Vanishing Dimensions and Planar Events at the LHC

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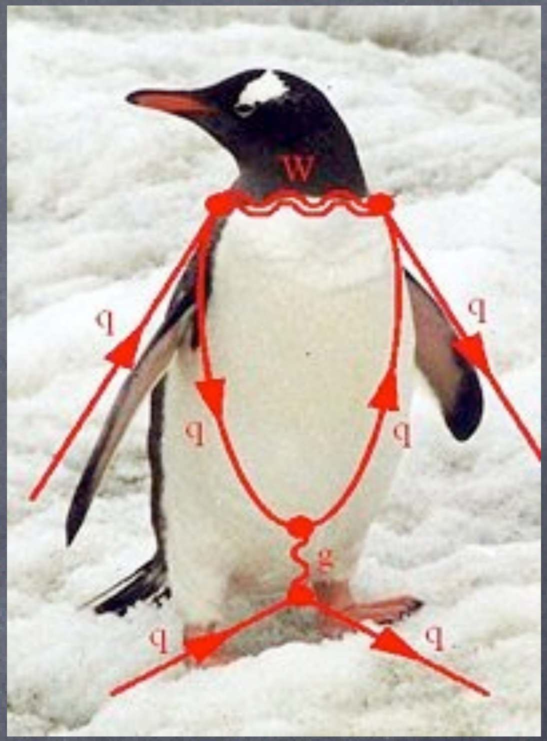
A conceptual new paradigm

The effective dimensionality
of the space we live in depends
on the length scale we are probing

As the length scale increases
new dimensions open up

At short scales space is lower D
at intermediate scales space is 3-D
at large scales space is higher D



LAA, De Chang Dai, Malcolm Fairbairn, Greg Landsberg and Dejan Stojkovic
arXiv:1003.5914

Outline

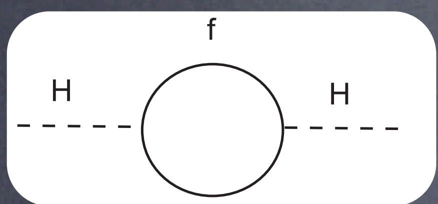
- Motivation
- Simplest example: ordered brane-lattice
- Lorentz violation
- Astrophysical probes: The usual suspects
- Collider signals
- Cosmology on the brane-lattice
- Conclusions

Gauge hierarchy problem

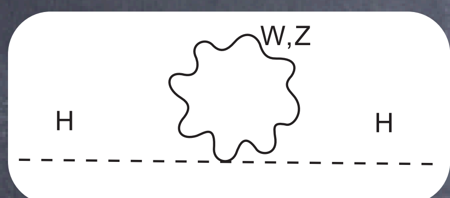
Higgs Lagrangian

$$\mathcal{L}_H = D_\nu \Phi^\dagger D^\nu \Phi - \mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 - \sum_f Y_f \, \Phi \, \bar{\psi}_f \psi_f$$

one-loop corrections to Higgs mass diagrams

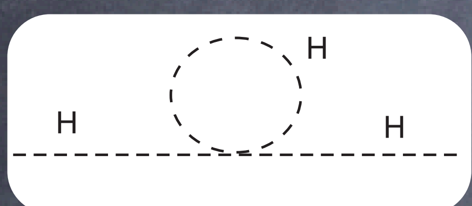


$$i\frac{Y_f^2}{2}\int^\Lambda \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\frac{i}{\not{k} - m_f}\frac{i}{\not{k} + \not{p} - m_f}\right) \sim -\Lambda^2 \, \mathrm{tr}(I_{3+1}) \, \frac{Y_f^2}{32\pi^2}$$



$$i\frac{g^2}{4}\int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_W^2} \sim \Lambda^2 \, \frac{g^2}{64\pi^2}$$

☛ $m_W \to m_Z$; $g^2 \to g^2 + g'^2$



$$i6\lambda \int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_H^2} \sim \Lambda^2 \, \frac{3\lambda}{8\pi^2}$$

summing over color and polarization degrees of freedom

$$\Delta\mu^2 \simeq \frac{3}{16\pi^2 v^2}\left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right)\Lambda^2$$

☛ $m_W^2 = \frac{1}{4}g^2v^2$ $\quad v = 246 \text{ GeV}$ $\quad m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$ $\quad m_t^2 = \frac{1}{2}Y_t^2 v^2$ $\quad m_H^2 = 2\lambda v^2$

☛ $I_{n+1} \equiv$ unit matrix in algebra of Dirac γ matrices of $(n+1)$ Minkowski

Unless m_H is fine-tuned to an accuracy $\mathcal{O}(10^{32})$ ↪
Δ_μ yields dangerous contribution to Higgs VEV destabilizing EW scale

Curing UV divergences through vanishing dimensions

SM works amazingly well by fixing Λ at EW scale

☛ evidence for new particles and laws of nature beyond 1TeV

Alternative approach exercise here: keep SM structure and change dimensionality of background where SM lives

in (2+1) dimensions

$$i\frac{Y_f^2}{2}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\mathrm{tr}\left(\frac{i}{\not k - m_f}\frac{i}{\not k + \not p - m_f}\right) \sim -\Lambda\,\mathrm{tr}(I_{2+1})\frac{Y_f^2}{4\pi^2}$$

$$i\frac{g^2}{4}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - m_W^2} \sim \Lambda\,\frac{g^2}{8\pi^2}$$

$$i6\lambda\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - m_H^2} \sim \Lambda\,\frac{3\lambda}{\pi^2}$$

in (1+1) dimensions

$$i\frac{Y_f^2}{2}\int^{\Lambda}\frac{d^2k}{(2\pi)^2}\mathrm{tr}\left(\frac{i}{\not k - m_f}\frac{i}{\not k + \not p - m_f}\right) \sim -\log(\Lambda/m_f)\,\mathrm{tr}(I_{1+1})\frac{Y_f^2}{4\pi}$$

$$i\frac{g^2}{4}\int^{\Lambda}\frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - m_W^2} \sim \log\left(\frac{\Lambda}{m_W}\right)\frac{g^2}{8\pi}$$

$$i6\lambda\int^{\Lambda}\frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - m_H^2} \sim \log\left(\frac{\Lambda}{m_H}\right)\frac{3\lambda}{\pi}$$

☛ alleviating the fine tuning problem

Let's talk about gravity

- (3+1) gravity is nonlinear and perturbatively non-renormalizable
- In any spacetime Riemann tensor $R_{\mu\nu\rho\sigma}$ may be decomposed into:

Ricci scalar   Ricci tensor   Conformally invariant Weyl tensor

- In 2D Weyl tensor vanishes

$$R_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\alpha}\epsilon_{\rho\sigma\beta}\left(R^{\alpha\beta} + \frac{1}{2}g^{\alpha\beta}R\right)$$

Any solution of vacuum Einstein's equations is locally flat

2D spacetimes have no local gravitational degrees of freedom

no gravitational waves in classical theory and no gravitons in quantum theory

Number of degrees of freedom is finite:

☛ quantum field theory reduces to quantum mechanics

☛ problem of non-renormalizability disappears

- (1+1) gravity is even more simple:

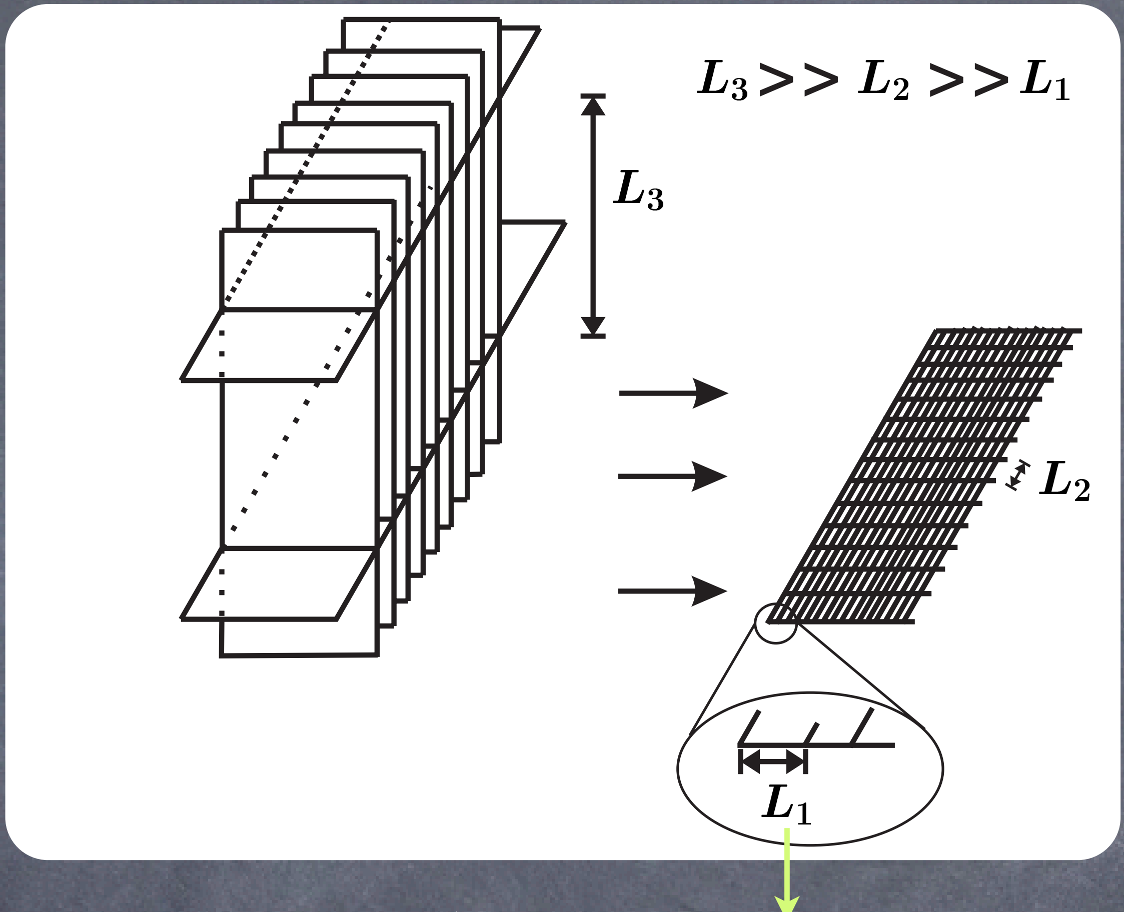
☛ Einstein action in 1D is Euler characteristic of manifold in question

☛ theory is trivial unless augmented by some additional fields

Lattice space(time)

Spacetime has an ordered lattice structure

☛ becomes anisotropic at very small distances



$L_3 = \Lambda_3^{-1}$

$L_2 = \Lambda_2^{-1}$

Fundamental quantization scale of space

Proposed set up is analogous to:

dimensional crossover in layered strongly correlated metals

- Insulating materials in direction perpendicular to layers at high T

☛ but become metal-like at low T

- Transport within layers remains metallic over whole T range

Lorentz violation

- For energies below Λ_3 particles propagate in 3D

Local Lorentz Invariance is preserved

☛ earliest verified landmarks (BBN and CMB) stay unaffected

- When de Broglie wavelength of fundamental particle becomes significantly shorter than L_3 in lattice rest frame

cosmic coordinates

☛ particle propagates locally in 2D rather than 3D

Brane lattice stochastic orientation

non-systematic Lorentz violation

Lorentz violation (cont'd)

- Straight propagation of high-energy gamma rays is not affected
  ☛ overall momentum of particle is preserved
  as it propagates through spatial lattice

- If lattice is rigid enough particle scatters coherently at its junctions
  and move along a jagged line preserving its original direction

- Similar to photon propagation straight through a crystal lattice
  despite being scattered elastically off the individual atoms
  via phonon exchange

  as long as energy of photon is small enough
  so that scattering is elastic
  ☛ propagation of electromagnetic wave through crystal preserves
  group velocity of photon and its direction on scales
  significantly larger than lattice spacing

Lorentz violation (cont'd)

- We now apply this analogy to scattering of a high-energy particle on brane junctions in the lattice
- Tension of brane lattice and relative sizes of sides of its primary cell determine refraction index
- At low energies ☛ refraction index of lattice is equal to unity and particle truly propagates in 3D
- Once energy of particle becomes higher than L_3
  ☛ it propagates via a jagged trajectory
  with degree of jaggedness given by L_2 to L_3 ratio

  which is the effective increase in the path length
- For elastic interactions with brane lattice
  ☛ refraction index for a high-energy particle becomes $1 + \Delta n$

$$\Delta n \sim L_2/L_3 \ll 1$$

- Dispersion relationship in brane lattice is very non-linear and is characterized by Fermi function with threshold $\sim 1/L_3 \sim 1 \text{ TeV}$

Astrophysical probes

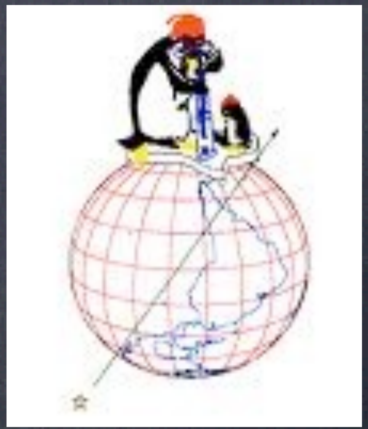
- Combination of threshold-like behavior and smallness of Δn allows us to elude all dispersion-like astrophysical constraints

- FermiLAT MAGIC and H.E.S.S. collaborations assume a dispersion relationship which affects all wavelengths
☛ low energies are used to constrain Lorentz violation at high energies

- For brane lattice dispersion relationship is essentially a Θ function
to see an effect one really has to look at multi-TeV photons
but there are simply not enough of them detected yet

- Measurements are problematic significant absorption of TeV γ rays due to pair production by IR background photons

- "macroscopic" UHECRs ☛ not a good probe
⇓
characteristic lengths > 1 fermi



- Waiting for IceCube detection of GRB's PeV neutrinos

Lattice QCD

✻ In hard scattering process exchanged gluon travels a distance $\sim r$

☛ uncertainty principle provides a lower bound on momentum transfer:

$$Q^2 \sim r^{-2}$$

✻ 3D nature of scattering process breaks down when $Q^2 \sim s > \Lambda_3^2$

☛ this corresponds precisely to impact parameter of QCD scattering $< L_3$

Consider a 2 → 2 scattering in brane-lattice model

❋ If Q^2 of the scattering becomes comparable to Λ_3^2

☛ mediating particle moves in 2D

❋ Effective impact parameter of the interaction is impact parameter in 2D plane defined by local lattice geometry

❋ Cross section of hard scattering processes decreases compared to that in the SM by the reduced phase space

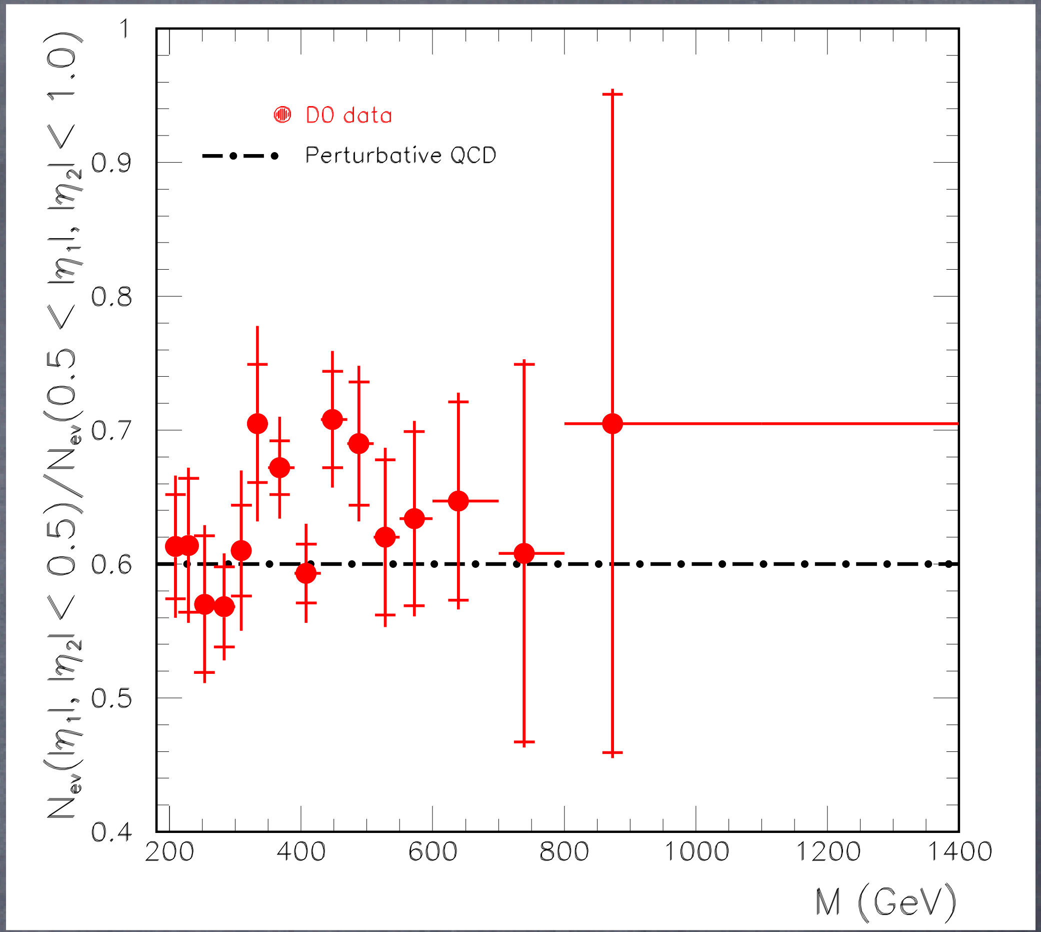
for randomly oriented planes ☛ reduction factor is 1/2

rough estimate as matrix element also changes in 2D

Dijet invariant mass spectra

- QCD cross sections are dominated by t-channel exchanges
  - ➢ dijets at large pseudorapidities $\eta_{1,2} = -\ln[\tan(\theta/2)]$

$$R = \frac{N_{\rm ev}(|\eta_1|, |\eta_2| < 0.5)}{N_{\rm ev}(0.5 < |\eta_1|, |\eta_2| < 1.0)}$$



- For QCD $R \simeq 0.6$
  independent of invariant mass M

D0 Collaboration, PRL 82, 2457 (1999)

- If lattice is randomly oriented ⤵
  its probability distribution is flat in θ
  But careful ☛ momentum should be conserved
  ☛ longitudinal boost of the dijet system
  would mask any planar behavior

Planar multi-jet events at the LHC

Consider a $2 \to 4$ scattering that involves several virtual particles

If Q^2 in each of the propagators is comparable with Λ_3^2
☛ spatial separation between incoming and outgoing particles
at time of interaction is comparable to size of lattice L_3

All virtual particles (propagators) must move in the same 2D space
transverse to the third dimension of lattice L_3

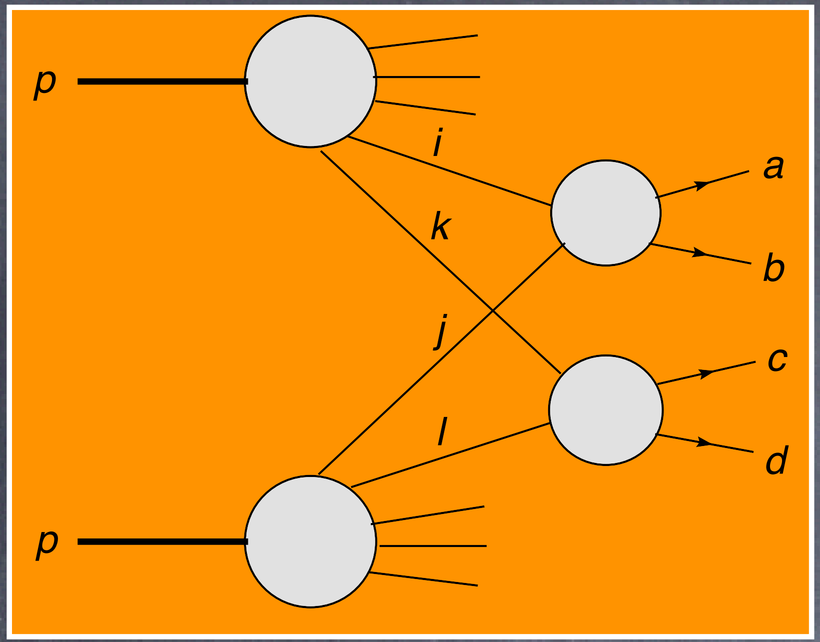
This results in outgoing four partons sharing same plane in c.m. frame
of the collision ☛ drastically different from 3D scattering
where four outgoing partons are in general acoplanar

Due to momentum conservation ☛ entire c.m. frame is boosted to
conserve longitudinal momentum of incoming partons in beam direction
☛ but does not affect initial planar configuration
per argument of photon propagation through lattice

LHC signal of vanishing dimensions

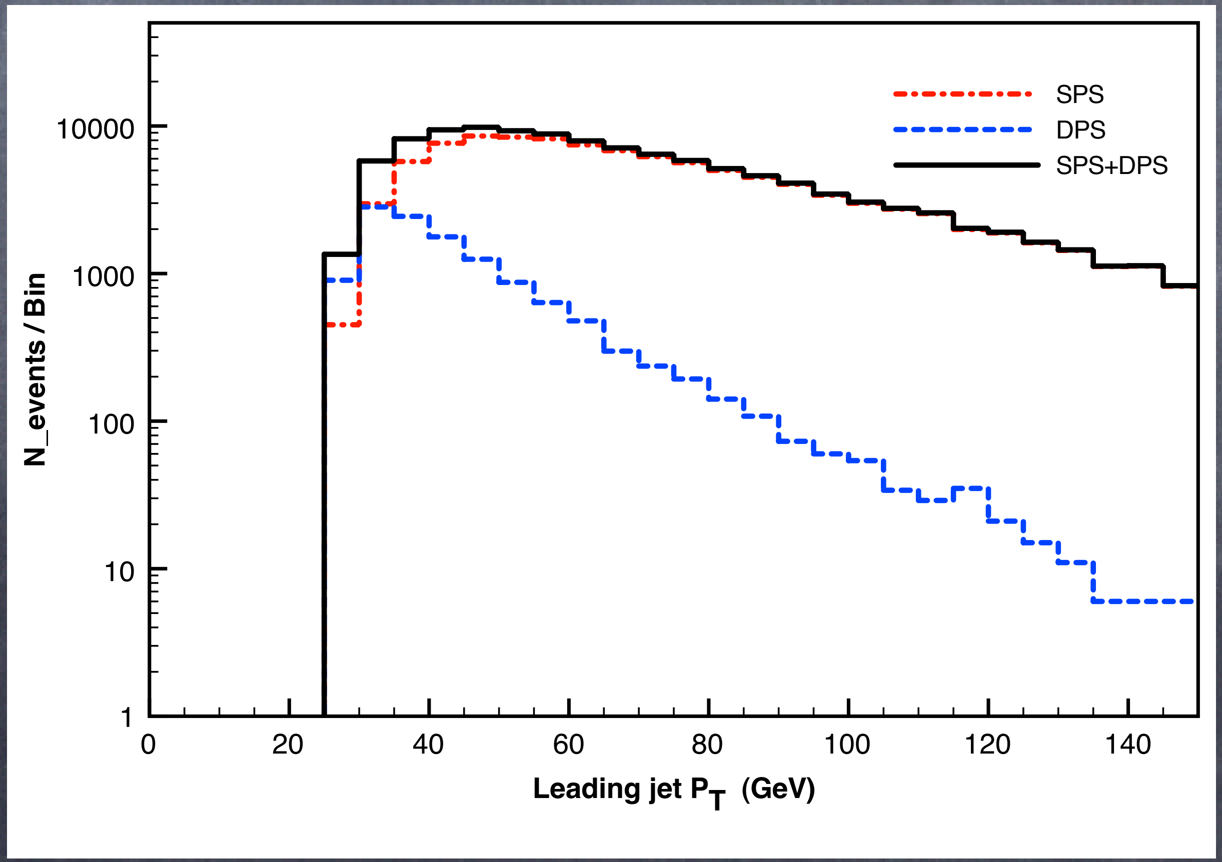
Multijet events with four or more jets at very high transverse momentum
become more and more planar as characteristic Q^2 approaches $\Lambda_3^2 \sim 1\text{TeV}^2$

Double parton scattering



For 4 jet final states
☛ transverse momentum p_T^{j1} of leading jet falls off significantly more rapidly in DPS than SPS

$\sqrt{s} = 10\ \mathrm{TeV}\ \&\ 10\ \mathrm{pb}^{-1}$



Berger, Jackson, Shaughnessy PRD 81, 014014 (2010)

For $\Lambda_3 > 1$ TeV ☛ DPS background is fairly small and can be safely ignored

Elliptic jets at the LHC

* If lattice structure is similar over distances comparable to $1/\Lambda_{\mathrm{QCD}}$
☛ individual jets at very high energy may become elliptic in shape

* This is due to nature of parton shower that is generally ordered in Q^2
☛ one expects largest Q^2 to happen at beginning of shower evolution

* If several successive shower splittings have $Q^2 \sim \Lambda_3^2$
and lattice orientation is preserved over the distance scale
of shower development ☛ the core of the jet will become planar

* After soft part of parton shower is finished
the resulting jets will be elliptic rather than round in shape

* It is not clear if this ellipticity will be large enough to be observable
particularly given fluctuations of parton shower within individual jets

☛ nevertheless we believe that looking at the individual jet ellipticity
as a function of jet energy may become
a striking experimental probe of models with vanishing dimensions

Lattice cosmology

- Increasing dimensionality of space-time at large distances would have some consequences for cosmology
- Assume that for distances $> L_4$ space-time becomes (4+1)D

  Vacuum solution $G_{AB} \equiv R_{AB} - \frac{1}{2} g_{AB} R = 0 \quad (A, B = 0, ...4)$

$$ds^2 = dt^2 - e^{2\sqrt{\Lambda/3t}}(dr^2 + r^2 d\Omega^2) - d\psi^2$$

$$\Lambda = 3/\psi^2$$

Ponce De Leon, GRG 20, 539 (1988)

- This metric reduces on ψ = constant hypersurfaces to 3D de Sitter metric with Λ = constant
- Observers from 3D lattice measure effective stress energy tensor with equation of state $p = -\rho$

  $\hookrightarrow \rho = \Lambda \bar{M}_{\rm Pl}$
- Observed vacuum energy density $\rho \approx (2.4 \times 10^{-3} \text{ eV})^4$

  corresponds to $\psi \approx 10^{61} M_{\rm Pl}^{-1} \approx 10^{26}$ m

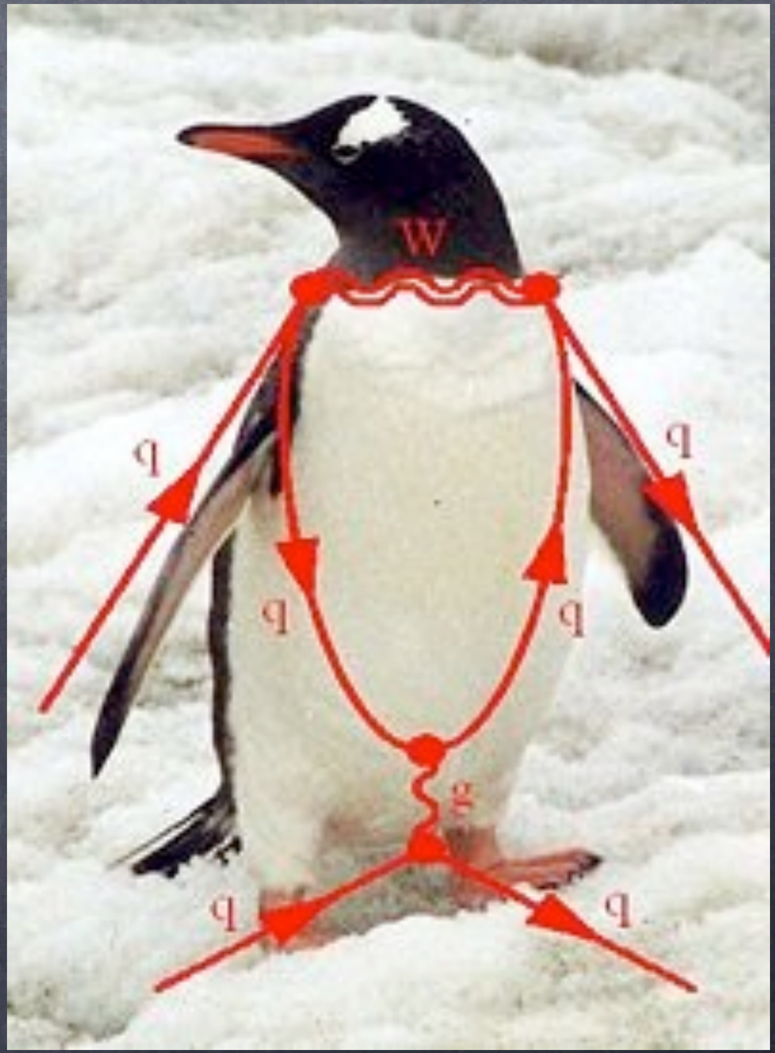
comparable to current horizon size ☛ comparable to characteristic distance between 3D sheets comprising 4D lattice structure

- If L_4 is of this scale ☛ represents minimum value of ψ maximum value of effective Λ experienced by observer on 3D sheet

It therefore sets the cosmological constant problem in a new context

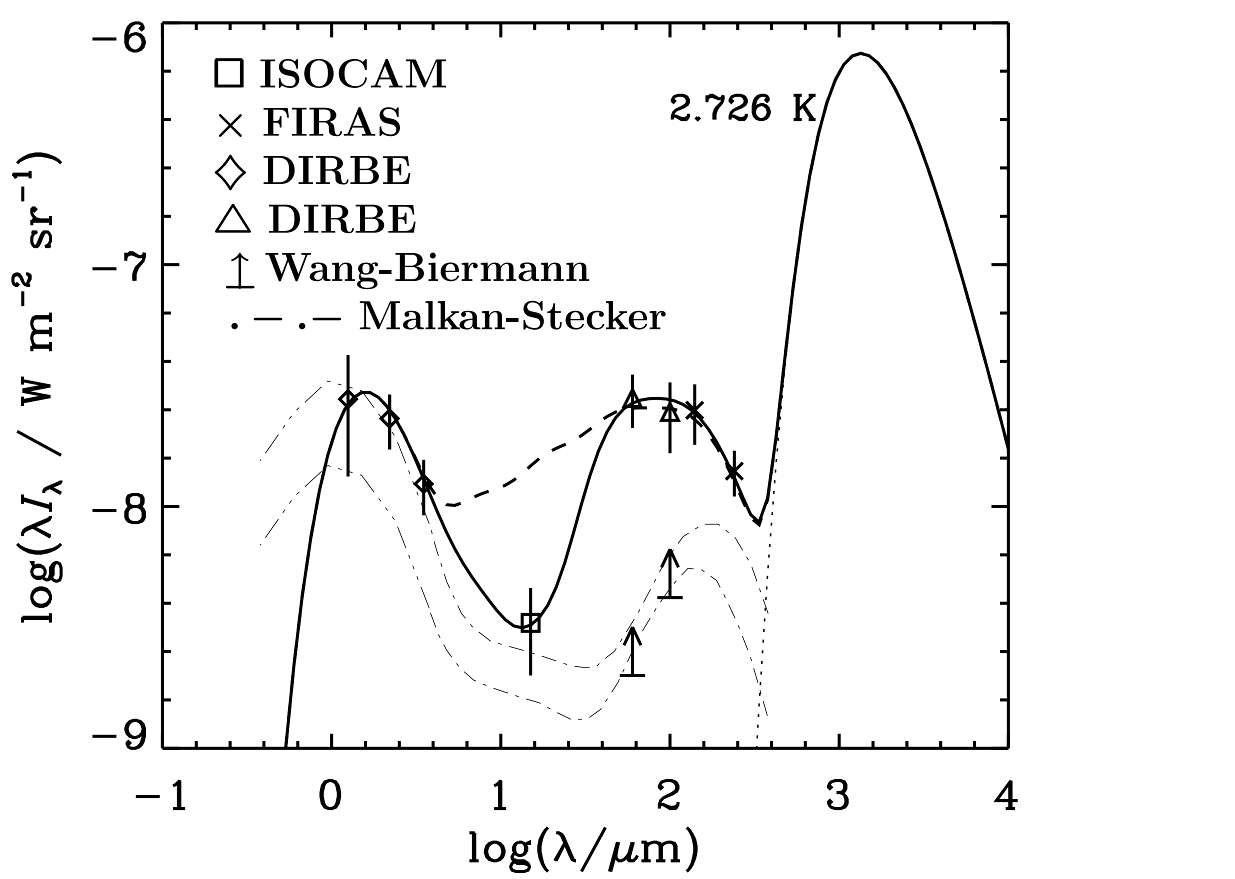
Conclusions

With the LHC just having achieved high-energy collisions at the half of the design energy and preparing for a decade of exciting explorations at the Terascale ☛ it's very timely to bring the paradigm of vanishing dimensions to the attention of the experimental and theoretical communities

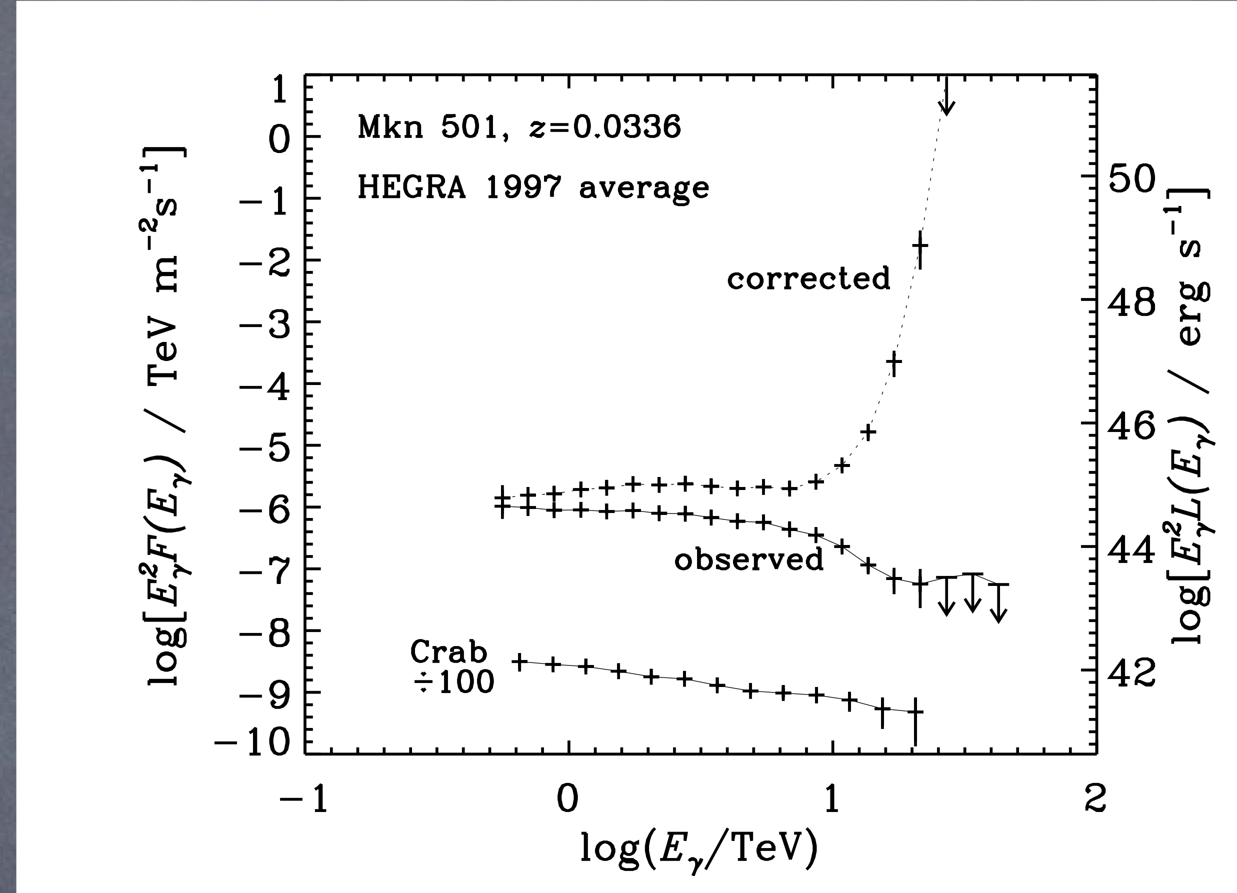


Epilogue

IR background - TeV γ ray crisis ☛ evidence of vanishing dimensions ?



_ _ ISOCAM taken as lower limit



observed spectrum luminosity

Protheroe, Meyer, PLB 493, 1 (2000)

Crisis exacerbated by H.E.S.S. measurements of blazars at $0.165 \leq z \leq 0.186$

H.E.S.S. Collaboration, Nature 440, 1018 (2006)

IR measurements by AKARI satellite consistent with COBE/DIRBE results

Matsuura et al, arXiv:1002.3674

Cross section suppression above Λ_3 enlarges $\gamma\gamma$ mean free path