



THE UNIVERSITY OF
CHICAGO

Irreducible Uncertainties in

$$\bar{B} \rightarrow X_s \gamma$$

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M. Benzke, S.J. Lee, M. Neubert, GP [arXiv:1003.5012]

S.J. Lee, M. Neubert, GP PRD **75** 114005 (2007)

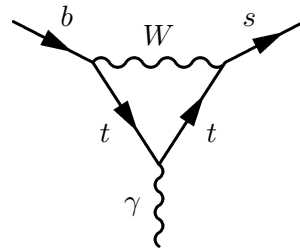
Introduction

$\bar{B} \rightarrow X_s \gamma$ in the SM

- $b \rightarrow s \gamma$ is a flavor changing neutral current (FCNC)

In SM no FCNC at tree level

Arises as a loop effect:



gives rise to the operator:

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b$$

part of the effective Hamiltonian

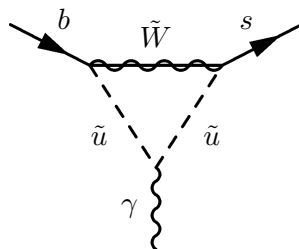
$$\mathcal{H}_{\text{eff}} \ni \frac{G_F}{\sqrt{2}} C_{7\gamma} Q_{7\gamma}$$

Constraints on New Physics: $\bar{B} \rightarrow X_s \gamma$

- $\bar{B} \rightarrow X_s \gamma$ is an important probe of new physics

$b \rightarrow s \gamma$ can have contribution from new physics e.g. SUSY

(only one diagram shown):



leads to same operator, modifies $C_{7\gamma}$

- BSM calculations are becoming advanced e.g.

Complete NLO SUSY MFV

SusyBSG package (Degrassi, Gambino, Slavich '07)

- Need to think seriously about errors!

How To Make a Photon?

- Produce it directly...

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How To Make a Photon?

- Produce it directly...

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} F^{\mu\nu} (1 + \gamma_5) b$$

- Or make a gluon or a quark pair

$$Q_{8g} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} G^{\mu\nu} (1 + \gamma_5) b$$

$$Q_1^q = (\bar{q}b)_{V-A} (\bar{s}q)_{V-A} \quad (p = u, c)$$

and convert them to a photon

How To Make a Photon?

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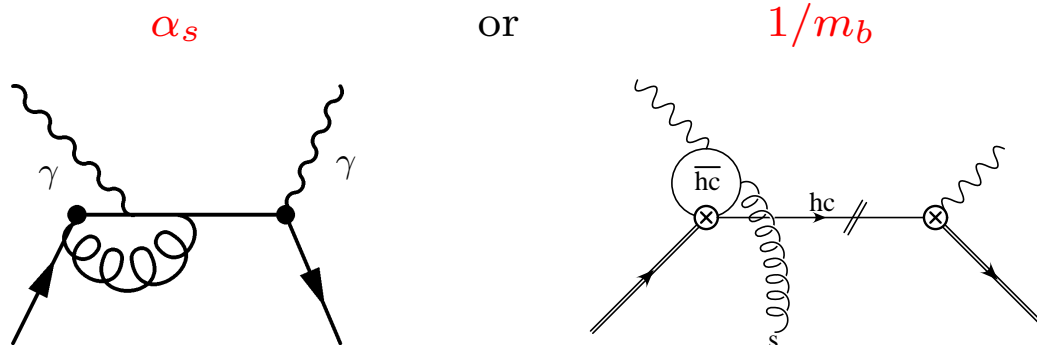
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and convert them to a photon

- But it will cost you..



Effective Hamiltonian

- For $\bar{B} \rightarrow X_s \gamma$ need Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

- At leading power only $Q_{7\gamma} - Q_{7\gamma}$ contribute
- At higher orders need other $Q_i - Q_j$ contributions
- Most important: $Q_{7\gamma}, Q_{8g}$, and Q_1

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b$$

$$Q_{8g} = \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b$$

$$Q_1^q = (\bar{q}b)_{V-A} (\bar{s}q)_{V-A} \quad (q = u, c)$$

- Ratio of Wilson coefficients:

$$C_1 \quad : \quad C_{7\gamma} \quad : \quad C_{8g}$$

$$3 \quad : \quad 1 \quad : \quad \frac{1}{2}$$

Total Rate

- Previous studies of $Q_i - Q_j$ contributions focus on $\Gamma(\bar{B} \rightarrow X_s \gamma)$
and mostly on α_s suppressed effects
- Common lore:
like $\Gamma(\bar{B} \rightarrow X_u l \bar{\nu})$ non perturbative effects arise at $1/m_b^2$

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- Common lore:
like $\Gamma(\bar{B} \rightarrow X_u l \bar{\nu})$ non perturbative effects arise at $1/m_b^2$
- Hints that not all is well
 - $Q_{8g} - Q_{8g}$ (Ali, Greub '95; Kapustin, Ligeti, Politzer '95)
 - $Q_1 - Q_{7\gamma}$ (Voloshin '96; Ligeti, Randall, Wise '97; Grant, Morgan, Nussinov, Peccei '97; Buchalla, Isidori, Rey '97)
 - No local OPE for $\Gamma(\bar{B} \rightarrow X_s \gamma)$ (Ligeti, Randall, Wise '97)

But effects were thought to be under control or small ...

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But effects were thought to be under control or small ...

- **Never** a systematic study!

In fact uncertainty from $Q_{7\gamma} - Q_{8g}$ was **missed!**

(Lee, Neubert, GP '06)

Non perturbative effects in $\Gamma(\bar{B} \rightarrow X_s \gamma)$ arise at $1/m_b$

- What do we find from a systematic analysis?

Factorization at Subleading Power
and
Irreducible Uncertainties
in
 $\bar{B} \rightarrow X_s \gamma$ Decay

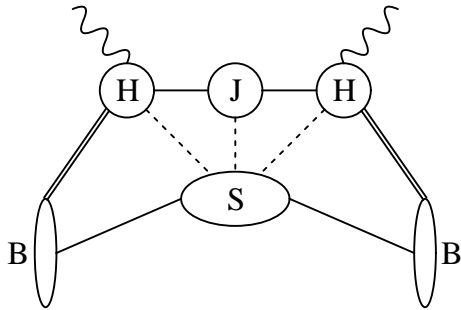
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New Factorization Formula: Schematically

At the endpoint region: $m_b - 2E_\gamma \sim \Lambda_{\text{QCD}}$

- Considering only $Q_{7\gamma} - Q_{7\gamma}$: factorization formula for $d\Gamma/dE_\gamma$
(Korchensky, Sterman '94; Bauer, Pirjol, Stewart '01)

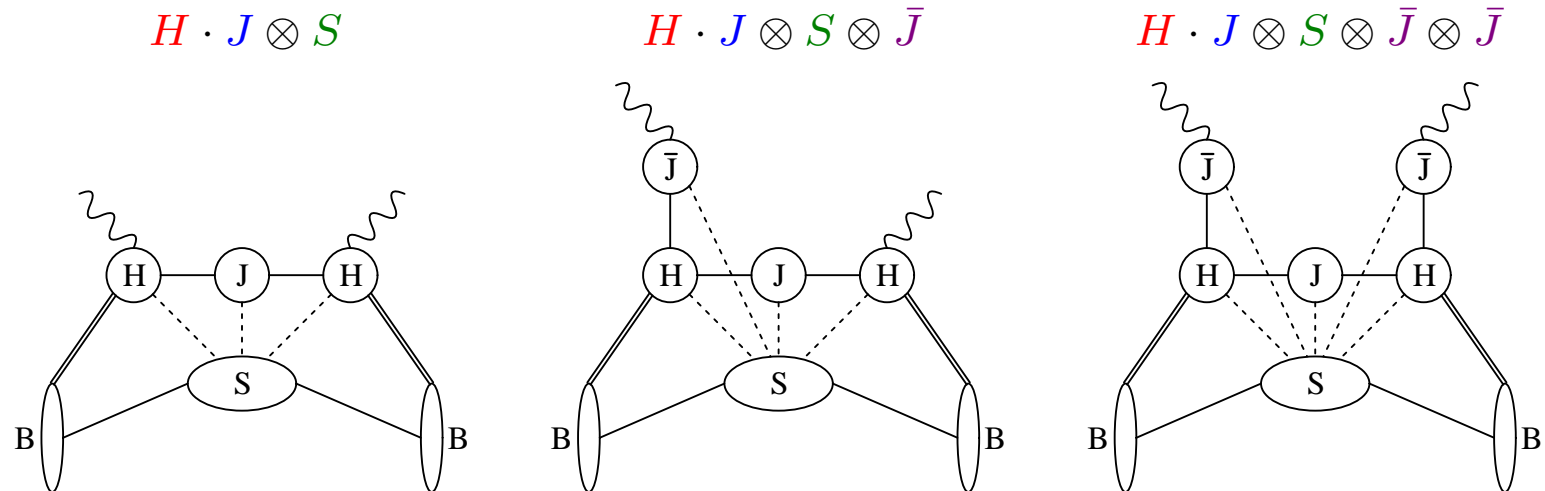
$$H \cdot J \otimes S$$



New Factorization Formula: Schematically

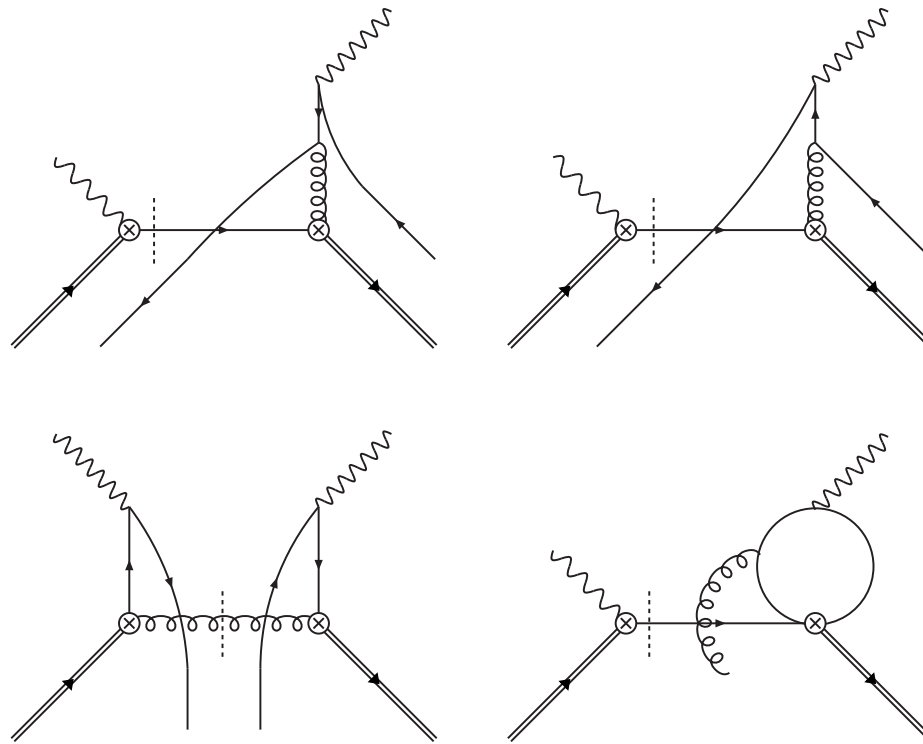
At the endpoint region: $m_b - 2E_\gamma \sim \Lambda_{\text{QCD}}$

- Considering only $Q_{7\gamma} - \bar{Q}_{7\gamma}$: factorization formula for $d\Gamma/dE_\gamma$ (Korchensky, Sterman '94; Bauer, Pirjol, Stewart '01)
- Considering also other operators \Rightarrow **new** factorization formula for $d\Gamma/dE_\gamma$ (Benzke, Lee, Neubert, GP '10)



- No analog for semileptonic decays
- New “resolved photon” contributions \Rightarrow $1/m_b$ corrections to $\Gamma(\bar{B} \rightarrow X_s \gamma)$
- What are they?

Resolved Photon Contributions



Top line:	$Q_{7\gamma} - Q_{8g}$
Bottom left:	$Q_{8g} - Q_{8g}$
Bottom right:	$Q_1 - Q_{7\gamma}$

- $Q_1 - Q_{8g}$ and $Q_1 - Q_1$ give a $1/m_b^2$ effect

Resolved Photon Contributions

- Non perturbative effects arise from “Resolved Photon Contributions”

They have the form

$$\Delta\Gamma \sim \begin{array}{ccc} \bar{J} & \otimes & h \\ \uparrow & & \uparrow \\ \text{Calc. in PT} & & \text{Non pert.} \end{array}$$

- The non perturbative functions h_{ij} are

$$h_{88}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots s(un) \bar{s}(r\bar{n}) \cdots b(0) | \bar{B} \rangle$$

$$h_{17}(\omega_1) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle$$

$$h_{78}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle$$

The Integrated Rate

- For a photon energy cut $E_\gamma > E_0$ define

$$\mathcal{F}_E(\Delta) = \frac{\Gamma(E_0) - \Gamma(E_0)|_{\text{OPE}}}{\Gamma(E_0)|_{\text{OPE}}},$$

where $\Delta = m_b - 2E_0$ and $\Gamma(E_0)|_{\text{OPE}}$ is the older calculation

- Assuming $\Delta \gg \Lambda_{\text{QCD}}$

$$\begin{aligned} \mathcal{F}_E(\Delta) = & \frac{C_1(\mu)}{C_{7\gamma}(\mu)} \frac{\Lambda_{17}(m_c^2/m_b, \mu)}{m_b} + \frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)} 4\pi\alpha_s(\mu) \frac{\Lambda_{78}^{\text{spec}}(\mu)}{m_b} \\ & + \left(\frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)} \right)^2 \left[4\pi\alpha_s(\mu) \frac{\Lambda_{88}(\Delta, \mu)}{m_b} - \frac{C_F\alpha_s(\mu)}{9\pi} \frac{\Delta}{m_b} \ln \frac{\Delta}{m_s} \right] + \dots, \end{aligned}$$

where **model independently**,

$$\Lambda_{17}\left(\frac{m_c^2}{m_b}, \mu\right) = e_c \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1}\right) + \frac{m_b \omega_1}{12m_c^2} \right] h_{17}(\omega_1, \mu),$$

$$\Lambda_{78}^{\text{spec}}(\mu) = \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\omega_2}{\omega_2 - i\varepsilon} h_{78}^{(5)}(\omega_1, \omega_2, \mu),$$

$$\begin{aligned} \Lambda_{88}(\Delta, \mu) = & e_s^2 \left[\int_{-\infty}^{\Lambda_{\text{UV}}} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\Lambda_{\text{UV}}} \frac{d\omega_2}{\omega_2 - i\varepsilon} 2h_{88}^{\text{cut}}(\Delta, \omega_1, \omega_2, \mu) \right. \\ & \left. - \frac{C_F}{8\pi^2} \Delta \left(\ln \frac{\Lambda_{\text{UV}}}{\Delta} - 1 \right) \right]. \end{aligned}$$

The Integrated Rate

- We need to estimate three parameters:

$$\Lambda_{17}\left(\frac{m_c^2}{m_b}, \mu\right) = e_c \operatorname{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1}\right) + \frac{m_b \omega_1}{12m_c^2} \right] h_{17}(\omega_1, \mu),$$

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- Naively, if $\Lambda_{ij} \sim \Lambda_{\text{QCD}} \sim 0.5 \text{ GeV}$

Effect on the rate can be up to 30%

- Fortunately, it is possible to constrain Λ_{17} and $\Lambda_{78}^{\text{spec}}$

We now discuss each of the three parameters

Λ_{17}

- We need to estimate

$$\Lambda_{17}\left(\frac{m_c^2}{m_b}, \mu\right) = e_c \operatorname{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1}\right) + \frac{m_b \omega_1}{12m_c^2} \right] h_{17}(\omega_1, \mu)$$

- Recall

$$h_{17}(\omega_1) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots G(s\bar{n}) \cdots b(0) | \bar{B} \rangle$$

where intuitively ω_1 is the soft gluon momentum

- $h_{17}(\omega_1)$
 - An even function of ω_1
 - Its normalization is $2\lambda_2 \approx 0.24 \text{ GeV}^2$
- Using an exponential or a Gaussian as a model for $h_{17}(\omega_1)$

$$h_{17}(\omega_1, \mu) = \frac{\lambda_2}{\sigma} e^{-\frac{|\omega_1|}{\sigma}} \quad \text{or} \quad h_{17}(\omega_1, \mu) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

- Varying σ
 - $(\Lambda_{17}^{\text{exp}})_{\text{max}} = -4.6 \text{ MeV}$ (for $\sigma = 0.51 \text{ GeV}$)
 - $(\Lambda_{17}^{\text{Gauss}})_{\text{max}} = -8.1 \text{ MeV}$ (for $\sigma = 0.77 \text{ GeV}$)
- Not a conservative bound!

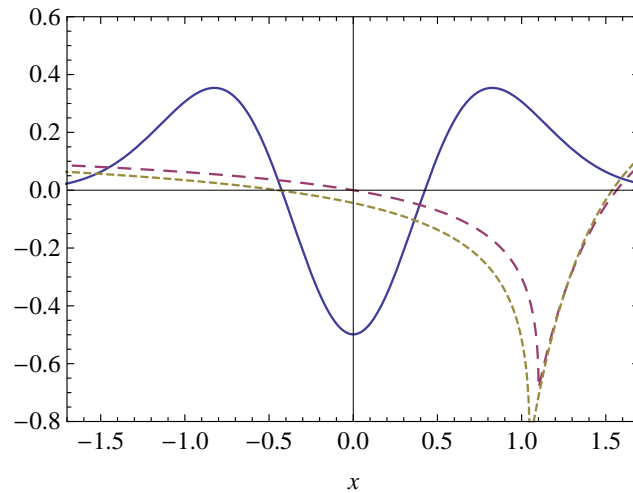
Λ₁₇

- We need to estimate

$$\Lambda_{17}\left(\frac{m_c^2}{m_b}, \mu\right) = e_c \operatorname{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1}\right) + \frac{m_b \omega_1}{12m_c^2} \right] h_{17}(\omega_1, \mu)$$

- $h_{17}(\omega_1)$ doesn't have to be positive, e.g.

$$h_{17}(\omega_1, \mu) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} \frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}$$



- Blue line: $\sigma = 0.5 \text{ GeV}$ and $\Lambda = 0.425 \text{ GeV} \Rightarrow \Lambda_{17} = -42 \text{ MeV}$
- Another choice: $\sigma = 0.5 \text{ GeV}$ and $\Lambda = 0.575 \text{ GeV} \Rightarrow \Lambda_{17} = 27 \text{ MeV}$.
- Final range: $-60 \text{ MeV} < \Lambda_{17} < 25 \text{ MeV}$

- We need to estimate

$$\Lambda_{78}^{\text{spec}}(\mu) = \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\omega_2}{\omega_2 - i\varepsilon} h_{78}^{(5)}(\omega_1, \omega_2, \mu)$$

- Recall

$$h_{78}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle$$

- Method I: Fierz and use VIA

$$h_{78}(\omega_1, \omega_2)|_{\text{VIA}} = -e_{\text{spec}} \frac{f_B^2 M_B}{8} \left(1 - \frac{1}{N_c^2}\right) \phi_+^B(-\omega_1) \phi_+^B(-\omega_2)$$

where $\phi_+^B(-\omega_1)$ is B-meson LCDA (“wave function”)

- Λ_{78} depends on LCDA’s inverse moment λ_B

$$\Lambda_{78}^{\text{spec}}|_{\text{VIA}} = -e_{\text{spec}} \left(1 - \frac{1}{N_c^2}\right) \frac{f_B^2 M_B}{8\lambda_B^2(\mu)} \in e_{\text{spec}} [-386 \text{ MeV}, -35 \text{ MeV}]$$

$$e_{\text{spec}} = \frac{1}{3} \text{ for } \bar{B}^0, \quad -\frac{2}{3} \text{ for } B^-$$

- We need to estimate

$$\Lambda_{78}^{\text{spec}}(\mu) = \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\infty} \frac{d\omega_2}{\omega_2 - i\varepsilon} h_{78}^{(5)}(\omega_1, \omega_2, \mu)$$

- Recall

$$h_{78}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots b(0) \sum_q e_q \bar{q}(r\bar{n}) \cdots q(s\bar{n}) | \bar{B} \rangle$$

- Method II: **Assume** $SU(3)$ flavor symmetry

⇒ $\Lambda_{78}^{\text{spec}}$ is determined by isospin asymmetry (Misiak '09)

$$\Delta_{0-} = \frac{\Gamma(\bar{B}^0 \rightarrow X_s \gamma) - \Gamma(B^- \rightarrow X_s \gamma)}{\Gamma(\bar{B}^0 \rightarrow X_s \gamma) + \Gamma(B^- \rightarrow X_s \gamma)}$$

measured by BaBar to be $\Delta_{0-} = (-1.3 \pm 5.9)\%$

- Including 30% $SU(3)$ flavor breaking gives

$$\Lambda_{78}^{\text{spec}} \approx -4.5 \text{ GeV} (e_{\text{spec}} \pm 0.05) \Delta_{0-}$$

- We need to estimate

$$\Lambda_{88}(\Delta, \mu) = e_s^2 \left[\int_{-\infty}^{\Lambda_{\text{UV}}} \frac{d\omega_1}{\omega_1 + i\varepsilon} \int_{-\infty}^{\Lambda_{\text{UV}}} \frac{d\omega_2}{\omega_2 - i\varepsilon} 2h_{88}^{\text{cut}}(\Delta, \omega_1, \omega_2, \mu) - \frac{C_F}{8\pi^2} \Delta \left(\ln \frac{\Lambda_{\text{UV}}}{\Delta} - 1 \right) \right]$$

- Recall

$$h_{88}(\omega_1, \omega_2) \quad \text{F.T. of} \quad \langle \bar{B} | \bar{b}(0) \cdots s(un) \bar{s}(r\bar{n}) \cdots b(0) | \bar{B} \rangle$$

- We model $\Lambda_{88}(\Delta, \mu)$ by

$$\Lambda_{88}(\Delta, \mu) \approx e_s^2 \Lambda(\mu), \quad \Lambda(\mu) > 0,$$

with $0 < \Lambda(\mu) < 1 \text{ GeV}$

Total Uncertainty

$$\mathcal{F}_E(\Delta) = \frac{C_1(\mu)}{C_{7\gamma}(\mu)} \frac{\Lambda_{17}(m_c^2/m_b, \mu)}{m_b} + \frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)} 4\pi\alpha_s(\mu) \frac{\Lambda_{78}^{\text{spec}}(\mu)}{m_b} \\ + \left(\frac{C_{8g}(\mu)}{C_{7\gamma}(\mu)} \right)^2 \left[4\pi\alpha_s(\mu) \frac{\Lambda_{88}(\Delta, \mu)}{m_b} - \frac{C_F\alpha_s(\mu)}{9\pi} \frac{\Delta}{m_b} \ln \frac{\Delta}{m_s} \right] + \dots,$$

- Using the above values for Λ_{17} and Λ_{88}

$$\mathcal{F}_E|_{17} \in [-1.7, +4.0] \%,$$

$$\mathcal{F}_E|_{88} \in [-0.3, +1.9] \%.$$

- While for Λ_{78} we have

$$\mathcal{F}_E|_{78}^{\text{VIA}} \in [-2.8, -0.3] \% \quad \text{or} \quad \mathcal{F}_E|_{78}^{\text{exp}} \in [-4.4, +5.6] \% \quad (95\% \text{ CL})$$

- “Scanning” over the ranges we have

$$-4.8\% < \mathcal{F}_E(\Delta) < +5.6\% \quad (\text{VIA for } \Lambda_{78}^{\text{spec}})$$

or

$$-6.4\% < \mathcal{F}_E(\Delta) < +11.5\% \quad (\Lambda_{78}^{\text{spec}} \text{ from } \Delta_{0-})$$

- Even if the error on Δ_{0-} was zero

$$-4.0\% < \mathcal{F}_E(\Delta) < +4.8\% \quad (\text{ideal case}).$$

$\Gamma(\bar{B} \rightarrow X_s \gamma)$ in SM

- Experiment

- Experimental value of $\text{Br}(\bar{B} \rightarrow X_s \gamma)$:

Extrapolated from measured $E_\gamma \sim 1.9$ GeV to $E_\gamma > 1.6$ GeV
(HFAG Average '08)

$$\text{Br}(\bar{B} \rightarrow X_s \gamma, E_\gamma > 1.6 \text{ GeV}) = (3.52 \pm 0.25) \cdot 10^{-4} \quad (\text{error } 7\%)$$

- Theory NNLO:

- **OPE**: Assume 1.6 GeV is in the OPE region

(Misiak et. al. '06)

$$\text{Br}(\bar{B} \rightarrow X_s \gamma, E_\gamma > 1.6 \text{ GeV}) = (3.15 \pm 0.23) \cdot 10^{-4} \quad (\text{error } 7\%)$$

- **MSOPE**: 1.6 GeV is still in MSOPE region

(Becher, Neubert '06)

$$\text{Br}(\bar{B} \rightarrow X_s \gamma, E_\gamma > 1.6 \text{ GeV}) = (2.98 \pm 0.26) \cdot 10^{-4} \quad (\text{error } 9\%)$$

- Largest error “non perturbative”: estimated **5%**

based on $Q_{7\gamma} - Q_{8g}$ (Lee, Neubert, GP '06) times 1.5

- Improved numerical estimate based on all $1/m_b$ contributions

(Benzke, Lee, Neubert, GP '10): **5%**

Conclusions and Outlook

Conclusions and Outlook

- New factorization formula for photon spectrum in endpoint region

$$\begin{aligned}
 d\Gamma(\bar{B} \rightarrow X_s \gamma) &= \overbrace{\sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)}}^{\text{known}} \\
 &+ \underbrace{\sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]}_{\text{new}}
 \end{aligned}$$

- New “resolved photon” contributions $\Rightarrow 1/m_b$ corrections to $\Gamma(\bar{B} \rightarrow X_s \gamma)$

- From a systematic study we find

An irreducible error of $\sim 5\%$ on $\Gamma(\bar{B} \rightarrow X_s \gamma)$

- This non perturbative error is the largest

\Rightarrow no prospect for an improvement on the theoretical prediction

- Future directions:

- New contributions to CP asymmetry in $\bar{B} \rightarrow X_s \gamma$
- Effect on the spectrum and the extraction of the HQET parameters