Higher Twist Scaling Violations

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NPAC

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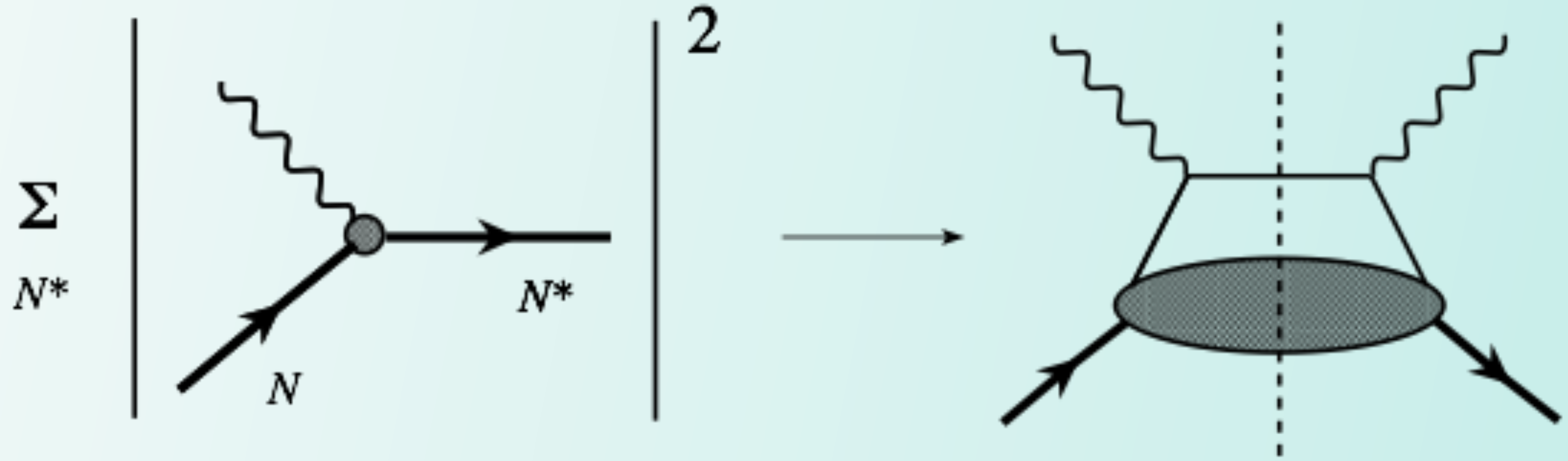
Outline

- Twist Expansion and Leading Twist:
- Motivation
- The Higher Twist
- Concluding Remarks

The Twist Expansion:



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4}\frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$$

$$\boxed{\mathrm{T}\{J_\mu(x)J_\nu(x)\} \sim \Gamma_{\mu\nu} \sum_{n,k} x^{\mu_1}\ldots x^{\mu_n} C_k^{(n)}(x^2)\mathcal{O}^{(n)}_{k,\mu_1\ldots\mu_n}(0)}$$

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The Cornwall-Norton Moment:

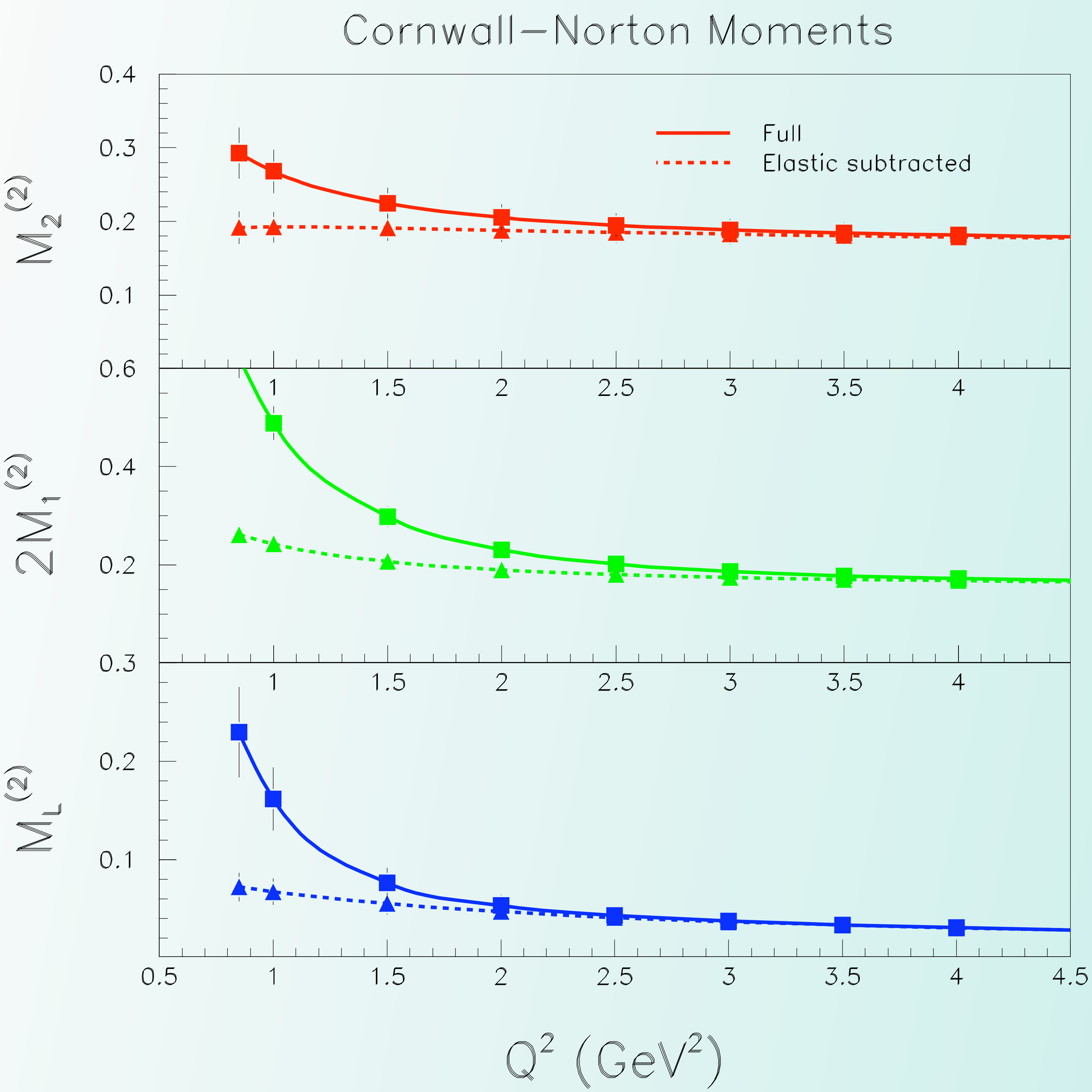
The moment of a structure function is a x-Bjorken weighted integral:

$$M_1^{(n)}(Q^2) = \int_0^1 dx_B \;\; x_B^{n-2} F_2(x_B, Q^2)$$

The moment can be computed using the momentum-space OPE:

$$\boxed{M_n(Q^2, g, \mu) = \int_0^1 dx_B \; x_B^{n-2} F_2(Q^2, g, \mu) \approx \sum_k \left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}} \tilde{c}_k^n(g, \mu) \; A_k^{(n)}}$$

Leading Moment data for F2: (Liang *et al.* JLAB Hall B - CLAS Collaboration)



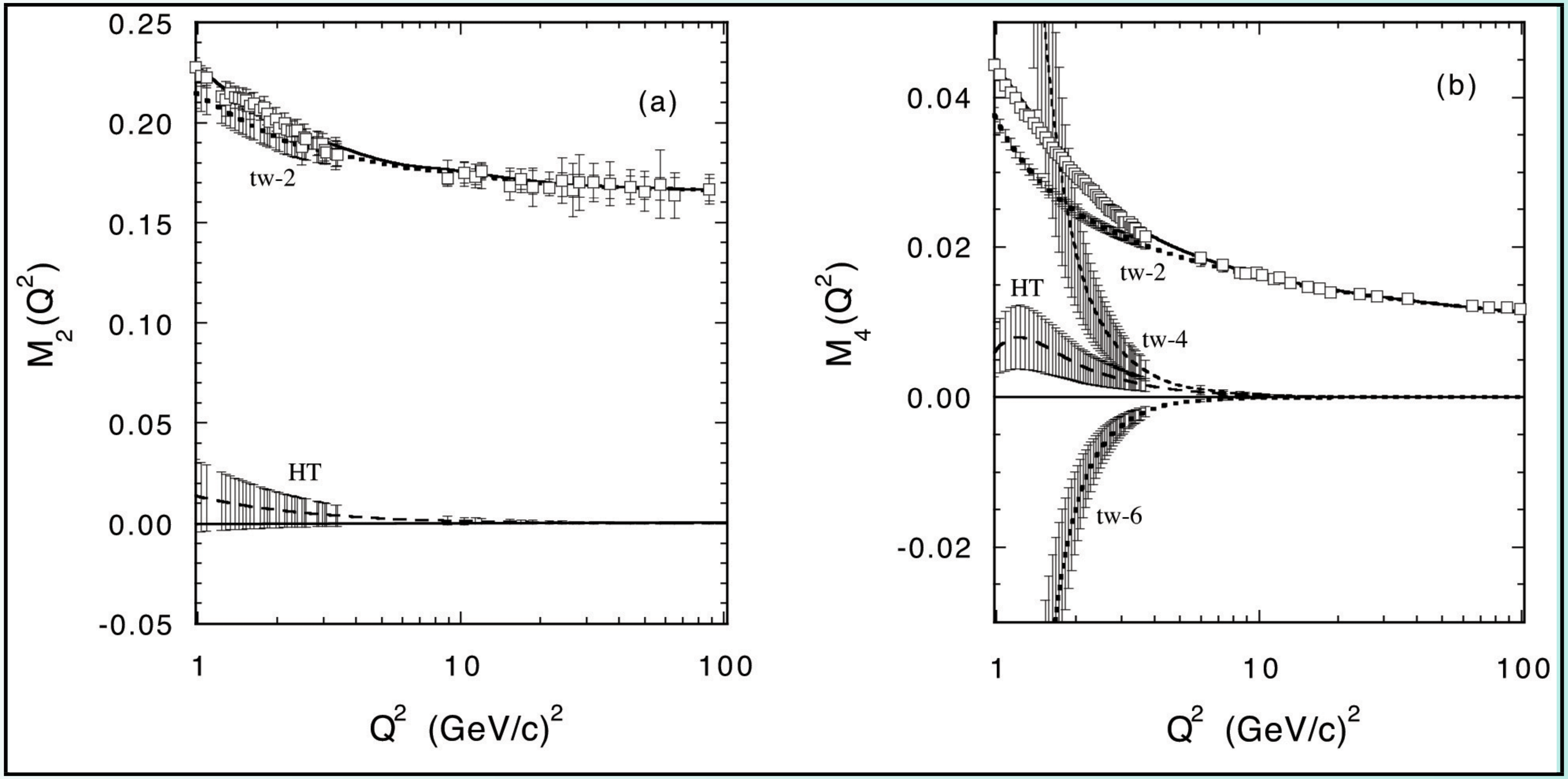
$$F_2^{\mathrm{EL}} = \frac{(G_E^2 + \tau G_M^2)\delta(x-1)}{1+\tau}$$

$$F_1^{\mathrm{EL}} = G_M^2\delta(x-1) \qquad \tau = \frac{q^2}{4M_p^2}$$

$$F_L^{\mathrm{EL}} = G_E^2\delta(x-1)$$

Puzzle: Where are the power corrections in the moment?

Cancellation of Higher Twist? (Liang *et al.* JLAB Hall C - CLAS Collaboration)



$$M_n(Q^2) = \eta_n(Q^2) + a_n^{(4)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(4)}} \frac{\mu^2}{Q^2} + a_n^{(6)} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(6)}} \frac{\mu^4}{Q^4}$$

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Twist-4 Operator Basis (R.L. Jaffe & M. Soldate - Phys. Rev. D V26 No.1)

- At twist-4, a “canonical” basis of operators has been proposed by Jaffe & Soldate:

- Twist-4 Operators must satisfy three conditions:
  a) Totally Symmetric
  b) Traceless
  c) Contain no contracted derivatives

- The first two conditions project out the highest spin portion of the operator.

- The last condition ensures the basis is not over-complete. These operators can be eliminated via QCD equations of motion.

The Operator Basis:

- 14 Operators appear at twist-4, which can be divided into two groups: 6 four-quark operators, 8 two-quark operators, and pure-gluonic operators:
- Operators of the following form appear at twist-4:

$$\triangle \cdot Q_n^{1(k,l)} = g[\bar{\psi}_R \triangle \overleftarrow{d}^{\,l} \overrightarrow{d}^{\,k} \psi_R][\bar{\psi}_R \triangle \overrightarrow{d}^{\,n-2-k-l} \psi_R]$$

$$\triangle \cdot Q_n^{8(k)} = i\bar{\psi} \overleftarrow{d}^{\,k} f \overrightarrow{d}^{\,n-1-k} \psi$$

$$\triangle \cdot G_n^{(k,l)} = \mathrm{Tr}[f_\alpha \overrightarrow{d}^{\,n-k-l} f^\alpha \overrightarrow{d}^{\,k} f_\beta \overrightarrow{d}^{\,l} f^\beta]$$

- Notation:

$$\triangle \cdot \mathcal{O}_n \equiv \triangle^{\mu_1} \ldots \triangle^{\mu_n} \mathcal{O}_{n,\mu_1 \ldots \mu_n} \; ; \qquad \triangle^2 = 0$$

$$d \equiv \triangle \cdot D$$

$$f^\beta \equiv F^{\rho\beta} \triangle_\rho$$

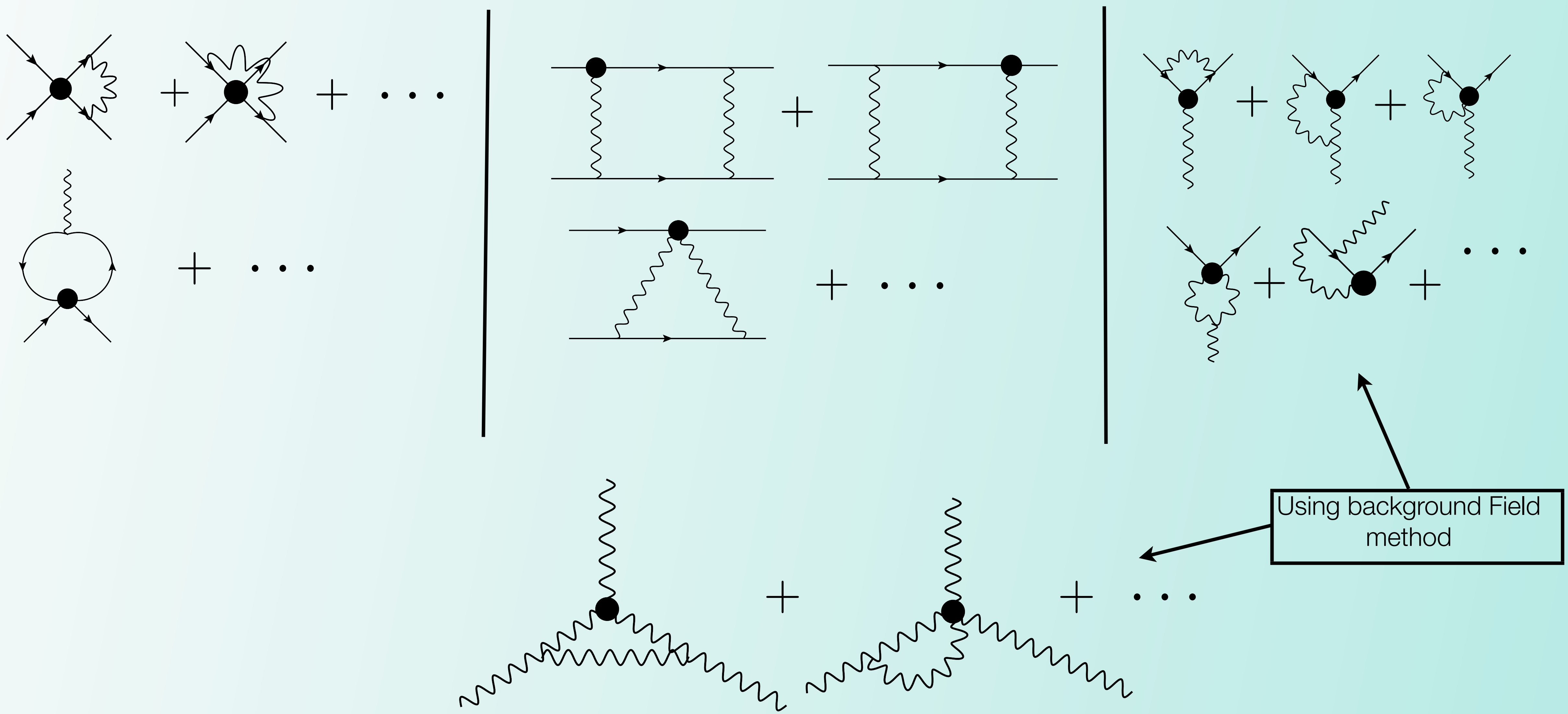
The Complete Set of Operators:

$$\Delta\cdot Q_n^{1(k,l)} = g\bar{\psi}_R \not{\Delta} \overleftarrow{d}^l \overrightarrow{d}^k \psi_R \bar{\psi}_R \not{\Delta} \overrightarrow{d}^{n-2-k-l} \psi_R ,$$
$$\Delta\cdot Q_n^{2(k,l)} = g\bar{\psi}_R \tau_a \not{\Delta} \overleftarrow{d}^l \overrightarrow{d}^k \psi_R \bar{\psi}_R \not{\Delta} \overrightarrow{d}^{n-2-k-l} \tau_a \psi_R ,$$
$$\Delta\cdot Q_n^{3(k,l)} = g\bar{\psi}_R \not{\Delta} \overleftarrow{d}^l \overrightarrow{d}^k \psi_R \bar{\psi}_L \not{\Delta} \overrightarrow{d}^{n-2-k-l} \psi_L ,$$
$$\Delta\cdot Q_n^{4(k,l)} = g\bar{\psi}_R \tau_a \not{\Delta} \overleftarrow{d}^l \overrightarrow{d}^k \psi_R \bar{\psi}_L \not{\Delta} \overrightarrow{d}^{n-2-k-l} \tau_a \psi_L ,$$
$$\Delta\cdot Q_n^{5(k,l)} = g\bar{\psi}_L \not{\Delta} \overleftarrow{d}^l \overrightarrow{d}^k \psi_L \bar{\psi}_L \not{\Delta} \overrightarrow{d}^{n-2-k-l} \psi_L ,$$
$$\Delta\cdot Q_n^{6(k,l)} = g\bar{\psi}_L \tau_a \not{\Delta} \overleftarrow{d}^l \overrightarrow{d}^k \psi_L \bar{\psi}_L \not{\Delta} \overrightarrow{d}^{n-2-k-l} \tau_a \psi_L ,$$
$$\Delta\cdot Q_n^{7(k)} = \bar{\psi} \overleftarrow{d}^k {}^* \not{f} \gamma_5 \overrightarrow{d}^{n-1-k} \psi ,$$
$$\Delta\cdot Q_n^{8(k)} = i\bar{\psi} \overleftarrow{d}^k \not{f} \overrightarrow{d}^{n-1-k} \psi ,$$
$$\Delta\cdot Q_n^{9(k,l)} = g\bar{\psi} \overleftarrow{d}^k f_a^{\alpha} (\overrightarrow{d}^l f_\alpha)_a \overrightarrow{d}^{n-3-k-l} \not{\Delta} \psi ,$$
$$\Delta\cdot Q_n^{10(k,l)} = igf_{abc} \bar{\psi} \overleftarrow{d}^k f_a^{\alpha} (\overrightarrow{d}^l f_\alpha)_b \overrightarrow{d}^{n-3-k-l} \not{\Delta} \tau_c \psi ,$$
$$\Delta\cdot Q_n^{11(k,l)} = gd_{abc} \bar{\psi} \overleftarrow{d}^k f_a^{\alpha} (\overrightarrow{d}^l f_\alpha)_b \overrightarrow{d}^{n-3-k-l} \not{\Delta} \tau_c \psi ,$$
$$\Delta\cdot Q_n^{12(k,l)} = ig\bar{\psi} \overleftarrow{d}^k {}^* f_a^{\alpha} (\overrightarrow{d}^l f_\alpha)_a \overrightarrow{d}^{n-3-k-l} \not{\Delta} \gamma_5 \psi ,$$
$$\Delta\cdot Q_n^{13(k,l)} = gf_{abc} \bar{\psi} \overleftarrow{d}^k {}^* f_a^{\alpha} (\overrightarrow{d}^l f_\alpha)_b \overrightarrow{d}^{n-3-k-l} \not{\Delta} \gamma_5 \tau_c \psi ,$$
$$\Delta\cdot Q_n^{14(k,l)} = igd_{abc} \bar{\psi} \overleftarrow{d}^k {}^* f_a^{\alpha} (\overrightarrow{d}^l f_\alpha)_b \overrightarrow{d}^{n-3-k-l} \not{\Delta} \gamma_5 \tau_c \psi .$$

$$
\begin{aligned}
\Delta\cdot O_n^{G1a(k,\ell)} &= \mathrm{Tr}[f_\alpha \overrightarrow{d}^{n-k-\ell} f^\alpha] \mathrm{Tr}[\overrightarrow{d}^k f_\beta \overrightarrow{d}^\ell f^\beta] \\
\Delta\cdot O_n^{G1b(k,\ell)} &= \mathrm{Tr}[f_\alpha \overrightarrow{d}^{n-k-\ell} f^\alpha \overrightarrow{d}^k f_\beta \overrightarrow{d}^\ell f^\beta] \\
\Delta\cdot O_n^{G2(k,\ell)} &= \mathrm{Tr}[f_\alpha \overrightarrow{d}^{n-k-\ell} f^\alpha \overrightarrow{d}^k f_\beta \overrightarrow{d}^\ell f^\beta] \\
\Delta\cdot O_n^{G3} &= \mathrm{Tr}[G^{\alpha\beta} \overrightarrow{d}^n G_{\alpha\beta}]
\end{aligned}
$$

The Anomalous Dimension For Twist 4:

- In order to RG evolve the moments of the structure functions, we must compute the one-loop corrections to each operator



Preliminary Results:

$$\gamma \sim \begin{pmatrix} {\color{blue}\text{Quark}} & \text{Quark} \to \text{Glue} \\ \text{Glue} \to \text{Quark} & \text{Glue} \end{pmatrix}$$

$$\gamma_{\color{blue}\text{Quark}} \sim \begin{pmatrix}
-\frac{g^2}{6\pi^2\epsilon} & -\frac{g^2}{8\pi^2\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{g^2}{36\pi^2\epsilon} & -\frac{7g^2}{72\pi^2\epsilon} & 0 & \frac{g^2}{18\pi^2\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{g^2}{6\pi^2\epsilon} & -\frac{g^2}{8\pi^2\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{g^2}{48\pi^2\epsilon} & \frac{g^2}{36\pi^2\epsilon} & \frac{g^2}{48\pi^2\epsilon} & 0 & -\frac{g^2}{36\pi^2\epsilon} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{g^2}{6\pi^2\epsilon} & -\frac{g^2}{8\pi^2\epsilon} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{g^2}{18\pi^2\epsilon} & -\frac{g^2}{36\pi^2\epsilon} & -\frac{g^2}{72\pi^2\epsilon} & 0 & 0 & 0 & 0 \\
\frac{65g^2}{96\pi^2\epsilon} & \frac{19g^2}{48\pi^2\epsilon} & \frac{485g^2}{432\pi^2\epsilon} & \frac{31g^2}{36\pi^2\epsilon} & \frac{163g^2}{288\pi^2\epsilon} & \frac{29g^2}{48\pi^2\epsilon} & \frac{239g^2}{576\pi^2\epsilon} & \frac{83g^2}{288\pi^2\epsilon} & -\frac{11g^2}{144\pi^2\epsilon} & \frac{11g^2}{144\pi^2\epsilon} \\
\frac{25g^2}{144\pi^2\epsilon} & \frac{47g^2}{576\pi^2\epsilon} & \frac{25g^2}{216\pi^2\epsilon} & \frac{67g^2}{288\pi^2\epsilon} & \frac{g^2}{16\pi^2\epsilon} & \frac{167g^2}{576\pi^2\epsilon} & -\frac{19g^2}{96\pi^2\epsilon} & -\frac{671g^2}{1152\pi^2\epsilon} & \frac{5g^2}{144\pi^2\epsilon} & -\frac{97g^2}{576\pi^2\epsilon} \\
-\frac{181g^2}{864\pi^2\epsilon} & \frac{59g^2}{576\pi^2\epsilon} & -\frac{181g^2}{432\pi^2\epsilon} & \frac{77g^2}{288\pi^2\epsilon} & -\frac{181g^2}{864\pi^2\epsilon} & \frac{59g^2}{576\pi^2\epsilon} & -\frac{11g^2}{192\pi^2\epsilon} & -\frac{g^2}{576\pi^2\epsilon} & \frac{175g^2}{288\pi^2\epsilon} & -\frac{g^2}{288\pi^2\epsilon} \\
\frac{97g^2}{432\pi^2\epsilon} & -\frac{55g^2}{576\pi^2\epsilon} & \frac{101g^2}{216\pi^2\epsilon} & -\frac{5g^2}{32\pi^2\epsilon} & \frac{97g^2}{432\pi^2\epsilon} & -\frac{55g^2}{576\pi^2\epsilon} & -\frac{g^2}{64\pi^2\epsilon} & -\frac{41g^2}{576\pi^2\epsilon} & \frac{35g^2}{144\pi^2\epsilon} & \frac{175g^2}{288\pi^2\epsilon}
\end{pmatrix}
\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7^{k=0} \\ Q_7^{k=1} \\ Q_8^{k=0} \\ Q_8^{k=1} \end{pmatrix}$$

$$\gamma_{\text{Glue}} \sim \begin{pmatrix} \frac{g^2}{3\pi^2\epsilon} & -\frac{g^2}{\pi^2\epsilon} \\ 0 & \frac{31g^2}{64\pi^2\epsilon} \end{pmatrix} \begin{pmatrix} \mathcal{G}_2^{2(1,1)} \\ \mathcal{G}_2^{3(1,1)} \end{pmatrix}$$

Concluding Remarks:

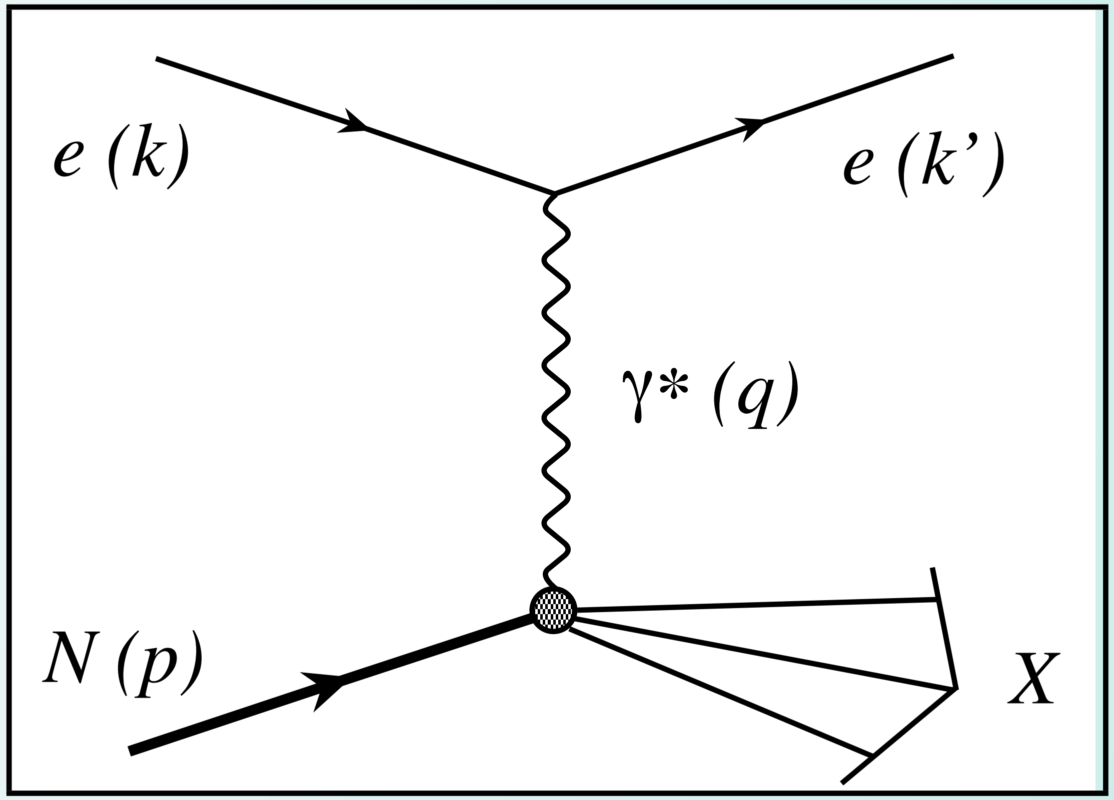
- DGLAP evolution equations can be incorporated by leading twist, but there is no DGLAP analog for higher twist, we expect power corrections to scaling at lower Q^2.

- At twist-4, these scaling violations include involved mixings among many operators.

- We hope to extend this analysis to arbitrary spin, employing a technique called non-local operator renormalization.

Thanks!

Kinematics of Deep Inelastic Scattering:



$$\nu = E - E'$$

$$Q^2 \equiv -q^2$$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\pi\alpha^2}{MQ^4}\left\{W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right\}$$

$$\left.\begin{array}{l} Q^2 \to \infty \;,\; \nu \to \infty \\ x_B = \dfrac{Q^2}{2M\nu} \end{array}\right\} \longrightarrow \begin{array}{r} 2MW_1(Q^2,\nu) \to F_1(x_B, Q^2) \\ \nu W_2(Q^2,\nu) \to F_2(x_B, Q^2) \end{array}$$

$$F_2(x) = 2xF_1(x) = x\sum_q e_q^2(q(x) + \bar{q}(x))$$

Physics on the Light Cone:

DIS corresponds to the kinematic limit: $\nu, Q^2 \to \infty, x_B = \frac{Q^2}{2P \cdot q}$ fixed.

In this limit, the forward Compton amplitude

$$T_{\mu\nu} = i \int d^4x e^{iq \cdot x} < p|T(J_\mu(x) J_\nu(0))|p >$$

Receives its greatest contributions from the light-cone, where:

$$x^2 \sim 0$$

Introduction to the OPE:

In general, time ordered products of operators at the same space-time point, contain singularities:

$$< 0|T(\phi(x)\phi(0))|0 >= -\frac{1}{4\pi}\delta(x^2) + \frac{m}{8\pi\sqrt{x^2}}\theta(x^2)\left[J_1(m\sqrt{x^2}) - iN_1(m\sqrt{x^2})\right]$$

Wilson hypothesized that this product can be expanded in a basis of operators:

$$\boxed{T\{J_\mu(x)J_\nu(x)\} \sim \Gamma_{\mu\nu}\Sigma_{n,k}x^{\mu_1}\dots x^{\mu_n}C_k^{(n)}(x^2)\mathcal{O}^{(n)}_{k,\mu_1\dots\mu_n}(0)}$$

It is useful to systematically organize these operators based on their scaling behavior

Twist Expansion

The scaling of operators is controlled by their twist.

Fourier transforming the OPE:

$$T\{J_\mu(x)J_\nu(x)\} \sim \Gamma_{\mu\nu}\Sigma_{n,k}x^{\mu_1}\dots x^{\mu_n}C_k^{(n)}(x^2)\mathcal{O}^{(n)}_{k,\mu_1\dots\mu_n}(0)$$

$$\tilde{C}_k^{(n)}(Q^2,g,\mu) = Q^{2n}\left(\frac{\partial}{\partial q^2}\right)^n \int \frac{d^4x}{(2\pi)^4}e^{iq\cdot x}C_k^{(n)}(x^2,g,\mu)$$

$$< p|\mathcal{O}^{(n)}_{k,\mu_1\dots\mu_n}(0)|p > \sim A_k^{(n)}p^{\mu_1}\dots p^{\mu_n} - \texttt{traces}$$

$$\boxed{T_{\mu\nu}(Q^2,\nu) = \Gamma_{\mu\nu}\sum_{k,n}(2i)^n\left(\frac{1}{x_B}\right)^n\left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}}\tilde{c}_k^{(n)}A_k^{(n)}}$$

"Twist" of an Operator:

To do this, we compare the mass dimension on both sides of the OPE:

$$T\{J_\mu(x)J_\nu(x)\} \sim \Gamma_{\mu\nu}\Sigma_{n,k}x^{\mu_1}\ldots x^{\mu_n}C_k^{(n)}(x^2)\mathcal{O}^{(n)}_{k,\mu_1\ldots\mu_n}(0)$$

$$\text{LHS} \to \left(\frac{1}{x^2}\right)^{\frac{1}{2}(2d_J)} \qquad \text{RHS} \to \left(\frac{1}{x^2}\right)^{\frac{1}{2}(-n+d_C+d_\mathcal{O})}$$

Solving for the dimension of the coefficient function:

$$[C] = \left(\frac{1}{x^2}\right)^{\frac{1}{2}(2d_J-\tau)}$$

We define the "twist" of an operator as: $\tau = d_\mathcal{O} - n$

Momentum Sum Rule:

Using Lorentz invariance and gauge invariance, one can write down a general expression for the hadronic tensor in DIS:

$$W^{\mu\nu} = W_1(\nu, Q^2)\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right) + \frac{W_2(\nu, Q^2)}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

This expression of the hadronic tensor allows us to relate the forward Compton amplitude to the structure functions W1 and W2.

Operator Scaling:

To determine the behavior of the moments at various values of Q^2, one must solve a Callan-Symanzik equation for the coefficient functions:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g}\right) C_{ij}^{(n)} = \sum_k \gamma_{jk}^{(n)} C_{ik}^{(n)}$$

$$\gamma_{jk}^{(n)} = \mu\frac{\partial}{\partial\mu}\ln Z_{jk}^{(n)}$$

$$\boxed{M_n(Q^2) \approx \sum_i \left(\frac{1}{Q^2}\right)^{\frac{\tau-2}{2}} \tilde{c}_j^n(Q^2, g(t), \mu) exp\left[-\int_0^t \gamma_{ij}^{(n)}(\bar{g}(t'))dt'\right] A_i^n}$$

Light-Cone Backup Slide:

$$q \cdot x = \tfrac{1}{2}(q_0 + q_3)(x_0 + x_3) + \tfrac{1}{2}(q_0 - q_3)(x_0 + x_3) - \vec{q}_T \cdot \vec{x}_T$$

$$p_\mu = (m_p, 0, 0, 0)$$

$$q_\mu = (\nu, 0, 0, (\nu^2 - q^2)^{1/2})$$

$$Q^2, \nu \to \infty$$

$$q \cdot x \sim \mathcal{O}(1)$$

$$x_0 + x_3 \sim \tfrac{\nu}{q^2} = \tfrac{1}{m x_B}$$

$$(x_0 + x_3)(x_0 - x_3) \sim \tfrac{1}{\nu}\tfrac{\nu}{q^2} = \tfrac{1}{q^2}$$

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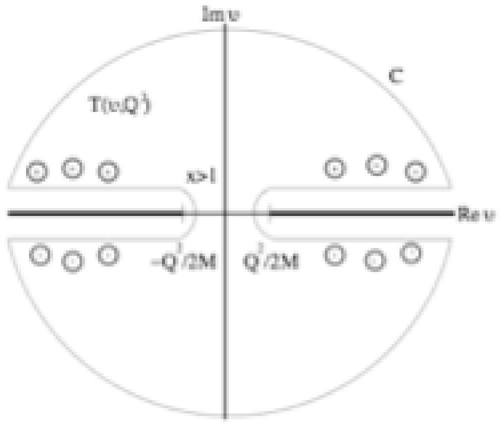
Analytic Continuation Backup (Momentum Sum Rule):



$$\tilde{T}_{\mu\nu}(q^2, \nu) = \sum_{i,j} \left(\tfrac{\nu}{Q^2}^{j}\right)\left(\tfrac{1}{Q^2}\right)^{\frac{1}{2}(\tau-2)} \tilde{c}_{i,j} a_{i,j}$$

OPE Region: $Q^2 >> \nu \to x_B > 1$ unphysical

$$I_n = \int \frac{d\nu}{(2\pi i)} \frac{1}{\nu^{n-2}} \left[\frac{W_2(\nu, Q^2)}{\nu}\right]$$

$$W_2 = \sum_f Q_f^2 \sum_n \tfrac{8}{Q^2} \tfrac{(2q \cdot P)^{n-2}}{(Q^2)^{(n-2)}} A_f^n$$

$$I_n = \sum_f Q_f^2 \tfrac{8}{(Q^2)^{n-1}} A_f^n$$

2nd contour $\longrightarrow$ $\frac{8}{(Q^2)^{n-1}} \int_0^1 dx x^{n-2} \frac{\nu}{4\pi} \mathrm{Im} W_2$

$$\boxed{\int_0^1 dx x^{n-1} f_i(x, Q^2) \sim A_f^n}$$

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RGE for Coefficient Functions:

$$A(x)B(0) \sim \sum_i C_i(x,g,\mu)\mathcal{O}_i$$

$$\Gamma_{AB} =< 0|T(A(x)B(0)\prod_k \phi_k(y_k))|0 > \qquad \Gamma_{\mathcal{O}_i} =< 0|T(\mathcal{O}_i)\prod_k \phi_k(y_k)|0 >$$

$$\left[D + \sum_k \gamma_k(g) - \gamma_A(g) - \gamma_B(g)\right]\Gamma_{AB}(g,\mu) = 0 \qquad \left[D + \sum_k \gamma_k(g) - \gamma_i(g)\right]\Gamma_{\mathcal{O}_i}(g,\mu) = 0$$

$$D = \mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}$$

$$[D + \gamma_A(g) + \gamma_B(g) - \gamma_i(g)]\, C_i(x,g,\mu) = 0$$

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DGLAP Equations:

$$P_{qq}(z) = \tfrac{4}{3}\left(\tfrac{1+z^2}{(1-z)_+} + \tfrac{3}{2}\delta(1-z)\right) \qquad \int_0^1 dx\,\tfrac{f(x)}{(1-x)_+} = \int_0^1 dx\,\tfrac{f(x)-f(1)}{(1-x)}$$

$$P_{gq}(z) = \tfrac{4}{3}\left(\tfrac{1+(1-z)^2}{z}\right)$$

$$P_{qg}(z) = \tfrac{1}{2}(z^2 + (1-z)^2)$$

$$P_{gg}(z) = 6\left(\tfrac{(1-z)}{z} + \tfrac{z}{(1-z)_+} + z(1-z) + \left(\tfrac{11}{12} - \tfrac{n_f}{18}\right)\delta(1-z)\right)$$

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Leading Twist and DGLAP:

- The scaling behavior of the parton distributions functions in QCD can be computed by solving the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations.

$$\longrightarrow \quad P_{qq}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+^2} + \frac{3}{2}\delta(1-z)\right]$$

$$\frac{d}{d\log Q} f_f(x,Q) = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{dz}{z}\left\{P_{q\leftarrow q}(z) f_f(\frac{x}{z},Q) + P_{q\leftarrow g}(z) f_g(\frac{x}{z},Q)\right\}$$

- The DGLAP equations in QCD predict the scaling behavior of the PDFs. Is there a correspondence between DGLAP and the twist expansion?

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Leading Twist and DGLAP:

- Defining the following moments and using the DGLAP equation:

$$f_f^-(x) = f_f(x) - f_{\bar{f}}(x) \longrightarrow M_{fn}^- = \int_0^1 dx x^{n-1} f_f^-(x)$$

- We arrive at a differential equation for the above moment:

$$\frac{d}{d\log Q} M_{nf}^- = \frac{\alpha_s(Q^2)}{\pi}\Big[\underbrace{\int_0^1 dz z^{n-1} P_{q\leftarrow q}(z)}_{d_{qq}^{(n)}}\Big] \underbrace{\int_0^1 dy y^{n-1} f_f^-(y)}_{M_{fn}^-}$$

- Solving the above differential equation for the moment, we arrive at the same scaling behavior for the moment as found in the operator analysis. Thus twist 2 is equivalent to the DGLAP method.

- However, there is not DGLAP analog for higher twist...

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Summary Of Goals:

Extend our understanding of QCD scaling violations past twist-2 to twist-4, which incorporates power corrections to scaling.

Develop a correct theoretical understanding of Higher Twist contributions to the leading moments of the structure functions.

A proper understanding of HT effects can also inform electroweak observables. Nuclear effects must be understood before claims of new physics can be made for NuTeV, as well as future JLAB semi-leptonic experiments.