Effective Operators
In Top Quark Production and Decay

Cen Zhang

Department of Physics
University of Illinois at Urbana-Champaign

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Outline

Effective Field Theory

When searching for new physics beyond the SM, one might

- see it directly (Z', etc.)
- see indirect effects (for example, the exchange of Z' between fermions appears as four-fermion interaction at low energy.)

In the latter case, we desire a model-independent approach to parameterize new physics.

Effective Field Theory

An effective field theory approach is a two step process.

- First, one integrates out all new heavy states and obtains effective interactions involving only fields of the SM.
- These effective higher-dimensional operators are then used to compute the deviations from the SM and compare with the experimental data.

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum c_i \frac{O_i^{(5)}}{\Lambda} + \sum c_i \frac{O_i^{(6)}}{\Lambda^2} + \cdots$$

Λ can be regarded as the scale of new physics.

Higher Dimensional Operators

- Dimension 5:

$$O_5 = \frac{c^{ij}}{\Lambda}\left(L^{iT}\epsilon\phi\right) C \left(\phi^T\epsilon L^j\right) + h.c.$$

(Weinberg 1979)

- Dimension 6:
  63 independent operators after the EOMs are applied.

  (Buchmuller and Wyler 1986, Aguilar-Saavedra 2009)

- There is no odd-dimensional operator that conserves baryon and lepton number.

Operators That Contribute to Top Quark Interaction

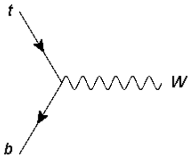
At leading order $\frac{1}{\Lambda^2}$, ignore bottom quark mass, we found

operator	process
$O_{\phi q} = i(\phi^+ \tau^i D_\mu \phi)(\bar{q}\gamma^\mu \tau^i q) + h.c.$	top decay, single top
$O_{tW} = (\bar{q}\sigma^{\mu\nu}\tau^i t)\tilde{\phi} W^i_{\mu\nu} + h.c.$	top decay, single top
$O_{tG\phi} = (\bar{q}\sigma^{\mu\nu}\lambda^A t)\tilde{\phi} G^A_{\mu\nu} + h.c.$	single top, $q\bar{q}, gg \to t\bar{t}$
$O_{G3} = g_s f_{ABC} G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$gg \to t\bar{t}$
$O_{\phi G} = \frac{1}{2}(\phi^+\phi) G^A_{\mu\nu} G^{A\mu\nu}$	$gg \to t\bar{t}$

and 8 four-fermions contact interactions such as $(\bar{u}\gamma_\mu u)(\bar{q}\gamma^\mu q)$.

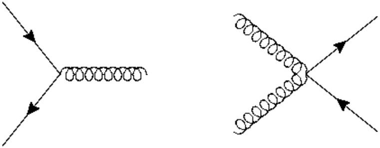
The *Wtb* coupling

- $O_{\phi q} = i(\phi^{+}\tau^{i} D_{\mu}\phi)(\bar{q}\gamma^{\mu}\tau^{i} q) + h.c.$
- $O_{tW} = (\bar{q}\sigma^{\mu\nu}\tau^{i} t)\tilde{\phi} W^{i}_{\mu\nu} + h.c.$



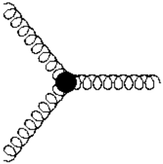
The G^3, Chromomagnetic Moment, and Higgs-Gluon Interactions

- $O_{tG\phi} = (\bar{q}\sigma^{\mu\nu}\lambda^A u)\tilde{\phi}G^A_{\mu\nu}$

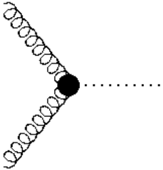


The G^3, Chromomagnetic Moment, and Higgs-Gluon Interactions

- $O_{G3} = g_s f_{ABC} G_{\nu}^{A\mu} G_{\rho}^{B\nu} G_{\mu}^{C\rho}$



- $O_{\phi G} = \frac{1}{2}(\phi^{+}\phi) G_{\mu\nu}^{A} G^{A\mu\nu}$



Top Quark Decay

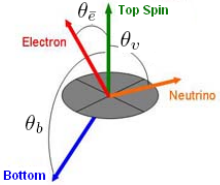
The W helicity fractions are given by:

$$
\begin{aligned}
F_0 &= \frac{m_t^2}{m_t^2 + 2m_W^2} - \frac{4\sqrt{2}C_{tW}v^2}{\Lambda^2}\frac{m_t m_W(m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2} \\
F_L &= \frac{2m_W^2}{m_t^2 + 2m_W^2} + \frac{4\sqrt{2}C_{tW}v^2}{\Lambda^2}\frac{m_t m_W(m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2} \\
F_R &= 0
\end{aligned}
$$

Polarized Decay Rate

- In the top quark rest frame:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_i} = \frac{1+\alpha_i\cos\theta_i}{2}$$



- Effects of new operators

$$\alpha_b = -\frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} + \frac{C_{tW}}{g\Lambda^2}\frac{16\sqrt{2}vm_t m_W^2(m_t^2 - m_W^2)}{(m_t^2 + 2m_W^2)^2}$$

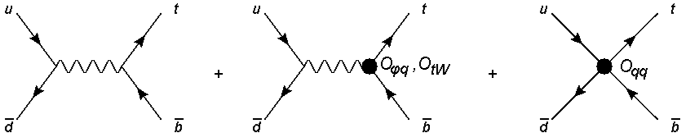
$$\alpha_\nu = \frac{m_t^6 - 12m_t^4 m_W^2 + 3m_t^2 m_W^4(3 + 8\ln(m_t/m_W)) + 2m_W^6}{m_t^6 - 3m_t^2 m_W^4 + 2m_W^6} - \frac{C_{tW}}{g\Lambda^2}\frac{24\sqrt{2}vm_t m_W^2(m_t^6 - 6m_t^4 m_W^2 + 3m_t^2 m_W^4(1 + 4\ln(m_t/m_W)) + 2m_W^6)}{(m_t^2 + 2m_W^2)^2(m_t^2 - m_W^2)^2}$$

$$\alpha_{\bar{e}} = 1$$

s,t-Channel Single Top

Cao, Wudka and Yuan, 2007

- Feynman Diagrams



- s-channel

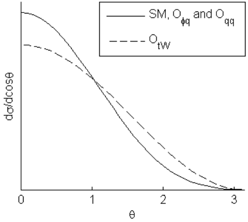
$$\frac{1}{4}\Sigma|M_{u\bar{d}\to t\bar{b}}|^2 = \frac{g^4 u(u-m_t^2)}{4(s-m_W^2)^2} + \frac{C_{\phi q}v^2}{2\Lambda^2}\frac{g^4 u(u-m_t^2)}{(s-m_W^2)^2} - \frac{\sqrt{2}C_{tW}m_t v}{\Lambda^2}\frac{g^3 su}{(s-m_W^2)^2} + \frac{2C_{qq}}{\Lambda^2}\frac{g^2 u(u-m_t^2)}{s-m_W^2}$$

- t-channel

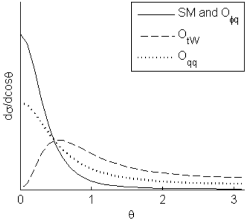
$$\frac{1}{4}\Sigma|M_{ub\to dt}|^2 = \frac{g^4 s(s-m_t^2)}{4(t-m_W^2)^2} + \frac{C_{\phi q}v^2}{2\Lambda^2}\frac{g^4 s(s-m_t^2)}{(t-m_W^2)^2} - \frac{\sqrt{2}C_{tW}m_t v}{\Lambda^2}\frac{g^3 st}{(t-m_W^2)^2} + \frac{2C_{qq}}{\Lambda^2}\frac{g^2 s(s-m_t^2)}{t-m_W^2}$$

$$\frac{1}{4}\Sigma|M_{\bar{d}b\to\bar{u}t}|^2 = \frac{g^4 u(u-m_t^2)}{4(t-m_W^2)^2} + \frac{C_{\phi q}v^2}{2\Lambda^2}\frac{g^4 u(u-m_t^2)}{(t-m_W^2)^2} - \frac{\sqrt{2}C_{tW}m_t v}{\Lambda^2}\frac{g^3 ut}{(t-m_W^2)^2} + \frac{2C_{qq}}{\Lambda^2}\frac{g^2 u(u-m_t^2)}{t-m_W^2}$$

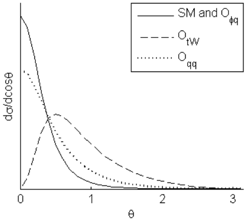
Angular Dependence (at $\sqrt{s} = 2m_t$)



s-channel



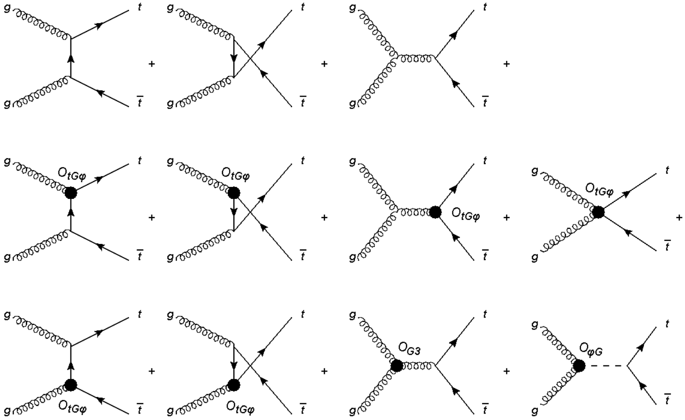
t-channel($ub \longrightarrow dt$)



t-channel($\bar{d}b \longrightarrow \bar{u}t$)

$gg \longrightarrow t\bar{t}$ Channel

- Feynman Diagrams



$gg \to t\bar{t}$ Channel

Cho and Simmons, 1994

- The squared amplitude:

$$\frac{1}{256}|M|^2 = \frac{3g_s^4}{4}\frac{(m^2-t)(m^2-u)}{s^2} - \frac{g_s^4}{24}\frac{m^2(s-4m^2)}{(m^2-t)(m^2-u)} + \frac{g_s^4}{6}\frac{tu - m^2(3t+u) - m^4}{(m^2-t)^2}$$
$$+ \frac{g_s^4}{6}\frac{tu - m^2(t+3u) - m^4}{(m^2-u)^2} - \frac{3g_s^4}{8}\frac{tu - 2m^2t + m^4}{s(m^2-t)} - \frac{3g_s^4}{8}\frac{tu - 2m^2u + m^4}{s(m^2-u)}$$
$$+ \frac{\sqrt{2}C_{tG\phi}g_s^3 vm}{3\Lambda^2}\frac{4s^2 - 9tu - 9m^2s + 9m^4}{(m^2-t)(m^2-u)} + \frac{9C_{G3}g_s^4}{8\Lambda^2}\frac{m^2(t-u)^2}{(m^2-t)(m^2-u)}$$
$$- \frac{C_{\phi G}g_s^2m^2}{16\Lambda^2}\frac{s^2(s-4m^2)}{(s-m^2)(t-m^2)(u-m^2)}$$

$u\bar{u}, d\bar{d} \to t\bar{t}$ Channel

- Result

$$\frac{1}{36}|M_{u\bar{u}\to t\bar{t}}|^2 = g_s^2(M_1 + M_2) + \frac{32\sqrt{2}C_{tG\phi}g_s^3 vm}{9\Lambda^2} + C_u^1\frac{s}{\Lambda^2}M_1 + C_u^2\frac{s}{\Lambda^2}M_2$$

$$\frac{1}{36}|M_{d\bar{d}\to t\bar{t}}|^2 = g_s^2(M_1 + M_2) + \frac{32\sqrt{2}C_{tG\phi}g_s^3 vm}{9\Lambda^2} + C_d^1\frac{s}{\Lambda^2}M_1 + C_d^2\frac{s}{\Lambda^2}M_2$$

where

$$M_1 = \frac{4g_s^2}{9s^2}(3m^4 - m^2(t+3u) + u^2)$$

$$M_2 = \frac{4g_s^2}{9s^2}(3m^4 - m^2(3t+u) + t^2)$$

$$C_u^1 = C_{4q}^1 + C_{4q}^2 + C_{4q}^3$$

$$C_u^2 = C_{4q}^5 + C_{4q}^7$$

$$C_d^1 = C_{4q}^1 - C_{4q}^2 + C_{4q}^4$$

$$C_d^2 = C_{4q}^6 + C_{4q}^7$$

$$O_{4q}^1 = \frac{1}{4}(\bar{q}^1\gamma_\mu\lambda^A q^1)(\bar{q}^3\gamma^\mu\lambda^A q^3)$$

$$O_{4q}^2 = \frac{1}{4}(\bar{q}^1\gamma_\mu\tau^i\lambda^A q^1)(\bar{q}^3\gamma^\mu\tau^i\lambda^A q^3)$$

$$O_{4q}^3 = \frac{1}{4}(\bar{u}^1\gamma_\mu\lambda^A u^1)(\bar{u}^3\gamma^\mu\lambda^A u^3)$$

$$O_{4q}^4 = \frac{1}{4}(\bar{d}^1\gamma_\mu\lambda^A d^1)(\bar{u}^3\gamma^\mu\lambda^A u^3)$$

$$O_{4q}^5 = (\bar{q}^3 u^1)(\bar{u}^1 q^3)$$

$$O_{4q}^6 = (\bar{q}^3 d^1)(\bar{d}^1 q^3)$$

$$O_{4q}^7 = (\bar{q}^1 u^3)(\bar{u}^3 q^1)$$

Anomalous Couplings Compared with Effective Operators

- *Wtb* vertex:

$$\bar{b}\Gamma^{\mu} tW_{\mu}^{-} = \frac{g}{\sqrt{2}}\bar{b}\gamma^{\mu}(V_L P_L + V_R P_R)tW_{\mu}^{-} + \frac{g}{\sqrt{2}}\bar{b}\frac{i\sigma^{\mu\nu}q_{\nu}}{M_W}(g_L P_L + g_R P_R)tW_{\mu}^{-}$$

$$V_L = 1 + C_{\phi q}^{*}\frac{v^2}{\Lambda^2} \qquad g_R = \sqrt{2}C_{tW}\frac{v^2}{\Lambda^2}$$

$$O_{\phi q} = i(\phi^{\dagger}\tau^{i}D_{\mu}\phi)(\bar{q}\gamma^{\mu}\tau^{i}q) \qquad O_{tW} = (\bar{q}\sigma^{\mu\nu}\tau^{i}t)\tilde{\phi}W_{\mu\nu}^{i}$$

(V_R and g_L do not enter at order $\frac{1}{\Lambda^2}$.)

- *gtt* vertex:

$$\bar{t}\Gamma^{\mu a}tG_{\mu}^{a} = g_s\bar{t}\frac{\lambda^a}{2}\gamma^{\mu}tG_{\mu}^{a} + g_s\bar{t}\lambda^a\frac{i\sigma^{\mu\nu}q_{\nu}}{m_t}(d_V + id_A\gamma_5)tG_{\mu}^{a}$$

$$d_V = \frac{\sqrt{2}}{g_s}\mathrm{Re}C_{tG\phi}\frac{vm_t}{\Lambda^2} \qquad d_A = \frac{\sqrt{2}}{g_s}\mathrm{Im}C_{tG\phi}\frac{vm_t}{\Lambda^2}$$

$$O_{tG\phi} = (\bar{q}\sigma^{\mu\nu}\lambda^a t)\tilde{\phi}G_{\mu\nu}^{a}$$

Summary

- The effective theory is a model-independent approach to parameterize new physics beyond the SM.
- The anomalous top quark interaction can be described using 13 dimensions-six operators.
- The anomalous couplings can be related to the coefficients of the effective operators.

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