Single Gravitino Coupling to Broken Gauge Theories

Full and Effective Lagrangian, and an MSSM Application

Feng Luo

collaborated with Keith Olive and Marco Peloso

University of Minnesota

May 11, 2010

Motivation

Gravitino polarization tensor is needed in calculating unpolarized square amplitudes for the gravitino scattering or decay processes

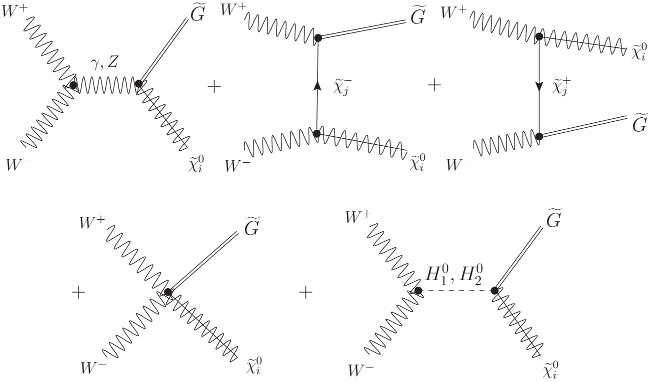
$$\Pi_{\mu\nu} \equiv \sum_{r=\pm\frac{1}{2},\pm\frac{3}{2}} \psi_\mu^{(r)}\,\bar{\psi}_\nu^{(r)} = \frac{2}{3}\,\frac{p_\mu\,p_\nu\,\not{p}}{m_{3/2}^2} + \mathrm{O}\left(\frac{1}{m_{3/2}}\right) \qquad (1)$$

The previous calculation are focus on unbroken gauge symmetries cases, and people always find that in the high energy limit $\overline{|\mathcal{M}|_i^2} = \frac{A}{M_p^2\,m_{3/2}^2}$, with A vanishing in the limit of exact susy.

Motivation

broken gauge symmetries case

$W^+ W^- \rightarrow \tilde{\chi}_i^0 + \tilde{G}$ was calculated [Ferrantelli, JHEP 0901 (2009)]



Motivation

- It was claimed that A that does not vanish in the limit of exact susy, and this discrepancy was attributed to the breaking of the electroweak symmetry.
- This is at odd with standard results.

So both the calculation and the theories need to be carefully checked.

$W^+ W^- \to \tilde{\chi}_i^0 + \tilde{G}$ Calculation

The interactions vertices between a single external gravitino and the MSSM fields

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= -\frac{i}{\sqrt{2}\,M_p}\,\bar{\psi}_\mu\,S^\mu_{\mathrm{MSSM}} + \mathrm{h.\,c.} \\
&= -\frac{i}{\sqrt{2}M_{\mathrm{Pl}}}\left[\mathcal{D}^{(\alpha)}_\mu \phi^{*i}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\chi^i_L - \mathcal{D}^{(\alpha)}_\mu\phi^i\overline{\chi}^i_L\gamma^\nu\gamma^\mu\psi_\nu\right] \\
&\quad -\frac{i}{8M_{\mathrm{Pl}}}\overline{\psi}_\mu[\gamma^\rho\,,\gamma^\sigma]\gamma^\mu\lambda^{(\alpha)}F^{(\alpha)}_{\rho\sigma}
\end{aligned}$$

The calculation when taking into account of the electroweak symmetry breaking becomes more complicated than that of the unbroken gauge symmetry case, due to various mixing in the transformation from gauge eigenstates to mass eigenstates and the additional vertices coming from the vev of the two neutral higgses

$W^+ W^- \rightarrow \tilde{\chi}_i^0 + \tilde{G}$ Calculation

- step 1: gauge eigenstates to mass eigenstates transformation
- step 2: collect all the relevant vertices, especially not forgetting the interaction terms that arise when expanding the vevs of the two higgses
- step 3: using FeynCalc to do the calculation

$W^+ W^- \to \tilde{\chi}_i^0 + \tilde{G}$ Calculation

Since several combinations of the physical masses and rotation parameters

$$M^2_{H_1^0},\ M^2_{H_2^0},\ \beta,\ \alpha,\ M_{\tilde{\chi}_j},\ U_{1j},\ U_{2j},\ V_{1j},\ V_{2j},\ M_{\tilde{\chi}_i^0},\ N_{i1},\ N_{i2},\ N_{i3},\ N_{i4}$$

appear in our (very lengthy) exact results for $\overline{|\mathcal{M}|_i^2}$, the behavior of the exact expressions in the limit of exact supersymmetry is not manifest.

$W^+ W^- \to \tilde{\chi}_i^0 + \tilde{G}$ Calculation

- step 4: expand the above parameters in terms of the relevant soft susy parameters

$$M_1 \sim M_2 \sim |m_{H_u}| \sim |m_{H_d}| \sim |\sqrt{b}| = \mathrm{O}\left(m_{\mathrm{susy}}\right)$$
$$m_{3/2} \ll m_{\mathrm{susy}} \ll M_W$$

e.g.,

$$\begin{aligned} b &= \left(\mu^2 + \frac{m_{H_u}^2 + m_{H_d}^2}{2}\right)\left[1 + \mathrm{O}\left(\frac{m_{\mathrm{susy}}^4}{M_W^4}\right)\right] \\ v_1 &= \frac{M_W}{g}\left[1 + \frac{\cos^2\theta_w}{2}\,\frac{m_{H_u}^2 - m_{H_d}^2}{M_W^2} + \mathrm{O}\left(\frac{m_{\mathrm{susy}}^4}{M_W^4}\right)\right] \end{aligned}$$

$W^+ W^- \rightarrow \tilde{\chi}_i^0 + \tilde{G}$ Calculation

the exact susy limit results can be formally written as

$$\overline{|\mathcal{M}|_i^2} = \frac{A + Bm_{\rm susy} + Cm_{3/2} + Dm_{\rm susy}^2 + Em_{\rm susy}m_{3/2} + Fm_{3/2}^2 + \ldots}{M_p^2 m_{3/2}^2}$$

We find that the "common lore" result

$$A = B = C = 0$$

is indeed recovered for all $i = 1, 2, 3, 4$

$W^+ W^- \rightarrow \tilde{\chi}_i^0 + \tilde{G}$ Calculation

The high energy limit (not lengthy any more)

$$\text{For } m_{3/2} \ll m_{\text{susy}} \ll M_W \ll \sqrt{|t|},\ \sqrt{s}:$$

$$\begin{aligned}
\overline{|\mathcal{M}|_1^2} &\simeq \frac{e^2 s\ M_1^2}{27\ \sin^2\theta_w\, M_p^2\, m_{3/2}^2} \\
\overline{|\mathcal{M}|_2^2} &\simeq \frac{e^2\, s\, \left[M_1\,(M_1 + 2\,M_2)\,\left(1 + 2\,\frac{t}{s}\right)^2 + M_2^2\left(17 + 36\,\frac{t}{s} + 36\,\frac{t^2}{s^2}\right)\right]}{108\, M_p^2\, m_{3/2}^2} \\
\overline{|\mathcal{M}|_3^2} &\simeq \overline{|\mathcal{M}|_4^2}
\end{aligned}$$

Since we consider the high energy limit ($m_{3/2} \ll \sqrt{s}$), we should expect the same results be given by gravitino Effective Lagrangian which describes the goldstino-matter coupling.

Effective Gravitino Lagrangian

replace $\psi_\mu \to \sqrt{\frac{2}{3}}\,\frac{\partial_\mu \chi}{m_{3/2}}$ in this high energy regime, therefore

$$
\begin{aligned}
\mathcal{L}_{\rm int} &= -\frac{i}{\sqrt{2}\,M_P}\,\bar{\psi}_\mu\,S^\mu_{\rm MSSM} + \mathrm{h.\,c.} \\
\to \mathcal{L}_{\rm D,\,eff} &= -\frac{i}{\sqrt{3}\,m_{3/2}\,M_P}\,\partial_\mu\bar{\chi}\;S^\mu_{\rm MSSM} + \mathrm{h.\,c.} \\
\mathcal{L}_{\rm ND,\,eff} &= \frac{i}{\sqrt{3}\,m_{3/2}\,M_P}\,\bar{\chi}\;\partial_\mu S^\mu_{\rm MSSM} + \mathrm{h.\,c.}
\end{aligned}
$$

Effective Gravitino Lagrangian

Methods and issues in going from the second line to the third line in the literature:

- Method one: write down S^{μ}_{MSSM} explicitly and do ∂_μ. Using e.o.m, so that $\partial_\mu \partial^\mu \rightarrow m^2_{soft}$ [Rychkov and Strumia, PRD, 75 (2007)] The results will be correct but the use of e.o.m for off-shell case is not justified.
- Method two: prove the equivalence of non-derivative and the derivative form of the Lagrangian, analyze each term, and see what is missing, so that they get the non-derivative form of the Effective Lagrangian [Lee and Wu, PLB, 447 (1999)]

Effective Gravitino Lagrangian

- Our method: not use the explicit expression for S^{μ}_{MSSM}, go directly from the second line to the third line, but we justify the use of equation of motion for off-shell case

Effective Gravitino Lagrangian

$$\delta\mathcal{L}_{\mathrm{MSSM}} = \frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\Phi_i}\,\delta\Phi_i + \frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\left(\partial_\mu\Phi_i\right)}\,\delta\partial_\mu\Phi_i$$

$$\delta\mathcal{L}_{\mathrm{MSSM}} = \left[\frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\Phi_i} - \partial_\mu\,\frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\left(\partial_\mu\Phi_i\right)}\right]\delta\Phi_i + \partial_\mu\left(\frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\left(\partial_\mu\Phi_i\right)}\,\delta\Phi_i\right)$$

$$\partial_\mu K^\mu \equiv \delta\mathcal{L}_{\mathrm{susy}} = \delta\mathcal{L}_{\mathrm{MSSM}} - \delta\mathcal{L}_{\mathrm{soft}} = \delta\mathcal{L}_{\mathrm{MSSM}} - \frac{\partial\mathcal{L}_{\mathrm{soft}}}{\partial\Phi_i}\,\delta\Phi_i$$

$$\frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\left(\partial_\mu\Phi_i\right)}\,\delta\Phi_i = \frac{\partial\mathcal{L}_{\mathrm{susy}}}{\partial\left(\partial_\mu\Phi_i\right)}\,\delta\Phi_i \equiv S^\mu_{\mathrm{MSSM}} + K^\mu$$

$$\partial_\mu\,S^\mu_{\mathrm{MSSM}} = \frac{\partial\mathcal{L}_{\mathrm{soft}}}{\partial\Phi_i}\,\delta\Phi_i - \left[\frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\Phi_i} - \partial_\mu\,\frac{\partial\mathcal{L}_{\mathrm{MSSM}}}{\partial\left(\partial_\mu\Phi_i\right)}\right]\delta\Phi_i$$

Effective Gravitino Lagrangian

Statement. $\left[\frac{\partial \mathcal{L}_{MSSM}}{\partial \Phi_i} - \partial_\mu \frac{\partial \mathcal{L}_{MSSM}}{\partial \partial_\mu \Phi_i}\right] \frac{\delta \Phi_i}{\epsilon}$ *does not contribute, at any order in perturbation theory, for arbitrary initial and final state, with one goldstino external line.*

$$
\begin{aligned}
\mathcal{L}_{\mathrm{int,\,eff}} = \quad & \frac{i\, m_{ij}^2}{\sqrt{3}\, M_p\, m_{3/2}} \left(\bar{\chi}\, \chi_L^i\, \phi^{*j} - \bar{\chi}_L^i\, \chi\, \phi^j\right) \\
& - \frac{i}{\sqrt{3}\, M_p\, m_{3/2}} \left[(AW)_i\ \bar{\chi}\, \chi_L^i - (AW)_i^*\ \bar{\chi}_L^i\, \chi\right] \\
& - \frac{M_\alpha}{4\sqrt{6}\, M_p\, m_{3/2}}\, F_{\mu\nu}^{(\alpha)a}\, \bar{\chi}\, [\gamma^\mu,\, \gamma^\nu]\ \lambda^{(\alpha)a} \\
& - \frac{i\, g_\alpha\, M_\alpha}{\sqrt{6}\, M_p\, m_{3/2}} \left(\phi^{*i}\ T_{ij}^a\, \phi^j\right) \bar{\chi}\, \gamma^5\, \lambda^{(\alpha)a}
\end{aligned}
$$

Using this Effective Lagrangian, we **precisely** *recovered the high energy limit results shown previously.*

Effective Gravitino Lagrangian

Proof. Consider

$$\langle f|\mathrm{T}\{\exp[i\int d^4x\mathcal{L}_{\text{int.}}]\int d^4y[\frac{\partial\mathcal{L}_{\text{MSSM}}}{\partial\Phi_i}-\partial_\mu\frac{\mathcal{L}_{\text{MSSM}}}{\partial(\partial_\mu\Phi_i)}]\frac{\delta\Phi_i}{\epsilon}\chi\}|i\rangle,$$

where $\mathcal{L}_{\text{MSSM}}=\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{int.}}$.
We have

$$\begin{aligned}
&\frac{\partial\mathcal{L}_{\text{MSSM}}}{\partial\Phi_i}-\partial_\mu\frac{\mathcal{L}_{\text{MSSM}}}{\partial(\partial_\mu\Phi_i)}\\
=\;&\left(\frac{\partial\mathcal{L}_{\text{free}}}{\partial\Phi_i}-\partial_\mu\frac{\mathcal{L}_{\text{free}}}{\partial(\partial_\mu\Phi_i)}\right)+\left(\frac{\partial\mathcal{L}_{\text{int.}}}{\partial\Phi_i}-\partial_\mu\frac{\mathcal{L}_{\text{int.}}}{\partial(\partial_\mu\Phi_i)}\right)\\
=\;&\text{free e.o.m. of }\Phi_i+\left(\frac{\partial\mathcal{L}_{\text{int.}}}{\partial\Phi_i}-\partial_\mu\frac{\mathcal{L}_{\text{int.}}}{\partial(\partial_\mu\Phi_i)}\right)
\end{aligned}$$

Effective Gravitino Lagrangian

Example:

Consider $\mathcal{L}_{\text{int.}} = A\Phi_i^4$, where A is coefficient not including Φ_i, but can include any other fields.

using $\Phi_i(z)$ free e.o.m. of $\Phi_i(y) = -(\Box + m^2)D(y-z) = i\delta(y-z)$

we get $$\int d^4z\mathcal{L}_{\text{int.}} \int d^4y \text{ free e.o.m. of } \Phi_i = i\int d^4y\, 4\, A\Phi_i^3(y)$$

$$= \quad i\int d^4y \left[\frac{\partial\mathcal{L}_{\text{int.}}}{\partial\Phi_i} - \partial_\mu \frac{\mathcal{L}_{\text{int.}}}{\partial(\partial_\mu\Phi_i)}\right].$$

This argument is also valid if the interaction term contains $\partial^\mu\Phi_i$, since one can always have $\int d^4x\partial^\mu\Phi_i A_\mu\Phi_i^n =$ $\int d^4x\frac{1}{n+1}\partial^\mu(\Phi_i^{n+1})A_\mu = -\int d^4x\frac{1}{n+1}\Phi_i^{n+1}\partial^\mu A_\mu$. This takes us back to the previous case.

Summary

- We calculate the exact unpolarized square amplitude for $W^+ W^- \rightarrow \tilde{\chi}_i^0 + \tilde{G}$ using the full Lagrangian of the gravitino interaction.
- We find the high energy limit results of this process, and we prove the previous claim of the bad high energy behavior in broken gauge theories is not correct.
- We come up with a simple derivation of the gravitino non-derivative Effective Lagrangian for the MSSM, and give a general proof of the equivalence of the non-derivative and derivative form of the Lagrangian.