Right-handed neutrino magnetic moments

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Effective Lagrangian and ν_R

- $\mathcal{L}_{\text{eff}}$ is parameterization of virtual heavy physics effects
- If ν_R is added, there may be new effective interactions involving the ν_R

New Interactions(dim-5)

in terms of Majorana mass eigenfields, *N*(heavy) and *ν*(light)

$$\bar{\nu}_R'^c \zeta \sigma^{\mu\nu} \nu_R' B_{\mu\nu} \Rightarrow \begin{array}{l} \zeta \times [N - N - (\gamma \text{ or } Z^0)] \\ \varepsilon\zeta \times [N - \nu - (\gamma \text{ or } Z^0)] \\ \varepsilon^2\zeta \times [\nu - \nu - (\gamma \text{ or } Z^0)] \end{array}$$

$|\varepsilon_{ij}| \sim \sqrt{\frac{m_\nu}{m_N}}$ small

$$-(\phi^\dagger \phi)\bar{\nu}_R'^c \xi \nu_R' \Rightarrow \begin{array}{l} \xi \times [N - N - (H \text{ or } H^2)] \\ \varepsilon\xi \times [N - \nu - (H \text{ or } H^2)] \\ \varepsilon^2\xi \times [\nu - \nu - (H \text{ or } H^2)] \end{array}$$

$(\bar{\tilde{\ell}}\phi)\chi(\tilde{\phi}^\dagger \ell)$ ⇒ ν_L-Majorana mass term

$$\nu_L' = P_L(\nu + \varepsilon N + \cdots),\ \nu_R' = P_R(N - \varepsilon^T \nu + \cdots),\ \varepsilon = M_D M_R^{-1}$$

$$\mathcal{L}_m = -\bar{e}M_e e - \tfrac{1}{2}\bar{\nu}M_\nu \nu - \tfrac{1}{2}\bar{N}M_N N,\quad M_R = M + \xi v^2,\ M_D = Y_\nu \frac{v}{\sqrt{2}}$$

Coefficient estimates from new physics models

- **From weakly coupled new physics:**

$(\phi^\dagger \phi)\bar{\nu}_R'^c \xi \nu_R'$ can be generated at the tree level $\Rightarrow$ $\xi \sim \frac{1}{M_{NP}}$

$\bar{\nu}_R'^c \zeta \sigma^{\mu\nu} \nu_R' B_{\mu\nu}$ can be generated only at the one-loop level $\Rightarrow$ $\zeta \sim \frac{1}{16\pi^2 M_{NP}}$

From LEP: $M_{NP} > 100$ GeV $\rightarrow$ $\zeta < \frac{1}{15\text{TeV}}$

- **If the new physics is strongly coupled :**

(If v_R participate in the new physics strong interactions)

$\times(4\pi)^2$ $\Rightarrow$ $\zeta \sim \frac{1}{M_{NP}}$

$\Rightarrow$ **Consider the strongly coupled case for the collider effects**

Collider effects

of heavy Majorana neutrinos

Heavy neutrino decay modes

• For simplicity, we considered only the lightest two heavy neutrinos, ***N_1*** and ***N_2*** ($m_2 > m_1$)

• If N_2 is produced at a collider, then it will dominantly decay into N_1 - γ or N_1 - Z^0 (if $m_2 < m_{\mathrm{Higgs}} + m_1$)

• If m_2 > 10GeV, the produced photon will be hard and this could be a signal for the N_2 decay. In addition, if the life time of N_2 is long, this can lead to a displaced photon vertex.

➡ *We did not have a full analysis.*

• ***N_1*** can only decay into the SM particles and the decay length will be usually longer than N_2

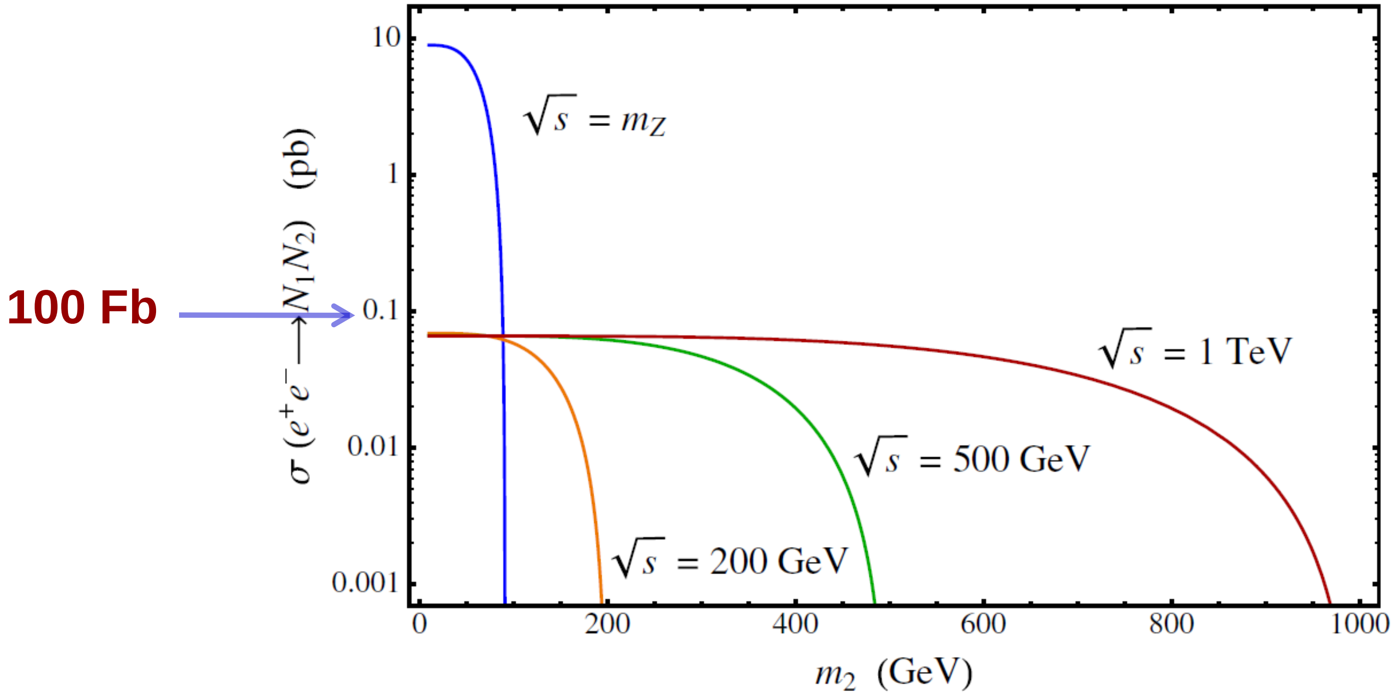
Bound on the new physics scale from LEP data

- If $m_1 + m_2 < m_Z$ we can obtain a limit on the new physics scale from the invisible Z-decay at the LEP

$$\Gamma(Z \to N_1 N_2) \leq \Gamma_{inv}(= 499.0 \pm 1.5\text{MeV (PDG 2008)}) - \Gamma_{SM}(Z \to \bar{\nu}\nu)$$

e.g. If $m_1 \sim m_2 \sim 35\text{GeV}$ then $\Lambda_{NP} > 1.9$ TeV, $\Lambda_{\rm NP} \sim \frac{1}{\zeta}$

Cross sections for $e^+e^- \to N_1 N_2$ at LEP and ILC



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Cross sections for heavy neutrino production as a function of m_2 for different $\sqrt{s}$

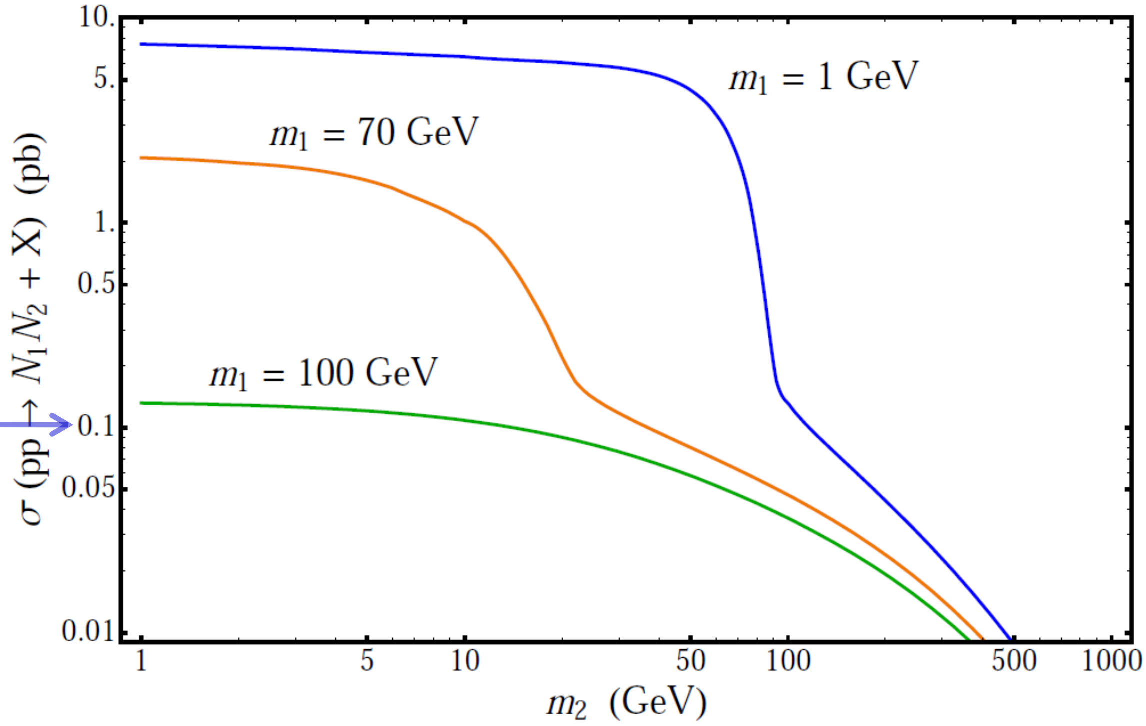
We took Λ_{NP}=10TeV, m_1~ 0

- 200 GeV is for the LEP. 500 GeV and 1 TeV are for the ILC
- Except for the collision at the *Z*-peak, cross sections are *quite* independent of $\sqrt{s}$ as long as the reactions are allowed

Heavy neutrino production at the LHC

though the Drell-Yan process $q\bar{q} \xrightarrow{\gamma \text{ or} Z} N_1 N_2$

100 Fb



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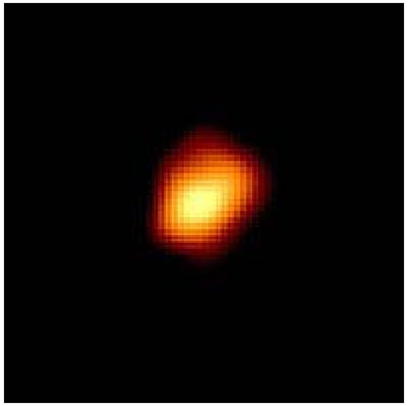
Cross sections for heavy neutrino production as a function of m_2 for several m_1 examples. We took $\sqrt{s}$ =14TeV and Λ_{NP}=10TeV.

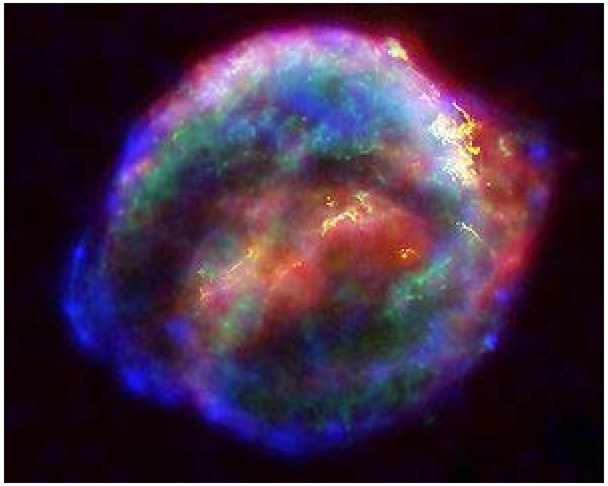
- σ > 100 Fb for $m_1+m_2 < m_Z$

Astrophysical and Cosmological Effects

of the magnetic moment coupling



The red giant Mira--Wikipedia



Multiwavelength X-ray, infrared, and optical compilation image of Kepler's Supernova Remnant, SN 1604. (Chandra X-ray Observatory)-Wikipedia.

Cooling of red giant stars and supernovae

- **In the plasma of a red giant star** a photon acquires a mass equal to the plasma frequency and decays into a pair of neutrinos by the magnetic moment coupling if the neutrino masses are smaller than the plasma frequency, ~10keV.
- If the neutrinos are produced, they will leave the star and contribute to the cooling rate of the star.

⇨ provides an upper limit on the coupling

- **A new mechanism for the cooling of supernovae:**

In a supernova a light neutrino can transform to a heavy neutrino by the magnetic moment coupling then the heavy neutrino will escape and contribute to the cooling of the supernova. ⇨ provides another limit on the coupling

Examples of the bounds

on the new physics scale

plasmon=massive photon
ω_P = plasma frequency
~ 8.6keV

1. Cooling of red giant stars

- If $m_N \ll \omega_P$: plasmon → NN

➡ $\frac{1}{\zeta} \sim \Lambda_{NP} \gtrsim 4\times10^6$ TeV

- If $m_N > \omega_P$: plasmon → $\nu\,\nu$

➡ $\Lambda_{NP} \gtrsim (m_\nu/m_N)^2\times4\times10^6$TeV

$\varepsilon^2\zeta \times [\nu - \nu - (\gamma \text{ or } Z^0)]$

2. Cooling of supernovae, $\gamma + \nu \to N$

➡ $\Lambda_{NP} \gtrsim (m_\nu/m_N)\times4\times10^6$TeV

$\varepsilon\zeta \times [N - \nu - (\gamma \text{ or } Z^0)]$

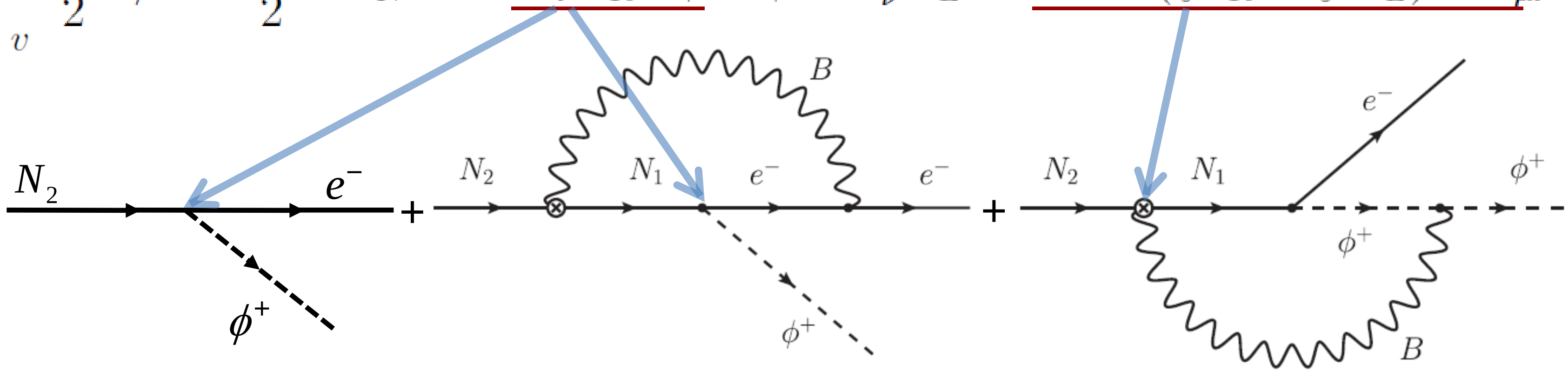
$$\Gamma(\text{plasmon} \to \sum_{i,j} N_i N_j) = \sum_{\substack{m_i+m_j<\omega_p \\ i>j}}^{\text{all allowed}} \frac{2c_w^2|\zeta_{ij}|^2}{3\pi}\frac{\omega_p^4}{\omega} f_Z(\omega_p, m_i, m_j)$$

CP asymmetries

- The magnetic moment coupling can contribute to the decay of $N \to e^{\pm}\phi^{\mp}$ in one-loop diagrams and results non zero CP asymmetries

$$\mathcal{L}_N = \frac{i}{2}\overline{N}\partial\!\!\!/ N - \frac{1}{2}\overline{N}M_N N - \underline{\bar{\ell}Y_\nu P_R N\tilde{\phi}} - \tilde{\phi}^\dagger \overline{N} Y_\nu^\dagger P_L \ell + \underline{\overline{N}\sigma^{\mu\nu}(\zeta P_R + \zeta^\dagger P_L)N\, B_{\mu\nu}}$$

$m_N \gg v$



$$\epsilon_{\not{CP}} \equiv \frac{\Gamma(N_2 \to e^-\phi^+) - \Gamma(N_2 \to e^+\phi^-)}{\Gamma(N_2 \to e^-\phi^+) + \Gamma(N_2 \to e^+\phi^-)}$$

$$= -\frac{g'}{2\pi}(m_2^2 - m_1^2)\frac{m_1}{m_2^3}\,\mathrm{Im}\left\{\frac{Y_{e2}Y_{e1}^*}{|Y_{e2}|^2}(\zeta_{12}^* m_2 + \zeta_{12} m_1)\right\},\ m_2 > m_1$$

- This could be relevant for leptogenesis when m_1 and m_2 are relatively close.

Summary of the bounds and prospects

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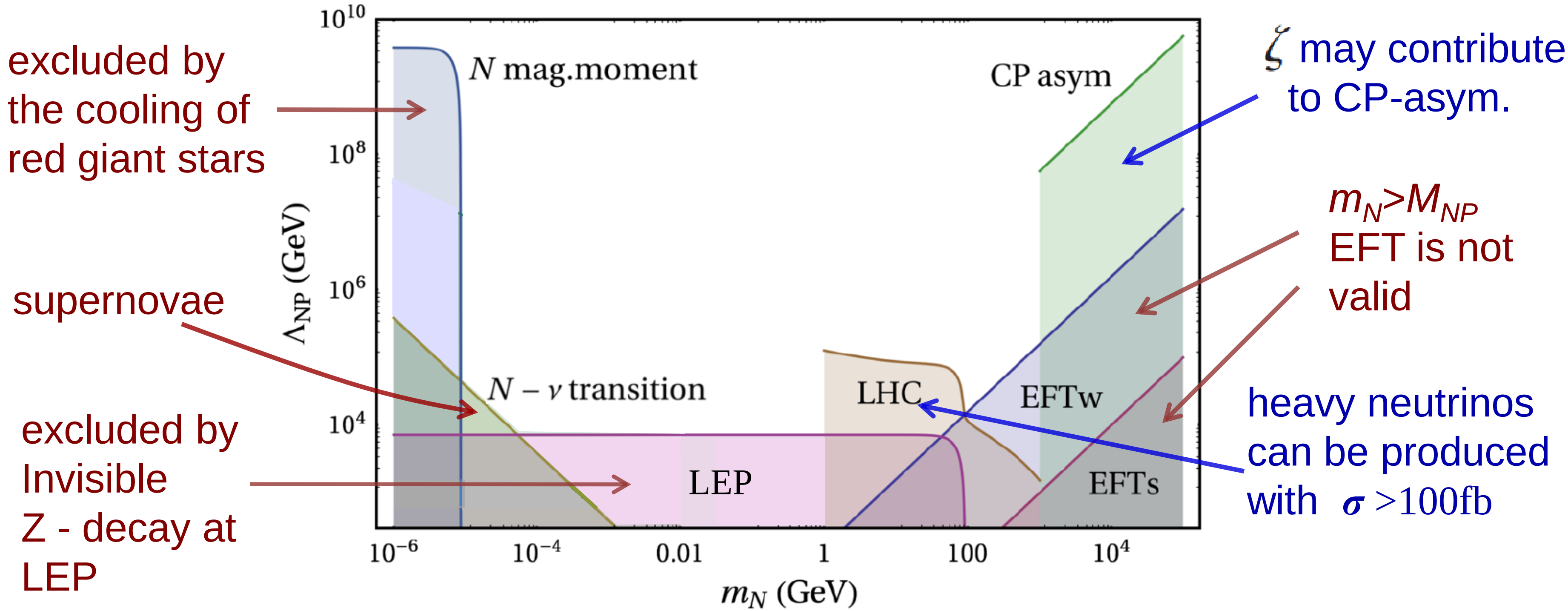
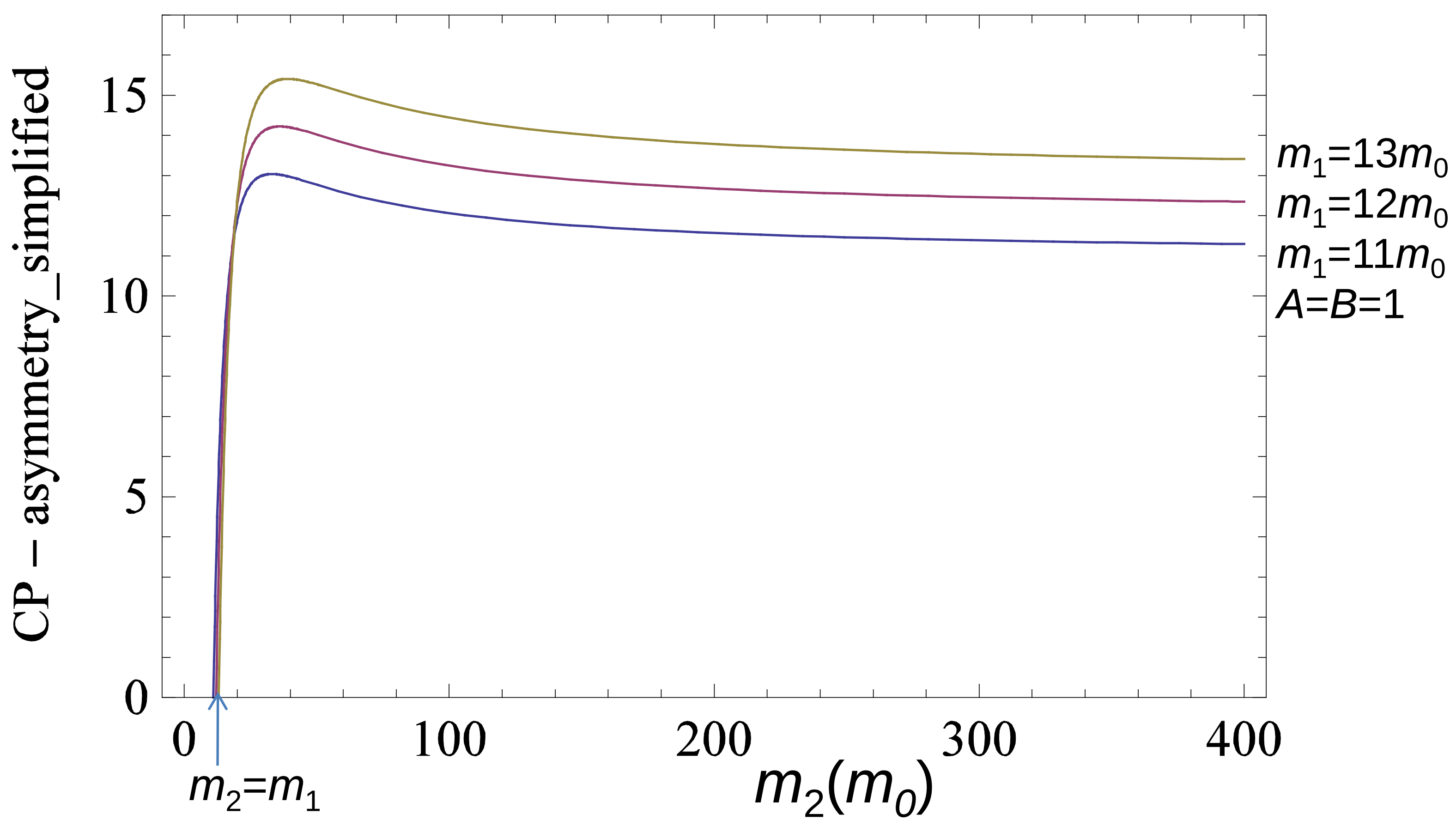


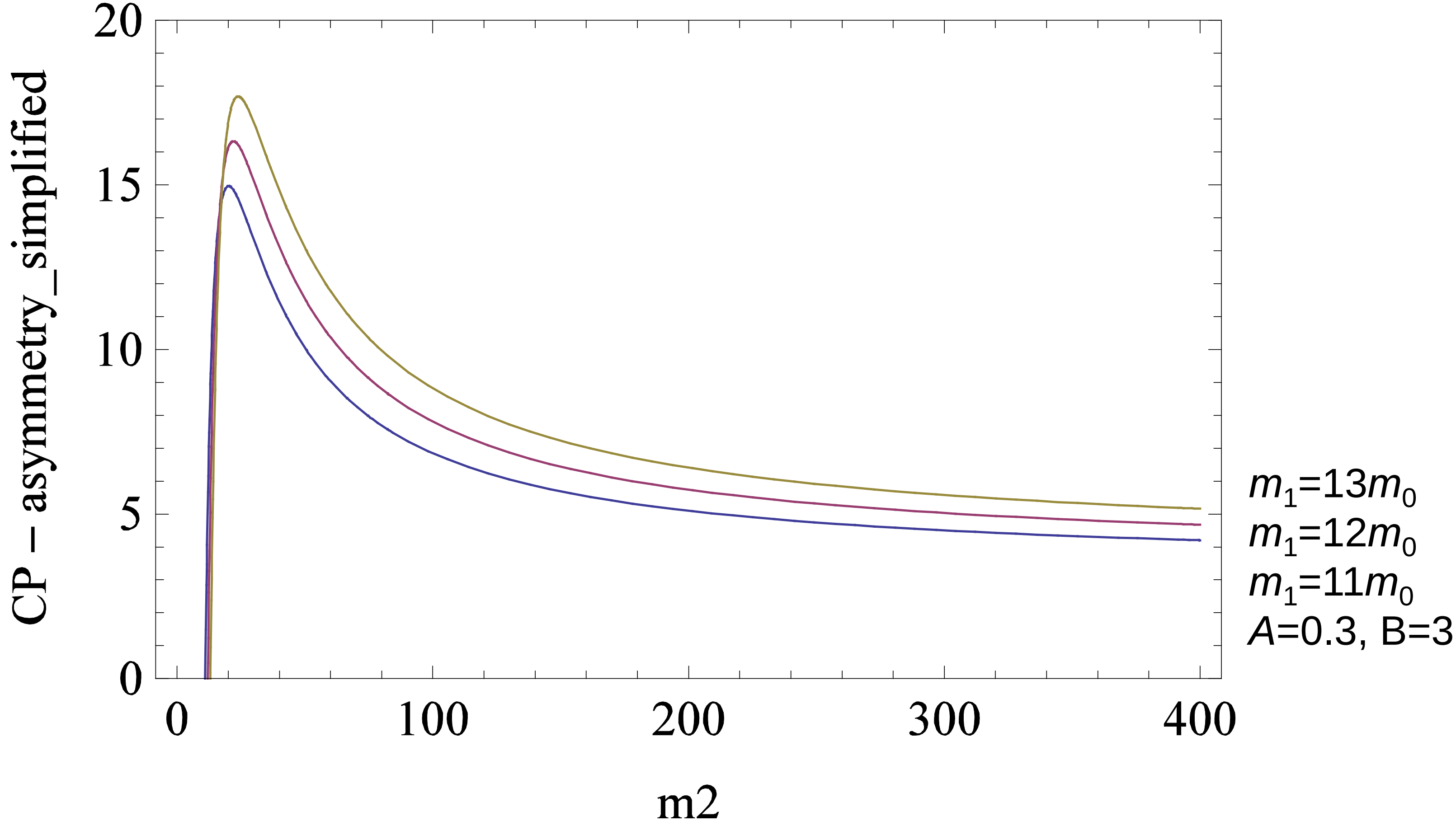
Excluded and interesting regions on $\Lambda_{NP} - m_N$ plane, m_ν=0.1eV

- Interesting Regions: LHC shade, CP-asym shade
- Excluded regions: all other shaded areas

Extra Slides



$$\epsilon_{C/P} = (m_2^2 - m_1^2)\frac{m_1}{m_2}(Am_2 + Bm_1)$$



$$\epsilon_{C\!/\!P} = (m_2^2 - m_1^2)\frac{m_1}{m_2}(Am_2 + Bm_1)$$

Effective Lagrangian up to dim-5 operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\nu_{\rm R}} + \mathcal{L}_5 + \cdots$$

$$\mathcal{L}_{\rm SM} = i\bar{\ell}\,\not{D}\,\ell + i\overline{e_R}\,\not{D}\,e_R - (\bar{\ell}Y_e e_R\phi + {\rm h.c.}) + \cdots$$

$$\mathcal{L}_{\nu_R} = i\overline{\nu'_R}\not{\partial}\,\nu'_R - \left(\frac{1}{2}\overline{\nu'^c_R}M\nu'_R + {\rm h.c.}\right) - \left(\bar{\ell}Y_\nu\nu'_R\,\tilde{\phi} + {\rm h.c.}\right)$$

$$\mathcal{L}_5 = \overline{\nu'^c_R}\zeta\sigma^{\mu\nu}\nu'_R B_{\mu\nu} + \left(\bar{\tilde{\ell}}\phi\right)\chi\left(\tilde{\phi}^\dagger\,\ell\right) - \left(\phi^\dagger\phi\right)\overline{\nu'^c_R}\xi\nu'_R + {\rm h.c.}$$

$$\ell = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$ e_R and ν_R = singlets, $\nu_R^c = C\bar{\nu}_R^T$

$C = i\gamma^2\gamma^0, \tilde{\ell} = \epsilon\ell^c, \phi = \epsilon\phi^*$

Flavor Matrices: $Y_e = 3\times 3$ comp., $M = n\times n$ comp. symm.,

$Y_\nu = 3\times n$ comp., $\chi = 3\times 3$ comp. symm.,

$\zeta = n\times n$ comp. antisymm., $\xi = n\times n$ comp. symm.

n =number of flavors of right handed neutrinos

Diagonalizing the mass matrices

$$M_R \gg M_D \gg M_L$$

$$\mathcal{L}_m = -\overline{e_L} M_e e_R - \overline{\nu'_L} M_D \nu'_R - \frac{1}{2}\overline{\nu'^c_L} M_L \nu'_L - \frac{1}{2}\overline{\nu'^c_R} M_R \nu'_R + \text{h.c.}$$

$$M_R = M + \xi v^2, \quad M_L = \chi v^2, \quad M_D = Y_\nu \frac{v}{\sqrt{2}}, \quad M_e = Y_e \frac{v}{\sqrt{2}}$$

$$\mathcal{L}_m = -\frac{1}{2}\left(\bar{\nu}'_L \bar{\nu}'^c_R\right) \begin{pmatrix} M_L^\dagger & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix} + h.c.$$

$$= -\frac{1}{2}\left(\bar{\nu}'_L \bar{\nu}'^c_R\right) U^* U^T \begin{pmatrix} M_L^\dagger & M_D \\ M_D^T & M_R \end{pmatrix} U U^\dagger \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix} + h.c.$$

$$\cong -\frac{1}{2}\left(\bar{n}_L \bar{N}^c_R\right) \begin{pmatrix} [U_\nu^T (M_L - M_D^* M_R^{-1*} M_D^\dagger) U_\nu]^\dagger & 0 \\ 0 & U_N^T M_R U_N \end{pmatrix} \begin{pmatrix} n^c_L \\ N_R \end{pmatrix} + h.c.$$

$$= -\frac{1}{2}\bar{\nu} M_\nu \nu - \frac{1}{2}\bar{N} M_N N, \quad \nu = n_L + n^c_L, \; N = N_R + N^c_R$$

$$\nu'_L = P_L(U_\nu \nu + \varepsilon U_N^* N + \cdots), \; \nu'_R = P_R(U_N N - \varepsilon^T U_\nu^* \nu + \cdots)$$

$$\varepsilon = M_D M_R^{-1}, \; |\varepsilon_{ij}| \lesssim \sqrt{\frac{m_\nu}{m_N}}$$

The Lagrangian in terms of mass eigenfields

$$\bar{\nu}'^{c}_{R}\zeta\sigma^{\mu\nu}\nu'_{R}B_{\mu\nu} = \left(\overline{N}U_N^{\dagger} - \overline{\nu}U_{\nu}^{\dagger}\varepsilon^{*}\right)\sigma^{\mu\nu}\left(\zeta P_R + \zeta^{\dagger}P_L\right)\left(U_N N - \varepsilon^T U_{\nu}\nu\right)\left(c_W F_{\mu\nu} - s_W Z_{\mu\nu}\right)$$

$$-(\phi^{\dagger}\phi)\bar{\nu}'^{c}_{R}\xi\nu'_{R} = -\frac{(v+H)^2}{2}(\bar{N} - \bar{\nu}\varepsilon^{*})\xi P_R(N - \varepsilon^T\nu) + \text{H.c.}$$

$$-\bar{\ell}\tilde{\phi}Y_{\nu}\nu'_R + \text{H.c.} = -\frac{1}{\sqrt{2}}(v+H)(\bar{\nu} + \bar{N}\varepsilon^{\dagger} + \cdots)Y_{\nu}P_R(N - \varepsilon^T\nu + \cdots) + \text{H.c.}$$

$$\bar{\ell}i\not{D}\ell = \frac{g}{\sqrt{2}}W_{\mu}^{-}\bar{e}_L U_e^{\dagger}\gamma^{\mu}P_L(U_{\nu}\nu + \varepsilon U_N^{*}N + \cdots) + \text{H.c.} + \frac{g}{2c_W}Z_{\mu}^{0}\bar{\nu}'_L\gamma^{\mu}\nu'_L + \cdots$$

- Majorana Fermions

  ν, N: Light and Heavy Neutrinos

- Flavor Matrices

  U_N, U_{ν}: Unitary Matrices

$\varepsilon = M_D M_R^{-1}$: $3\times n, n =$ number of ν_R

$$|\varepsilon_{ij}| \sim \sqrt{\frac{m_{\nu}}{m_N}}$$

New Interactions

heavy , light Majorana

$$\bar{\nu}_R'^c \zeta \sigma^{\mu\nu} \nu_R' B_{\mu\nu} \Rightarrow \zeta \times [N - N - (\gamma \text{ or } Z^0)],\ \varepsilon\zeta \times [N - \nu - (\gamma \text{ or } Z^0)],$$

small

$$\varepsilon^2 \zeta \times [\nu - \nu - (\gamma \text{ or } Z^0)]$$

$$-(\phi^\dagger \phi) \bar{\nu}_R'^c \xi \nu_R' \Rightarrow \xi \times [N - N - (H \text{ or } H^2)],\ \varepsilon\xi \times [N - \nu - (H \text{ or } H^2)],$$

$$\varepsilon^2 \xi \times [\nu - \nu - (H \text{ or } H^2)]$$

$$i\bar{\ell} \not{D} \ell \Rightarrow \varepsilon \times [N - W - \ell],\ [\nu - W - \ell],\ \varepsilon \times [Z^0 - N - \nu],\ [Z^0 - \nu - \nu]$$

$$\varepsilon^2 \times [Z^0 - N - N]$$

$$\bar{\ell} Y_\nu \nu_R' \tilde{\phi} \Rightarrow Y_\nu \times [H - N - \nu],\ \varepsilon Y_\nu [H - N - N],\ \varepsilon Y_\nu \times [H - \nu - \nu]$$

$$\mathcal{L}_m = -\bar{e} M_e e - \tfrac{1}{2} \bar{\nu} M_\nu \nu - \tfrac{1}{2} \bar{N} M_N N \qquad Y_\nu \sim \frac{\sqrt{2}\varepsilon M_N}{v}$$

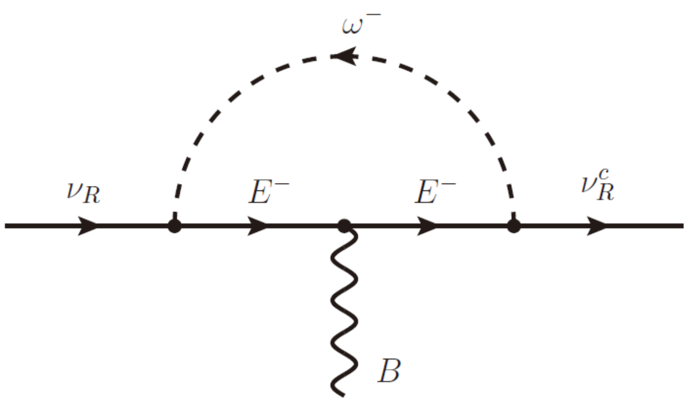
$\varepsilon = M_D M_R^{-1}$: Mixing between between heavy and light neutrinos

$$|\varepsilon_{ij}| \sim \sqrt{\frac{m_\nu}{m_N}}$$

Heavy physics example for $\bar{\nu}'^{c}_{R}\zeta\sigma^{\mu\nu}\nu'_{R}B_{\mu\nu}$

$$\mathcal{L} = \lambda'_i \bar{\nu}^c_{Ri} E\omega^* + \lambda_i \bar{E}\nu_{Ri}\omega + \text{H.c.} + \bar{E} i \not{D} E$$

λ'_i and λ_i are real. E and ω are (1,1,y)



$$\zeta \sim \frac{g' y \lambda^2}{16\pi^2} \frac{m_{\text{fermion}}}{\max(m^2_{\text{fermion}}, m^2_{\text{boson}})} < \frac{g' y \lambda^2}{16\pi^2 m_{\text{fermion}}} \sim \frac{1}{16\pi^2 M_{NP}}$$

Vector and fermion pair: $W'_\mu \bar{E}\gamma^\mu \nu'_R$ and $W'_\mu \bar{E}\gamma^\mu \nu'^{c}_R$

Heavy physics example for $(\bar{\tilde{\ell}}\phi)\chi(\tilde{\phi}^\dagger\ell)$

$$\mathcal{L} = -\bar{\ell}\lambda\epsilon\phi^* F - \frac{1}{2}\bar{F}^c M F + \text{H.c.}$$

$$\frac{\delta\mathcal{L}}{\delta F_k} = \bar{\ell}_i\lambda_{ik}\epsilon\phi^* + \bar{F}_i^c M_{ik} = 0, F = M^{-1}\phi^\dagger\lambda^T\epsilon\ell^c$$

$$\mathcal{L}_{\text{eff}} = \bar{\tilde{\ell}}\epsilon\phi\left(-\frac{1}{2}\lambda M^{-1}\lambda^T\right)^\dagger\tilde{\phi}^\dagger\ell + \text{H.c.}$$

$$\chi = \left(-\frac{1}{2}\lambda M^{-1}\lambda^T\right)^\dagger \sim \frac{1}{M_{NP}}$$

Heavy physics example for $(\bar{\tilde{\ell}}\phi)\chi(\tilde{\phi}^{\dagger}\ell)$

$$\mathcal{L} = \bar{\tilde{\ell}}\lambda\Phi_I\sigma^I\ell - \mu\tilde{\phi}^{\dagger}\Phi_I^*\sigma^I\phi + \mathrm{H.c.} - M^2Tr[(\Phi_I\sigma^I)^{\dagger}(\Phi_I\sigma^I)]$$

$$\Phi_I\sigma^I = \begin{pmatrix} \Phi_3 & \Phi_1 - i\Phi_2 \\ \Phi_1 + i\Phi_2 & -\Phi_3 \end{pmatrix} = \begin{pmatrix} \Phi_+/\sqrt{2} & \Phi_{++} \\ \Phi_0 & -\Phi_+/\sqrt{2} \end{pmatrix}$$

$$\phi_2 \xrightarrow{\mathrm{VEV}} \frac{v}{\sqrt{2}}, \Phi_0 \xrightarrow{\mathrm{VEV}} \frac{v_\Phi}{\sqrt{2}}, v_\Phi = \frac{\mu v^2}{\sqrt{2}M^2}$$

$$\chi \sim \frac{1}{M_{NP}}$$

Heavy neutrino decay rates

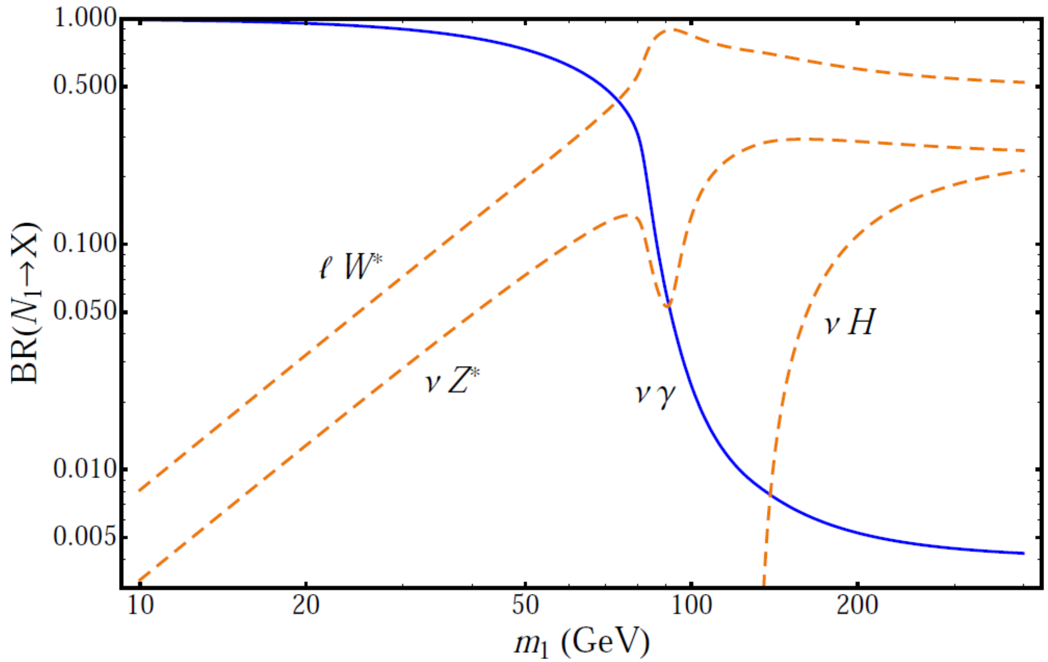


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Figure 1: Decay branching ratios of N_1. Solid for $N_1 \to \nu\gamma$ and dashed for $N_1 \to eW^* \to e$+fermions, $N_1 \to \nu Z^* \to \nu$+fermions and $N_1 \to \nu H$ (see text). We take $\varepsilon \sim 10^{-6}$, $\Lambda_{NP} = 10\,\mathrm{TeV}$ and $m_H = 130\,\mathrm{GeV}$.

- $m_1 < m_W : \Gamma(N_1 \to \nu\gamma)$ dominates
- $m_H \ll m_1 : \Gamma(N_1 \to \nu Z^*) = \Gamma(N_1 \to \nu H) = \frac{1}{2}\Gamma(N_1 \to \ell W^*)$

Bound on the new physics scale from LEP data

$$\Gamma(Z \to N_1 N_2) \leq \Gamma_{inv}(= 499.0 \pm 1.5\text{MeV (PDG 2008)}) - \Gamma_{SM}(Z \to \bar{\nu}\nu)$$

$$\Gamma(Z \to N_1 N_2) < 0.48 \times 1.5\,\text{MeV} = 0.72\,\text{MeV} \qquad 90\%\ \text{CL}$$

$$\Lambda_{NP} = \frac{1}{|\zeta_{12}|} > 7\sqrt{f_Z(m_Z, m_1, m_2)}\,\text{TeV}$$

For example, $\Lambda_{NP} > 1.9$ TeV if $m_1 = m_2 = 35$ GeV

Heavy neutrino decay lengths

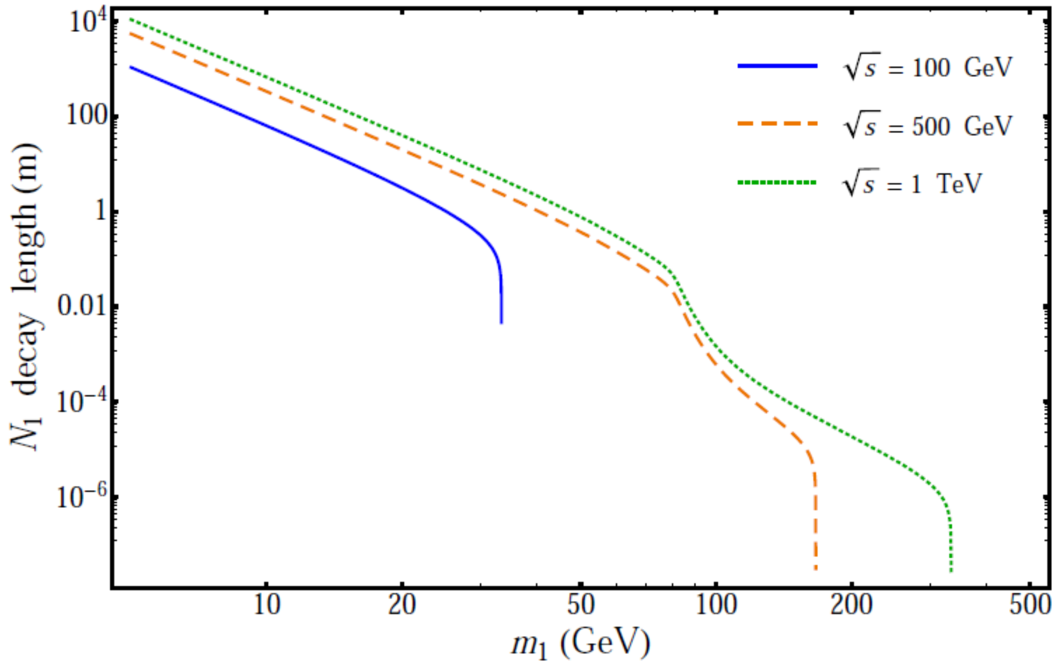


Figure 2: N_1 decay lengths for a N_1 produced together with a N_2 at CM. We present results for CM energies of $\sqrt{s} = 100\,\text{GeV}$ (solid), $500\,\text{GeV}$ (dashed), and $1\,\text{TeV}$ (dotted); we took $m_2 = 2m_1$, $\Lambda_{NP} = 10\,\text{TeV}$ and $\varepsilon = 10^{-6}$. Ref.: Phys. Rev. D 80, 013010

Higgs decays into heavy neutrinos

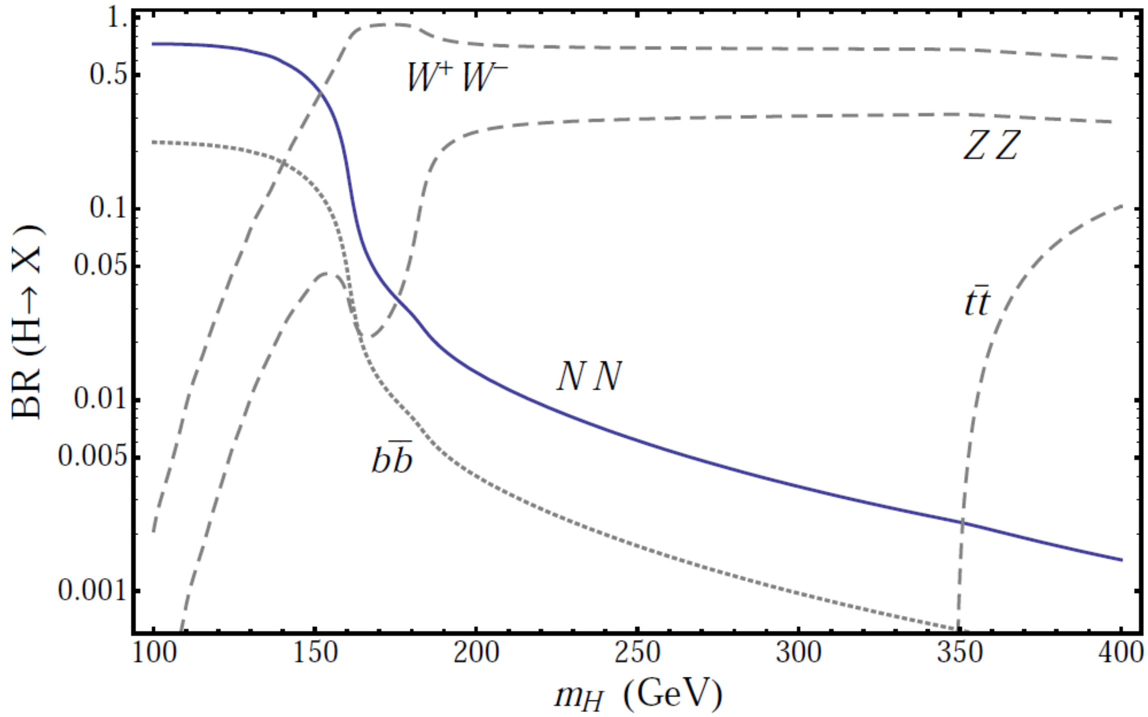


Figure 6: Estimated branching ratios for Higgs decays with the new-physics scale at $1/\xi = 10$ TeV. Heavy neutrino masses have been neglected. Ref.: Phys. Rev. D 80, 013010

$$f_Z(m_Z, m_i, m_j) = \frac{\sqrt{\lambda(m_Z^2, m_i^2, m_j^2)}}{m_Z^6}[m_Z^2(m_Z^2 + m_i^2 + m_j^2 - 6m_i m_j \cos 2\delta_{ij}) - 2(m_i^2 - m_j^2)^2].$$

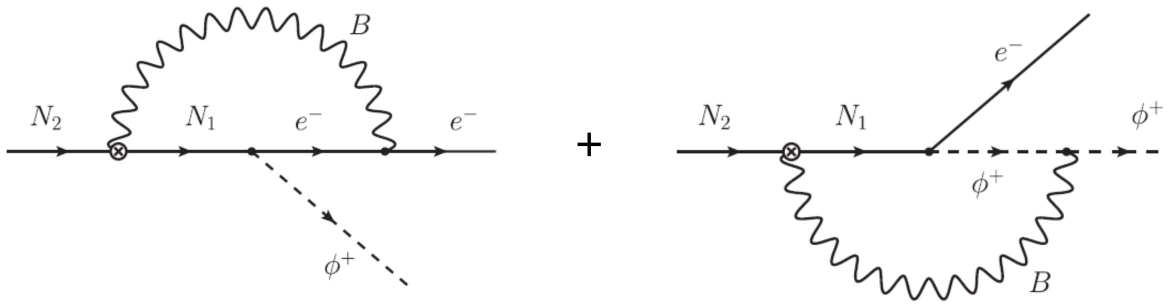
$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$

CP asymmetries

Assume that $m_N \gg v$

$$\mathcal{L}_N = \frac{i}{2}\overline{N}\partial\!\!\!/ N - \frac{1}{2}\overline{N}M_N N - \bar{\ell}Y_\nu P_R N\tilde{\phi} - \tilde{\phi}^\dagger \overline{N} Y_\nu^\dagger P_L \ell + \overline{N}\sigma^{\mu\nu}(\zeta P_R + \zeta^\dagger P_L)N\, B_{\mu\nu}$$

Tree diagram +



$$\epsilon_{\not{C}\not{P}} \equiv \frac{\Gamma(N_2 \to e^-\phi^+) - \Gamma(N_2 \to e^+\phi^-)}{\Gamma(N_2 \to e^-\phi^+) + \Gamma(N_2 \to e^+\phi^-)}$$

$$= -\frac{g'}{2\pi}(m_2^2 - m_1^2)\frac{m_1}{m_2^3}\,\mathrm{Im}\left\{\frac{Y_{e2}Y_{e1}^*}{|Y_{e2}|^2}(\zeta_{12}^* m_2 + \zeta_{12} m_1)\right\}$$

For $m_1 \ll m_2$, $\epsilon_{\not{C}\not{P}} \sim -\frac{g'}{2\pi}\frac{m_1}{\Lambda_{\mathrm{NP}}}\,\mathrm{Im}\left\{\frac{Y_{e2}Y_{e1}^*}{|Y_{e2}|^2}e^{-i\delta_{12}}\right\}$