Improving the precision of light quark masses

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Based on
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T. Izubuchi, C.T. Sachrajda and A. Soni

Introduction

- Light quark masses (up-, down- and strange-quark masses) can be determined non-perturbatively with lattice simulations in QCD
- Result from RBC/UKQCD Coll., domain-wall fermions:

$$m_{ud}^{\overline{\text{MS}}}(2\text{ GeV}) = 3.72(0.16)_{\text{stat}}(0.18)_{\text{syst}}(0.33)_{\text{ren}}\text{ MeV}$$
$$m_{s}^{\overline{\text{MS}}}(2\text{ GeV}) = 107.3(4.4)_{\text{stat}}(4.9)_{\text{syst}}(9.7)_{\text{ren}}\text{ MeV}$$

C. Allton et al.

- Error (11%) from renormalization dominates (>60% of tot.)
- Pert. calculations are performed in dim. reg. $\rightsquigarrow$ not directly amenable to lattice calculations
- Direct calculation of bare quantity with lattice spacing acting as ultra-violet cutoff in some particular discretization of QCD instead of space-time dimension $d \neq 4$
- Minimal subtraction(MS) à la dim. reg. not directly possible

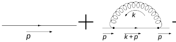
Introduction

Regularization invariant momentum subtraction schemes

- Use for renormalization regularization invariant(RI) scheme, which removes ultraviolet divergences at a certain momentum point(subtraction point) $\rightsquigarrow$ RI/MOM-scheme
  Martinelli et al. '93-'95

- Determine QCD parameters: $m_R = Z_m m_B,\ \Psi_R = Z_q^{1/2}\Psi_B, ..$
  $\rightsquigarrow$ fix renormalization constants, define scheme in PT:

$$S_R^{-1} = Z_q^{-1} S_B^{-1} \propto \not{p}\,\Sigma_R^V(p^2) - m_R \Sigma_R^S(p^2) \quad \Leftrightarrow$$



$+ \dots$

$$\lim_{m_R\to 0} \frac{1}{12 m_R} \mathrm{Tr}[S_R^{-1}(p)]\Big|_{p^2=-\mu^2} = 1 \qquad \lim_{m_R\to 0} \frac{1}{48} \mathrm{Tr}\left[\gamma^\mu \frac{\partial S_R^{-1}(p)}{\partial p^\mu}\right]\Bigg|_{p^2=-\mu^2} = -1 \qquad \text{RI/MOM}$$

$$\lim_{m_R\to 0} \frac{1}{12 m_R} \mathrm{Tr}[S_R^{-1}(p)]\Big|_{p^2=-\mu^2} = 1 \qquad \lim_{m_R\to 0} \frac{1}{12 p^2} \mathrm{Tr}[S_R^{-1}(p)\,\not{p}]\Big|_{p^2\to-\mu^2} = -1 \qquad \text{RI'/MOM}$$

Introduction

Ward-Takahashi identities

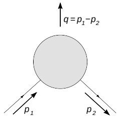
- Ward-Takahashi identities(WI)

$$q_\mu \Lambda^\mu_{V,B}(p_1,p_2) = S_B^{-1}(p_2) - S_B^{-1}(p_1)$$

$$-iq_\mu \Lambda^\mu_{A,B}(p_1,p_2) = 2m_B \Lambda_{P,B}(p_1,p_2) - i\gamma_5 S_B^{-1}(p_1) - S_B^{-1}(p_2) i\gamma_5$$

- WI valid for renorm. quantities: $O_R = Z_O O$, $\Lambda_{O,R} = \frac{Z_O}{Z_q}\Lambda_{O,B}$
- Renormalization condition on $S \Leftrightarrow$ condition on Λ_O

$$\frac{1}{N}\mathrm{Tr}\left[\Lambda_{O,R}(p_1,p_2)P_O\right]\Big|_{mom.conf.} = 1$$



$\rightsquigarrow$ study quark bilinear operators with vector(γ^μ), axial-vector($\gamma_5\gamma^\mu$), pseudo-scalar(γ_5) and scalar($\mathbf{1}$) operators

- Renormalization constants related:
  $Z_A = 1 = Z_V$, $Z_P = Z_S$, $Z_P = 1/Z_m$

Motivation

- RI/MOM

$$\lim_{m_R\to 0}\frac{1}{48}\mathrm{Tr}\left[\Lambda^{\mu}_{V,R}(p_1,p_2)\gamma_\mu\right]\Big|_{asym}=1,\quad \lim_{m_R\to 0}\frac{1}{48}\mathrm{Tr}\left[\Lambda^{\mu}_{A,R}(p_1,p_2)\gamma_5\gamma_\mu\right]\Big|_{asym}=1$$

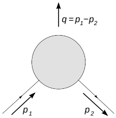
$$\lim_{m_R\to 0}\frac{1}{12}\mathrm{Tr}\left[\Lambda_{S,R}(p_1,p_2)\mathbf{1}\right]\Big|_{asym}=1,\quad \lim_{m_R\to 0}\frac{1}{12i}\mathrm{Tr}\left[\Lambda_{P,R}(p_1,p_2)\gamma_5\right]\Big|_{asym}=1$$

- Asymmetric/exceptional momentum config.(MOM):

$$p_1^2=p_2^2=-\mu^2,\quad \mu^2>0,\quad p_1=p_2,\quad q=0$$

Symmetric/nonexceptional momentum config(SMOM):

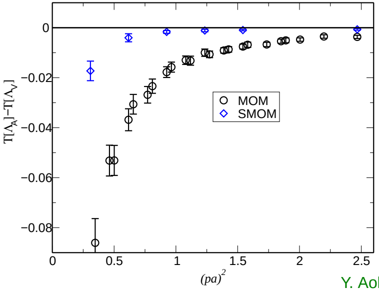
$$p_1^2=p_2^2=q^2=-\mu^2,\quad \mu^2>0,\quad q=p_1-p_2$$



- Renormalization constants need to be determined through simulation
- Need to introduce renormalization scale μ typically $\mu\sim 2$ GeV for this problem

Motivation

- Symmetric subtraction point implies a lattice simulation with suppressed contamination from infrared effects
- For asymmetric subtraction point effects of chiral symmetry breaking vanish slowly like $1/p^2$ for large ext. momenta



Y. Aoki

- For SMOM infrared effects better behaved, vanishing with larger powers of p N.H. Christ, et al.

Motivation

RI-scheme $\Longrightarrow$ $\overline{\text{MS}}$-scheme

- Conversion/Matching factor:

$$m_R^{\overline{\text{MS}}} = C_m^{\text{RI/MOM}} m_R^{\text{RI/MOM}} \qquad (C_m \text{ in general gauge dependent})$$

- RI/MOM scheme intermediate scheme before conversion to the $\overline{\text{MS}}$ scheme
- C_m can be computed in cont. PT, e.g. RI/MOM, RI'/MOM: known up to 3-loop G. Martinelli et al.; Franco, Lubicz; Chetyrkin, Retey; Gracey

$$C_m^{\text{RI/MOM}} = 1.0 - 0.1333 - 0.0759 - 0.0557$$

$$C_m^{\text{RI'/MOM}} = 1.0 - 0.1333 - 0.0816 - 0.0603$$

$\alpha_s(2\text{ GeV})/\pi \sim 0.1$

$n_f = 3$

- Observation:
  Size of NLO, N^{2}LO, N^{3}LO contr. amount $\sim$ 13%, $\sim$ 8%, $\sim$ 6%
  $\rightsquigarrow$ poor convergence $\rightsquigarrow$ big error in renormalization
- Matching to pert. theo.: reduce truncation error: large μ
  $\rightsquigarrow$ window problem

Concepts & Framework

RI/SMOM

- Task: Find a RI/MOM type scheme which is "close" to $\overline{\text{MS}}$ scheme, e.g. which has a matching factor with small corrections $\rightsquigarrow$ Small expansion coefficients
- Idea: Use subtraction point with symmetric momenta

RI/SMOM conditions:

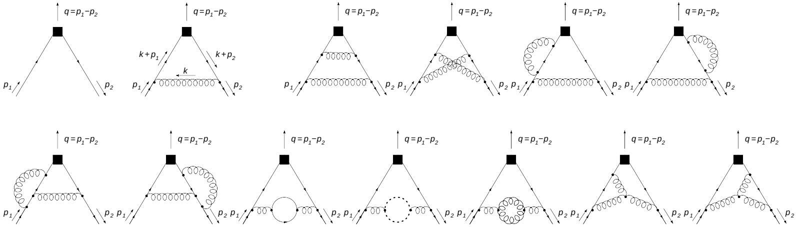
$$\lim_{m_R \to 0} \frac{1}{12} \text{Tr}\left[\Lambda_{S,R}(p_1,p_2)\mathbf{1}\right]\Big|_{sym} = 1, \quad \lim_{m_R \to 0} \frac{1}{12i} \text{Tr}\left[\Lambda_{P,R}(p_1,p_2)\gamma_5\right]\Big|_{sym} = 1$$

$$\lim_{m_R \to 0} \frac{1}{12q^2} \text{Tr}\left[q_\mu \Lambda^\mu_{V,R}(p_1,p_2)\not{q}\right]\Big|_{sym} = 1, \quad \lim_{m_R \to 0} \frac{1}{12q^2} \text{Tr}\left[q_\mu \Lambda^\mu_{A,R}(p_1,p_2)\gamma_5 \not{q}\right]\Big|_{sym} = 1$$

- Alternative scheme (RI/SMOM$_{\gamma_\mu}$) using different projectors

$$\lim_{m_R \to 0} \frac{1}{48} \text{Tr}\left[\Lambda^\mu_{V,R}(p_1,p_2)\gamma_\mu\right]\Big|_{sym} = 1, \quad \lim_{m_R \to 0} \frac{1}{48} \text{Tr}\left[\Lambda^\mu_{A,R}(p_1,p_2)\gamma_5\gamma_\mu\right]\Big|_{sym} = 1$$

- Need to compute the matching factor C_m in the new RI/SMOM schemes
- 1 diagram at LO, 1 at NLO and 11 at NNLO:



- Calculation:
  Two steps: A) IBP/Laporta's algorithm
  K.G. Chetyrkin, F.V. Tkachov / S. Laporta, E. Remiddi
  $\rightsquigarrow$ reduction to small(7) set of MI
  B) Solve MI, here known Davydychev et al.

Results at NNLO

Results & Comparison of the conversion factors for $n_f = 3$, $\alpha_s/\pi \simeq 0.1$, scale of ~ 2 GeV

- RI′/MOM $\Longleftrightarrow$ RI/SMOM:

$$C_{m,L}^{\mathrm{RI'/MOM}} = 1 - 0.1333333\ldots - 0.07585848\ldots - 0.0556959\ldots$$

$$C_{m,L}^{\mathrm{RI/SMOM}} = 1 - 0.0161380\ldots - 0.00660442\ldots \leftarrow \textit{New}$$

$\hookrightarrow$ in agreement with Jaeger, Gorbahn

- RI/MOM $\Longleftrightarrow$ RI/SMOM$_{\gamma_\mu}$:

$$C_{m,L}^{\mathrm{RI/MOM}} = 1 - 0.1333333\ldots - 0.0815876\ldots - 0.0602759\ldots$$

$$C_{m,L}^{\mathrm{RI/SMOM}_{\gamma_\mu}} = 1 - 0.0494713\ldots - 0.0228421\ldots \leftarrow \textit{New}$$

- Matching factors for schemes with symmetric subtraction point show better convergence behavior
  $\rightsquigarrow$ significant reduction of syst. error on light quark masses
- Informative to have multiple schemes
  $\rightsquigarrow$ better assessment of syst. errors

Summary & Conclusion

- Framework + concepts of renorm. of quark bilinear operators in the RI/SMOM schemes has been discussed
- Results can be used to convert light quark masses in this scheme to the $\overline{\text{MS}}$ scheme
- The conversion factors in the RI/SMOM schemes are now available up to NNLO + show a better convergence behavior than in the traditional RI/MOM schemes
- The pert. truncation error is smaller than in RI/MOM
- RI/SMOM schemes are less sensitive to infrared effects in latt. simulation

$\rightsquigarrow$ The use of the RI/SMOM schemes will reduce the systematic error and improve precision of light quark mass determinations from lattice simulations obtained in this approach