p_T of the $t\bar{t}$ system, and the η, ϕ, and m for each of the 4 jet-producing partons.

- QMC is used for 18 of these 19 variables. The leptonic m_W^2 requires special treatment to avoid potential phase space singularities.

- To save time, we quickly identify permutations with lower likelihood and spend less time integrating these permutations.

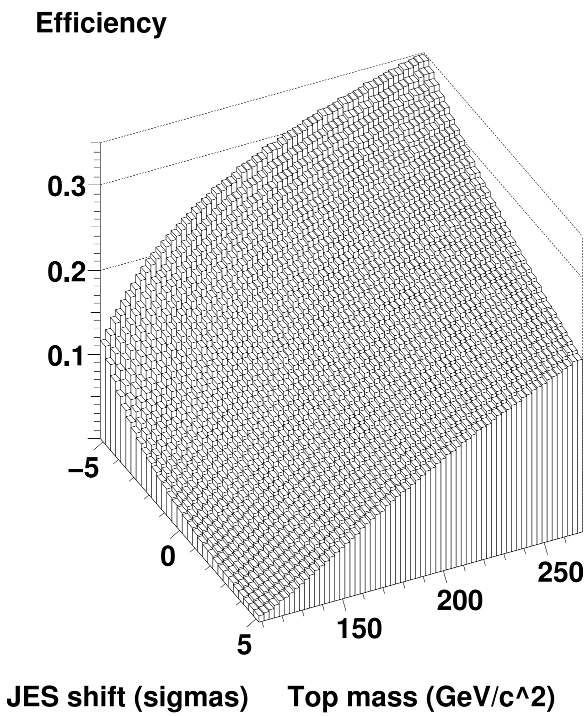
- The integration terminates when either a preset convergence target or a timeout (currently 2 hours) is reached. About 2/3 of the events reach their target before timing out.

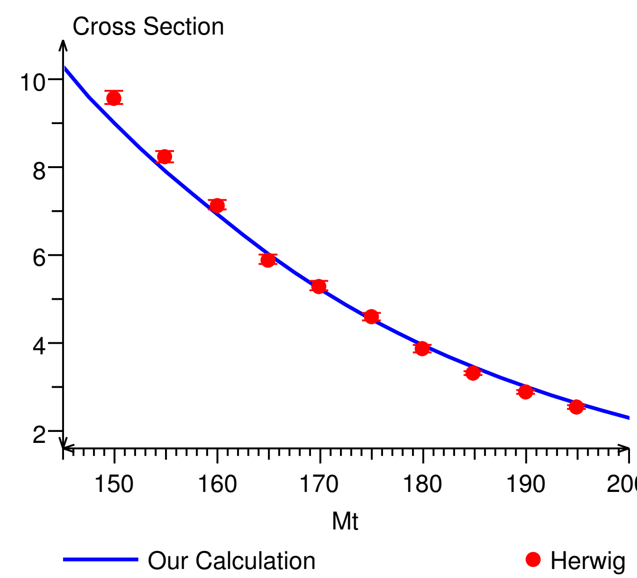
- Overall, the average integration time is about 80 min/event — long but doable.

Acceptance and Normalization

- Acceptance (left) accounts for the changes in detector acceptance with m_t and JES.

- Normalization (right) is obtained by integrating the matrix element and PDFs over the parton phase space.

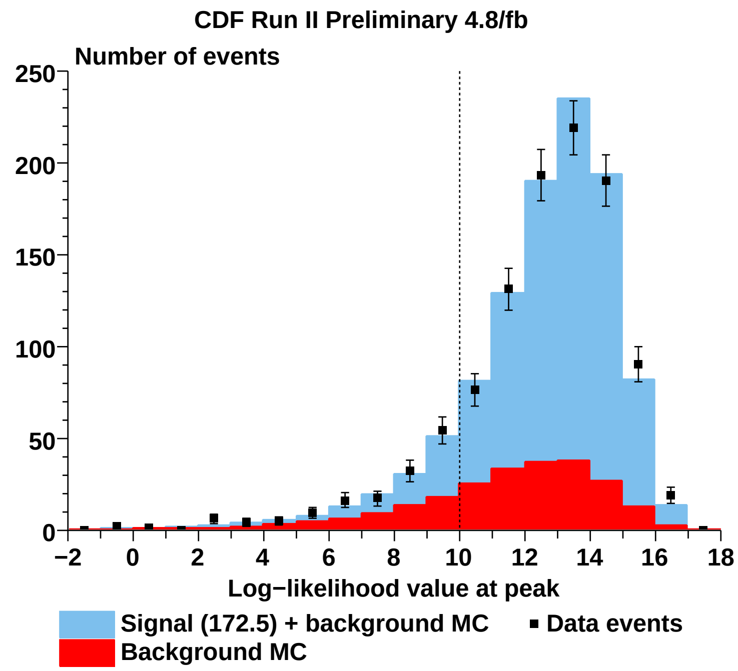




Systematics Overview

- Generator accounts for the differing parton shower models in PYTHIA and HERWIG.
- Color reconnection is a new systematic accounting for the effect of color connection between the $t\bar{t}$ partons and the remaining incoming partons.
- Residual JES accounts for the fact that the JES uncertainty contains several different uncertainties, each with their own p_T and η dependence.
- b-JES accounts for the differing JES uncertainties for b-jets.
- Background modeling includes our background fraction and composition, and our background subtraction technique.

Likelihood Comparisons



- Comparison between MC and data of log-likelihood value of likelihood peaks for all events.