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Decay rates and invariant mass spectra for $\eta \to \pi_0 \gamma\gamma$ and $\eta' \to (\pi_0, \eta)\gamma\gamma$

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Introduction

- **We will present the branching ratios and the invariant mass spectra for three processes:**

  $\eta \rightarrow \pi_0\gamma\gamma$

  $\eta' \rightarrow \pi_0\gamma\gamma$

  $\eta' \rightarrow \eta\gamma\gamma$

- **First process is well accounted for both theoretically and experimentally.**

- **We hope in the future all these decays to be measured at GAMS, CLEO, VES, KLOE-2, Crystal Ball, Crystal Barrel, WASA or BES-III.**

- **We will give a first indication about the second and third processes decay widths and invariant spectra here.**

- **Our approach is based on a combination of linear sigma model and vector meson dominance model.**

Experimental results

- **Before the 1980's there were 13 experiments which studied the decay rate $\eta \to \pi_0\gamma\gamma$ with conflicting results.**

- **In 1984 GAMS measured the partial widths for three reactions including $\eta \to \pi_0\gamma\gamma$ (this was joint experiment between IHEP, USSR and CERN, Switzerland):**

  $BR = (7.1 \pm 1.4) \times 10^{-4}$

  $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.84 \pm 0.17$ eV

- **Another experiment: Crystal Ball multiphoton spectrometer at AGS (uses a beam of negative ions). They obtained:**

  $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.45 \pm 0.12$ eV later improved to
  $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.29 \pm 0.059 \pm 0.022$ eV.

Experimental results

- **Kloe experiment is based at Frascati and it is an e^+e^- collider which works at $\sqrt{s} \sim 1020$ MeV.**

  The eta's are produced through electromagnetic decay of the Φ meson, $\Phi \to \eta\gamma$.

  $BR(\eta \to \pi_0\gamma\gamma) = (8.4 \pm 2.7 \pm 1.4) \times 10^{-5}$

  $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.108 \pm 0.0.35 \pm 0.029$ eV.

- **It is expected that Daphne 2 will improve the precision for this reaction and allow a better study of the invariant mass spectrum for it.**

- **One of the latest results comes again from Crystal Ball at AGS:**

  $BR(\eta \to \pi_0\gamma\gamma) = 2.21 \pm 0.53 \times 10^{-4}$

  $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.285 \pm 0.068_{\text{tot}}$ eV.

- **We conclude by giving the PDG result for the process $\eta \to \pi_0\gamma\gamma$:**

  $BR(\eta \to \pi_0\gamma\gamma) = 2.7 \pm 0.5 \times 10^{-4}$

  $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.35 \pm 0.07_{\text{tot}}$ eV.

Theoretical results

- **There are no established results for the reactions $\eta' \to \pi_0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$ although the decay rate for the first process can be obtained directly from the decay rate for the process $\eta \to \pi_0\gamma\gamma$ by replacing some angles($\cos\Phi_p \to \sin\Phi_p$ and $\sin\Phi_p \to -\cos\Phi_p$) and by integrating over the corresponding phase space which is significantly larger.**

- **Most theoretical approaches are an admixture of vector meson dominance model and chiral perturbation theory. The vector meson dominance results agrees with each other but adding other contributions or interference terms can lead to differences.**

- **In chiral perturbation theory the tree level contributions are zero at order $O(p^2)$ and $O(p^4)$ and the process $\eta \to \pi_0\gamma\gamma$ can take place at order $O(p^4)$ through charged kaon loops or at order $O(p^6)$.**

- **Since counterterms at order $O(p^6)$ are not determined in ChPT and are model dependent it turns out that this decay is a very useful laboratory for testing ChPT.**

Method

- **We use a linear sigma model to quantify the scalar meson exchanges and VMD for the vector meson ones and allow the amplitudes to interfere.**
- The vector meson model is standard; the only possible variations that appear stem from the numerical values of the couplings and of the pseudoscalar mixing angle Φ_p.

- In a linear sigma model the processes take place in the s, t and u channels only at one loop level.
- The full amplitudes are written as products of two factors:

$$\begin{aligned}
\mathcal{A}^{LSM}_{\eta\to\pi_0\gamma\gamma} &= M^{\mu\nu}_{K^+K^-}\epsilon_\mu(k_1)\epsilon_\nu(k_2)\mathcal{A}^{LSM}_{K^+K^-\to\pi_0\eta} \\
\mathcal{A}^{LSM}_{\eta'\to\pi_0\gamma\gamma} &= M^{\mu\nu}_{K^+K^-}\epsilon_\mu(k_1)\epsilon_\nu(k_2)\mathcal{A}^{LSM}_{K^+K^-\to\pi_0\eta'} \\
\mathcal{A}^{LSM}_{\eta'\to\eta\gamma\gamma} &= M^{\mu\nu}_{K^+K^-}\epsilon_\mu(k_1)\epsilon_\nu(k_2)\mathcal{A}^{LSM}_{K^+K^-\to\eta\eta'} + \\
&+ M^{\mu\nu}_{\pi^+\pi^-}\epsilon_\mu(k_1)\epsilon_\nu(k_2)\mathcal{A}^{LSM}_{\pi^+\pi^-\to\eta\eta'}
\end{aligned} \tag{1}$$

Linear sigma model

- **The linear sigma model is introduced by the lagrangian.**

$$\mathcal{L} = -\frac{1}{2}Tr(D_\mu M D_\mu M^\dagger) - V_0(M) - V_{SB} \tag{2}$$

- **The field M contains both scalar and pseudoscalar states and we use for the symmetric part of the potential:**

$$V_0 = \frac{\mu^2}{2}Tr(MM^\dagger) + \frac{\lambda}{4}Tr(MM^\dagger MM^\dagger) + \frac{\lambda'}{4}Tr(MM^\dagger)Tr(MM^\dagger) \tag{3}$$

- **The symmetry breaking potential contain two terms, the first one leading to the chiral symmetry breaking and the second one to the explicit symmetry breaking of $U(1)_A$:**

$$V_{SB} = 2Tr(AS) + \beta[\det(M) + \det(M^\dagger)] \tag{4}$$

S denotes the scalar matrix while A is a diagonal matrix proportional to the quark masses(we assume isospin invariant limit where $A_1 = A_2$).

The gauge covariant derivative is given by

$$D_\mu M = \partial_\mu M - ieA_\mu[Q, M] \tag{5}$$

where $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ is the charge matrix.

Scalar meson exchange

- **The processes $\eta(\eta') \to \pi_0\gamma\gamma$ occur through kaon loops while $\eta' \to \eta\gamma\gamma$ occurs through pion and kaon loops, the former being prevalent. The loop contributions take place through three diagrams which added together give a finite result. Thus the matrix elements corresponding to kaon and pion loops are:**

$$
\begin{aligned}
M^{\mu\nu}_{K^+K^-} &= \frac{-i}{16\pi^2}2e^2\frac{1}{s}[g^{\mu\nu}s - 2k_2^\mu k_1^\nu] \\
&\times[1 + 2\frac{m_K^2}{s}F(s)] \\
M^{\mu\nu}_{\pi^+\pi^-} &= \frac{-i}{16\pi^2}2e^2\frac{1}{s}[g^{\mu\nu}s - 2k_2^\mu k_1^\nu] \\
&\times[1 + 2\frac{m_\pi^2}{s}F(s)]
\end{aligned}
\tag{6}
$$

- Here k_1 and k_2 are the 4-momenta of the two photons and $F(s)$ is the loop function defined as follows:

$$
\begin{aligned}
F(a) &= \frac{1}{2}[\ln(\frac{1+(1-4m^2/a)^{1/2}}{1-(1-4m^2/a)^{1/2}}) - i\pi] \text{ for } a > 4m^2 \\
F(a) &= -\frac{1}{2}[\pi - 2\arctan[\frac{4m^2}{a} - 1]^{1/2}]^2 \text{ for } 0 < a < 4m^2
\end{aligned}
\tag{7}
$$

Scalar meson exchange

- **The amplitude $\mathcal{A}^{LSM}_{\eta\pi_0 \to K^+K^-}$ is a sum of three contributions corresponding to the s(a_0), t and u (κ) channels.**

$$\mathcal{A}^{LSM}_{\eta\pi_0 \to K^+K^-} = g_{\pi_0 \eta K^+ K^-} - g_{\pi_0 a_0 \eta} g_{a_0 K^+ K^-} \frac{1}{s - m^2_{a_0}}$$
$$+ g_{\pi_0 \kappa K^+} g_{\eta \kappa K^-} [\frac{1}{t - m^2_\kappa} + \frac{1}{u - m^2_\kappa}] \tag{8}$$

- In the soft pion limit where $p = 0$ and $s = m^2_\eta$ and $t = u = m^2_K$ the amplitude must be equal to zero. This fixes the values of the coupling constant to:

$$g_{\pi_0 \eta K^+ K^-} = \frac{g_{\pi_0 a_0 \eta} g_{a_0 K^+ K^{-1}}}{m^2_\eta - m^2_{a_0}} + \frac{g_{\pi_0 \kappa K^+} g_{\eta \kappa K^+}}{m^2_K - m^2_\kappa} + \frac{g_{\pi_0 \kappa K^+} g_{\eta \kappa K^+}}{m^2_K - m^2_\kappa} \tag{9}$$

Scalar meson exchange

- **The full s dependent amplitude $\mathcal{A}^{LSM}_{\eta\pi_0\to K^+K^-}$ is obtained by replacing the t and u contributions with there limit when all scalar masses are taken to infinity.**

$$\mathcal{A}^{L\sigma M}_{K^+K^-\to\pi_0\eta} = \frac{1}{2f_\pi f_K}[(s-m_\eta^2)\frac{m_K^2-m_{a_0}^2}{s-m_{a_0}^2}\cos\Phi_p] + \frac{1}{4f_\pi f_K}[-s+m_\eta^2+m_\pi^2](\cos\Phi_p-\sqrt{2}\sin\Phi_p) \qquad (10)$$

In a similar way we obtain:

$$\mathcal{A}^{LSM}_{K^+K^-\to\pi_0\eta'} = \frac{1}{2f_\pi f_K}[(s-m_\eta^2)\frac{m_K^2-m_{a_0}^2}{s-m_{a_0}^2}\sin\Phi_p] + \frac{1}{4f_\pi f_K}[-s+m_\eta^2+m_\pi^2](\sin\Phi_p+\sqrt{2}\cos\Phi_p) \qquad (11)$$

Amplitude for the process $\eta' \to \eta\gamma\gamma$

$$\mathcal{A}^{LSM}_{\eta\eta'\to\pi^+\pi^-} = \frac{\sin 2\Phi_p}{2f_{\pi^2}}[(m^2_\eta \cos^2\Phi_p + m^2_{\eta'}\sin^2\Phi_p - m^2_a) \times$$
$$(\cos\Phi_s + \sqrt{2}\sin\Phi_s(\frac{2f_K}{f_\pi} - 1))$$
$$-(m^2_{\eta'} - m^2_\eta)(\cos\Phi_s \cos 2\Phi_p - \frac{1}{2}\sin\Phi_s \sin 2\Phi_p] \cos\Phi_s \frac{s - m^2_\pi}{s - m^2_\sigma} +$$
$$+\frac{\sin 2\Phi_p}{2f_{\pi^2}}[(m^2_\eta \cos^2\Phi_p + m^2_{\eta'}\sin^2\Phi_p - m^2_a) \times$$
$$(\sin\Phi_s - \sqrt{2}\cos\Phi_s(\frac{2f_K}{f_\pi} - 1))$$
$$-(m^2_\eta - m^2_{\eta'})(\sin\Phi_s \cos 2\Phi_p - \frac{1}{2}\cos\Phi_s \sin 2\Phi_p] \sin\Phi_s \frac{s - m^2_\pi}{s - m^2_{f_0}} +$$
$$\frac{\sin 2\Phi_p}{2f^2_\pi}[2m^2_\pi - s] \quad (12)$$

$$
\begin{aligned}
&\mathcal{A}^{LSM}_{\eta\eta'\to K^+K^-} = \frac{\sin 2\Phi_p}{4 f_\pi f_K}[(m_\eta^2\cos^2\Phi_p + m_{\eta'}^2\sin^2\Phi_p - m_a^2)\times \\
&(\cos\Phi_s + \sqrt{2}\sin\Phi_s(\frac{2f_K}{f_\pi}-1)) \\
&-(m_{\eta'}^2 - m_\eta^2)(\cos\Phi_s\cos 2\Phi_p - \frac{1}{2}\sin\Phi_s\sin 2\Phi_p]\times \\
&[\cos\Phi_s - \sqrt{2}\sin\Phi_s]\frac{s-m_K^2}{s-m_\sigma^2} + \\
&+\frac{\sin 2\Phi_p}{2f_\pi}[(m_\eta^2\cos^2\Phi_p + m_{\eta'}^2\sin^2\Phi_p - m_a^2)\times \\
&(\sin\Phi_s - \sqrt{2}\cos\Phi_s(\frac{2f_K}{f_\pi}-1)) \\
&-(m_{\eta'}^2 - m_\eta^2)(\sin\Phi_s\cos 2\Phi_p - \frac{1}{2}\cos\Phi_s\sin 2\Phi_p] \\
&[\sin\Phi_s + \sqrt{2}\cos\Phi_s]\frac{s-m_K^2}{s-m_{f_0}^2} + \\
&\frac{1}{4f_K^2}[-\sin\Phi_p\cos\Phi_p + \sqrt{2}(\cos^2\Phi_p - \sin^2\Phi_p)][2m_K^2 - s] \quad (13)
\end{aligned}
$$

Vector meson exchange

- **The relevant interactions are described by the Lagrangians:**

$$\mathcal{L}_{VVP} = \frac{G}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle\partial_\mu V_\nu \partial_\alpha V_\beta P\rangle$$
$$L_{V\gamma} = -4f^2 egA_\mu\langle QV^\mu\rangle \qquad (14)$$

Here $G = \frac{3g^2}{4\pi^2 f}$ is the $\omega\rho\pi$ coupling constant and $|g| = 4.0$ from ρ and ω decay data.

- **We consider constant decay widths in the propagators for Ω and Φ,**

$$D(\omega) = \frac{1}{q^2 - m_{\omega 2}^2 - im_\omega\Gamma_\omega}$$
$$D(\Phi) = \frac{1}{q^2 - m_\Phi^2 - im_\phi\Gamma_\phi}$$
$$D(\rho) = \frac{1}{q^2 - m_\rho^2 - im_\rho\Gamma_\rho(q^2)} \qquad (15)$$

whereas for the rho we will consider a momentum dependent width:

$$\Gamma_\rho(q^2) = \frac{m_\rho}{\sqrt{q^2}}\Gamma_\rho(\frac{q^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2})^{3/2}\theta(q^2 - 4m_\pi^2) \qquad (16)$$

Vector meson contributions

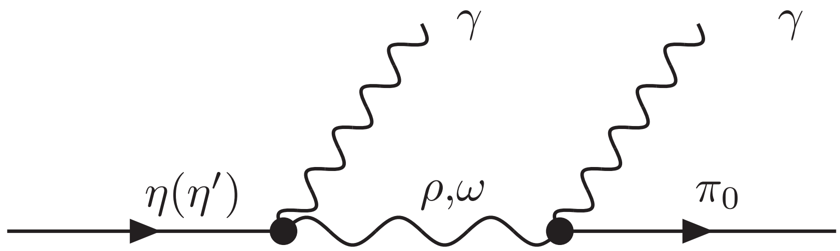


Fig. 3.Eta decay in VMD models.

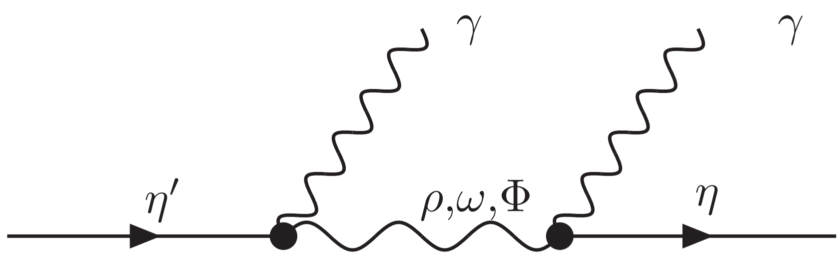


Fig 4. $\eta' \to \eta\gamma\gamma$ *in VMD models.*

Vector meson exchange

- **The amplitude for the process $\eta \to \pi_0 \gamma\gamma$ reads:**

$$\mathcal{A}^{VMD} = C(P^0 P^0 \gamma\gamma) \frac{2G^2 e^2}{g^2} \begin{vmatrix} q \cdot q & q \cdot k_2 & q \cdot \epsilon_2 \\ k_1 \cdot q & k_1 \cdot k_2 & k_1 \cdot \epsilon_2 \\ \epsilon_1 \cdot q & \epsilon_1 \cdot k_2 & \epsilon_1 \cdot \epsilon_2 \end{vmatrix} \times$$

$$[\frac{1}{q^2 - M_\rho^2 + iM_\rho \Gamma(q^2)} + (k_1 \leftrightarrow k_2, q \leftrightarrow q') + (\rho \to \omega) \quad (17)$$

Here $q = P - k_1$ and $q' = P - k_2$ are the momenta of the vector mesons in the t and u channels while P, k_1 and k_2 are the momenta for the eta and the two photons.

$$C = \frac{1}{12} \cos \Phi_p \quad (18)$$

- **The amplitude for the process $\eta' \to \pi_0 \gamma\gamma$ is obtained by replacing $\cos \Phi_p$ with $\sin \Phi_p$.**

- **The amplitude for the process $\eta' \to \eta\gamma\gamma$ reads:**

$$\mathcal{A}^{VMD}_{\eta'\to\eta\gamma\gamma} = C(\rho)\frac{32G^2 f^4 e^2 g^2}{M_\rho^4}\begin{vmatrix} q\cdot q & q\cdot k_2 & q\cdot\epsilon_2 \\ k_1\cdot q & k_1\cdot k_2 & k_1\cdot\epsilon_2 \\ \epsilon_1\cdot q & \epsilon_1\cdot k_2 & \epsilon_1\cdot\epsilon_2 \end{vmatrix} \times$$
$$[\frac{1}{q^2 - M_\rho^2 + iM_\rho\Gamma(q^2)} + (k_1 \leftrightarrow k_2, q \leftrightarrow q')$$
$$+(\rho\to\omega) + (\rho\to\Phi)] \tag{19}$$

Here $q = P - k_1$ and $q' = P - k_2$ are the momenta of the vector mesons in the t and u channels while P, k_1 and k_2 are the momenta for the eta and the two photons.

$$\begin{aligned} C(\rho) &= \frac{1}{4}\sin\Phi_p\cos\Phi_p \\ C(\omega) &= \frac{1}{36}\sin\Phi_p\cos\Phi_p \\ C(\Phi) &= -\frac{1}{9}\sin\Phi_p\cos\Phi_p \end{aligned} \tag{20}$$

Decay rates

- **The amplitude for the decay rate is the sum between the scalar and vector meson contributions:**

$$|\bar{\mathcal{A}}|^2 = |\bar{\mathcal{A}}_{LSM}|^2 + |\bar{\mathcal{A}}_{VMD}|^2 + 2\mathrm{Re}\mathcal{A}^*_{VMD}\mathcal{A}_{LSM} \tag{21}$$

- **We use the standard decay rates formulas.**

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\bar{\mathcal{A}}|^2 dm^2_{12} dm^2_{23} \tag{22}$$

where:

$$\begin{aligned} m^2_{12} &= m^2_{\gamma\gamma} = k_1^2 + k_2^2 + 2k_1k_2 = 2k_1k_2 \\ m^2_{23} &= k_2^2 + p_3^2 + 2k_2p_3 = m_3^2 + 2k_2p_3 \end{aligned} \tag{23}$$

- **We are interested in the differential decay rate with respect the invariant mass distribution $m^2_{\gamma\gamma}$ and the integrated ones.**

Numerical results

We list below the decay rates as a whole and decomposed into scalar, vector and interference contribution.

Decay rates	$\eta \to \pi_0\gamma\gamma$	$\eta' \to \pi_0\gamma\gamma$	$\eta' \to \eta\gamma\gamma$
Γ_{VMD} (MeV)	2.53×10^{-7}	1.14×10^{-3}	8.45×10^{-5}
Γ_{LSM} (MeV)	2.70×10^{-7}	8.42×10^{-6}	2.65×10^{-7}
$\Gamma_{\rm int}$ (MeV)	-2.97×10^{-7}	4.10×10^{-6}	4.47×10^{-7}
$\Gamma_{\rm teor}$ (MeV)	2.25×10^{-7}	1.16×10^{-3}	8.52×10^{-5}
$\Gamma_{\rm exp}$ (MeV)	$3.5 \pm 0.7 \times 10^{-7}$	-	-
$B_{\rm teor}$	1.73×10^{-4}	5.7×10^{-3}	4.18×10^{-4}
$B_{\rm exp}$	$2.7 \pm 0.5 \times 10^{-4}$	$< 8 \times 10^{-4}$	-

Table 1: Theoretical decay rates and branching ratio as compared to the experimental ones(when they are known).

Differential decay rate for the process $\eta \to \pi_0\gamma\gamma$

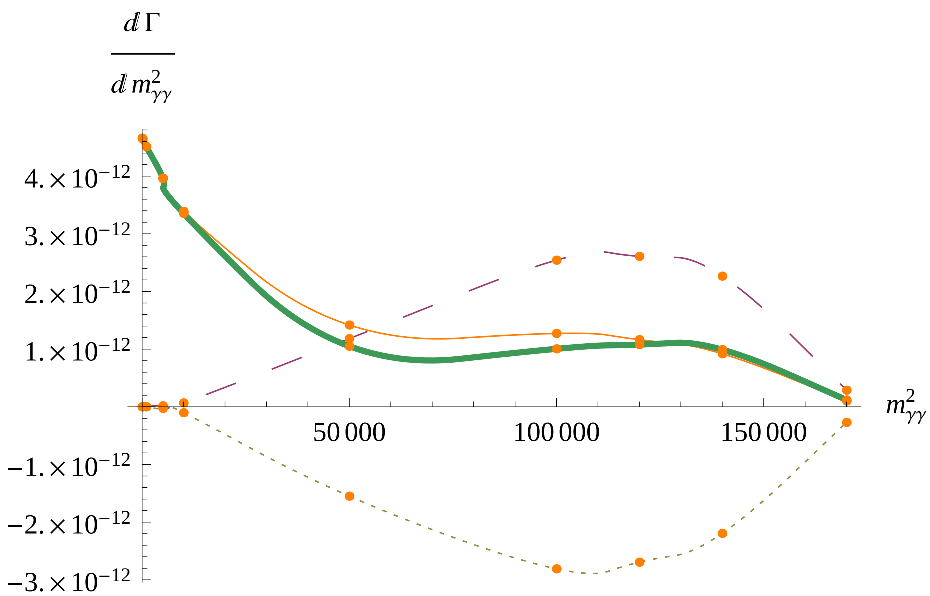
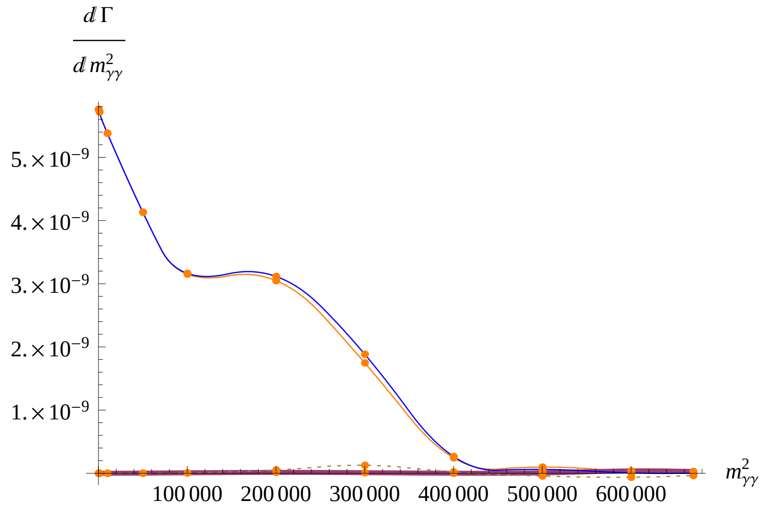


Figure 1: $\frac{d\Gamma}{dm^2_{\gamma\gamma}}$ (MeV) for the decay $\eta \to \pi_0\gamma\gamma$ The contributions are: total decay rate (thick line); vector meson contributions (thin line); scalar meson contribution (large dashed line); the interference term (small dashed line).

Scalar and vector meson contribution are almost equally important.

Differential decay rate for the process $\eta' \to \pi_0\gamma\gamma$



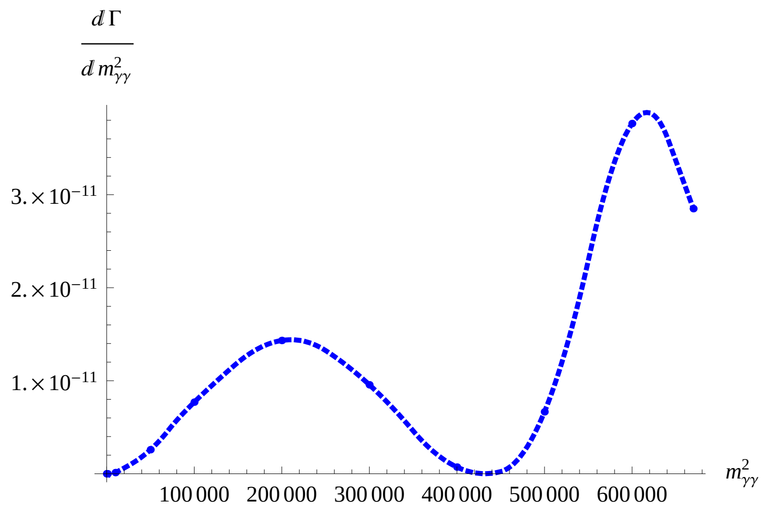


Figure 2: $\frac{d\Gamma}{dm^2_{\gamma\gamma}}$ (MeV) for the decay $\eta' \to \pi_0\gamma\gamma$. Top figure contains all the contributions: total decay rate (blue line); vector meson (orange line); scalar mesons (thick line), interference term (dashed line). Down figure: scalar meson contribution.

Differential decay rate for the process $\eta' \to \eta\gamma\gamma$

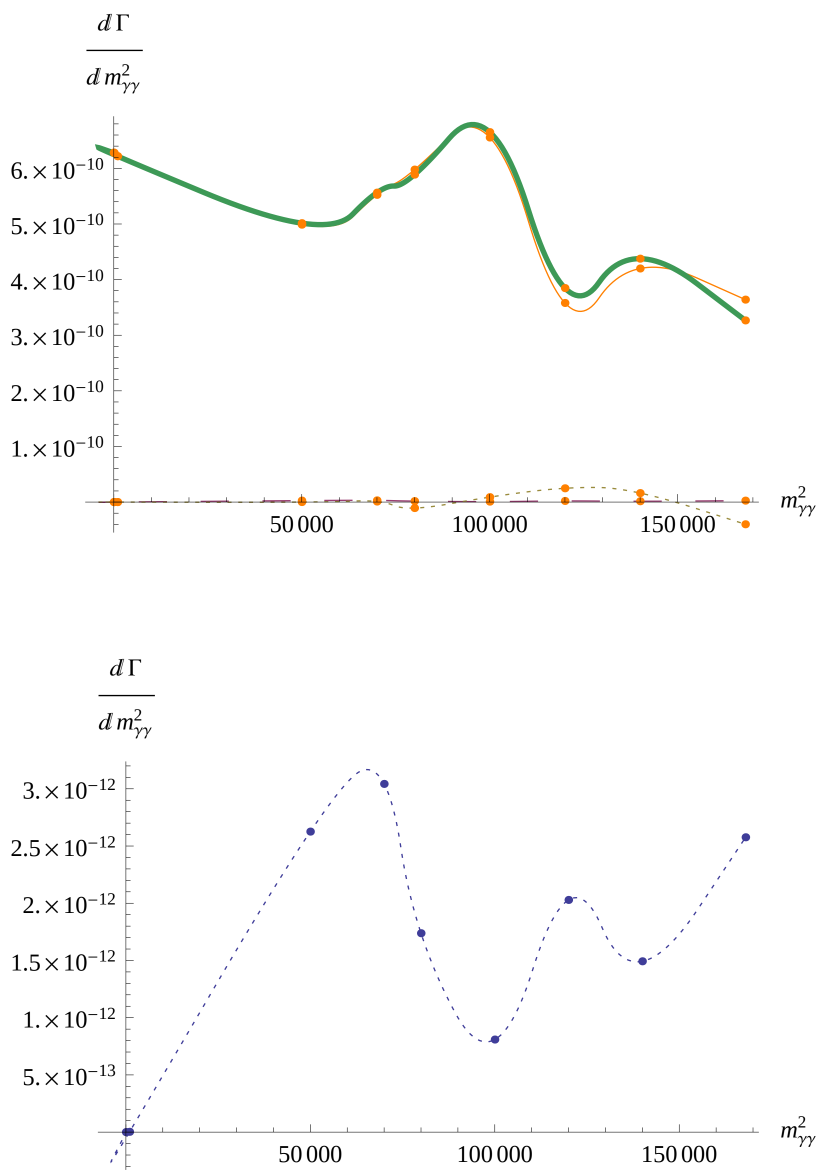


Figure 3: $\frac{d\Gamma}{dm^2_{\gamma\gamma}}$ (MeV) for the decay $\eta' \to \eta\gamma\gamma$. Top figure contains all the contributions: total (thick line); vector mesons (large dashed line); interference term (small dashed line). Down figure contains scalar meson contribution.

Conclusions

- **We have obtained the invariant mass spectrum and the decay rates for three processes. The first one $\eta \to \pi_0\gamma\gamma$ has been studied both from theoretical and experimental point of view. Our decay width $\Gamma_{\eta\to\pi_0\gamma\gamma} = 2.25 \times 10^{-7}$ MeV compares well with the experimental one, $\Gamma_{\eta\to\pi_0\gamma\gamma} = 3.5 \pm 0.7 \times 10^{-7}$ MeV, although on the lower side.**

- **The latest theoretical result is coming from Oset et al (arXiv:0801.2633) within the chiral unitary approach and VMD, where for example vector meson contribution amounts to $\Gamma^{VMD}_{\eta\to\pi_0\gamma\gamma} = 3.0 \pm 0.6 \times 10^{-7}$ MeV with normalized coupling. Our result for this contribution is $\Gamma^{VMD}_{\eta\to\pi_0\gamma\gamma} = 2.53 \times 10^{-7}$ MeV is close to this, although various couplings and the pseudoscalar mixing angle, Φ_p are slightly different.**

- **The full latest theoretical result is $\Gamma_{\eta\to\pi_0\gamma\gamma} = 0.33 \pm 0.08$ eV. Our result is within this range** (Note that in our work the interference between the scalar meson exchange term and the vector meson exchange one is destructive).

Conclusions

- There are not established theoretical or experimental results for the processes $\eta' \to \pi_0(\eta)\gamma\gamma$.

- For $\eta' \to \pi_0\eta\eta$ the vector meson decay width $\Gamma^{VMD}_{\eta'\to\pi_0\gamma\gamma} = 1.14 \times 10^{-3}$ MeV is much bigger than the scalar one $\Gamma^{LSM}_{\eta'\to\pi_0\gamma\gamma} = 8.42 \times 10^{-6}$ MeV the latter contributing only as fractions of percent to the final result $\Gamma_{\eta'\to\pi_0\gamma\gamma} = 1.16 \times 10^{-3}$ MeV.

- For the reaction $\eta' \to \eta\gamma\gamma$ the vector meson term is $\Gamma^{VMD}_{\eta'\to\eta\gamma\gamma} = 8.45 \times 10^{-5}$ MeV while the scalar one is $\Gamma^{LSM}_{\eta'\to\eta\gamma\gamma} = 2.65 \times 10^{-7}$ MeV. The full decay width is $\Gamma_{\eta'\to\eta\gamma\gamma} = 8.52 \times 10^{-5}$ MeV thus scalars contributing only as percents.