Range of Majorana Neutrino Mixing Parameters Including Yukawa Couplings

Northwestern University
Wei-Chih Huang

In collaboration with André de Gouvêa, Shashank Shalgar

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Motivations

- To find the physical range of parameters in the Maki-Nakagawa-Sakata (MNS) matrix.
- To understand the effect of Yukawa couplings on the physical range of parameters.
- Application to the low energy effective theory in the case of Casas-Ibarra parametrization.

The MNS Matrix

$$\mathcal{L}_{\nu SM} \supset -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\nu}_\alpha \gamma^\mu \ell_\alpha W_\mu^+ + \bar{\ell}_\alpha \gamma^\mu \nu_\alpha W_\mu^-\right)$$

$$-\frac{g}{2\cos\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu \nu_\alpha Z_\mu - \sum_{\alpha=e,\mu,\tau} m_\alpha \bar{e}_\alpha \ell_\alpha - \frac{1}{2} \sum_i \bar{\nu}_i^c m_i \nu_i.$$

$$\nu_\alpha = U_{\alpha i} \nu_i$$

The MNS Matrix

$$\mathcal{L}_{\nu SM} \supset \sum_{\alpha,i} \bar{\ell}_\alpha \gamma^\mu U_{\alpha i} \nu_i W_\mu^- - \frac{g}{2\cos\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu \nu_\alpha Z_\mu - \sum_{\alpha=e,\mu,\tau} m_\alpha \bar{e}_\alpha \ell_\alpha - \frac{1}{2} \sum_i \bar{\nu}_i^c m_i \nu_i.$$

The Lagrange is invariant under the following transformations:
(3 active neutrinos and N right-handed neutrinos)

$$\begin{aligned} U'(\theta',\phi') &= \prod_{\alpha=1}^{3+n} P^\alpha(\pi) U(\theta,\phi) \prod_{i=2}^{3+n} P^i(\pi) \\ \bar{\ell'}_\alpha &= \prod_{\alpha=1}^{3+n} P^\alpha(\pi) \bar{\ell}_\alpha \\ \nu'_i &= \prod_{i=2}^{3+n} P^i(\pi) \nu_i \end{aligned}$$

where $P^{i(\alpha)}(\phi)$ is a diagonal $(3+N)\times(3+N)$ matrix whose diagonal entries are all unity except for the i, i element, given by $e^{i\phi}$. Note that $P^{i(\alpha)}(\phi)^2$ is the unit matrix

A simple 1+1 case

$$U = \begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix} = \mathbf{R^{12}}(\theta)\mathbf{P^2}(\phi) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

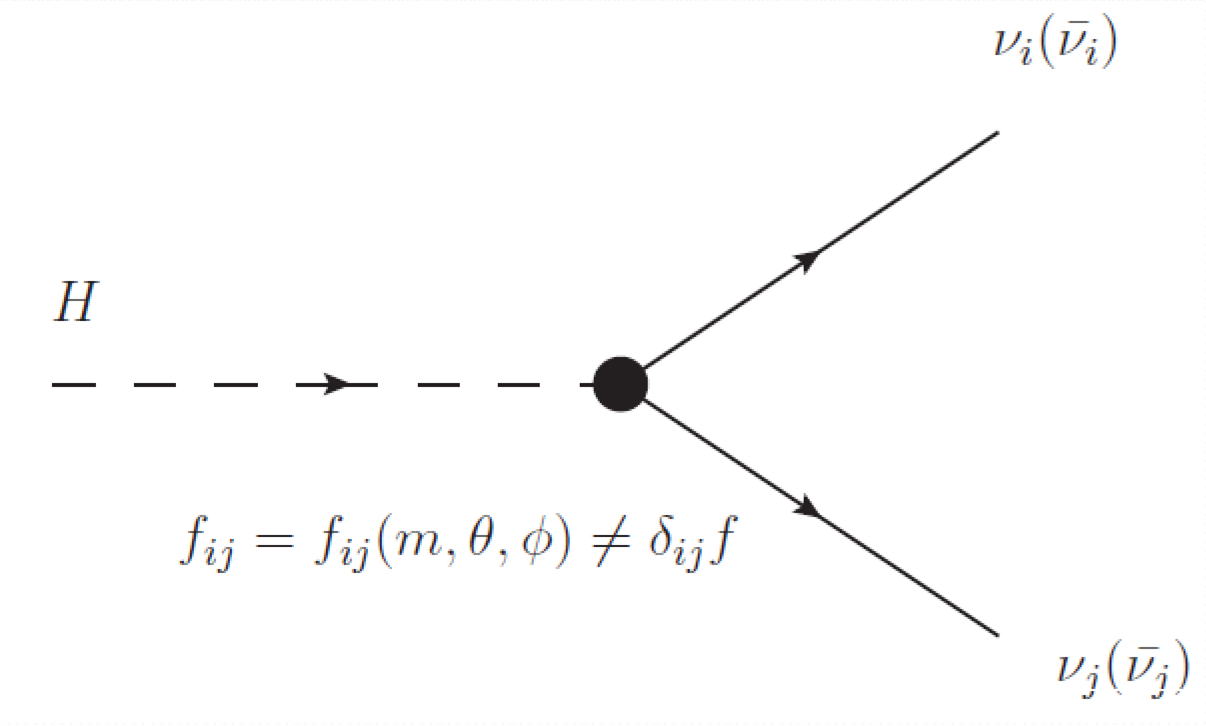
$$\begin{pmatrix} \cos(\pi+\theta) & \sin(\pi+\theta) \\ -\sin(\pi+\theta) & \cos(\pi+\theta) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

It shows $\theta + \pi$ is physically equivalent to θ.

Results

- All Majorana phases $\phi_i \in [0, \pi]$, $i = 2, \ldots, 3 + N$;
- All Dirac phases $\delta, \delta_{ij} \in [-\pi, \pi[$;
- All mixing angles can be constrained to the first quadrant: $\cos\theta_{ij}, \sin\theta_{ij} > 0$.

In the Presence of Yukawa Interactions



In the Presence of Yukawa Interactions

$$\mathcal{L} \supset y^{\alpha\beta}\bar{N}^{\alpha}\nu^{\beta}H + \text{h.c.} + \frac{M}{2}\bar{N}^{c}N$$

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \qquad m_\nu = U M^D U^\dagger$$

For Type-I seesaw, 3 active and 3 right-handed neutrinos, we can rewrite Yukawa interactions in the mass basis in terms of angles and phases.

$$\begin{aligned} \nu_i y_{ij} \nu_j \;=\; & \frac{1}{v}\begin{pmatrix} \nu_1 & \dots & \nu_6 \end{pmatrix} U^T \begin{pmatrix} 1_{3\times3} & 0 \\ 0 & 0_{3\times3} \end{pmatrix} U^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_6 \end{pmatrix} U^\dagger \begin{pmatrix} 0_{3\times3} & 0 \\ 0 & 1_{3\times3} \end{pmatrix} U \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix} \\ & + \frac{1}{v}\begin{pmatrix} \nu_1 & \dots & \nu_6 \end{pmatrix} U^T \begin{pmatrix} 0_{3\times3} & 0 \\ 0 & 1_{3\times3} \end{pmatrix} U^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_6 \end{pmatrix} U^\dagger \begin{pmatrix} 1_{3\times3} & 0 \\ 0 & 0_{3\times3} \end{pmatrix} U \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix} \end{aligned}$$

In the Presence of Yukawa Interactions

- We express Yukawa couplings in terms of angles, phases and masses.

$$\begin{aligned} \nu_\alpha y^{\alpha\beta}(\theta_1+\pi,\ldots)\nu_\beta &= \nu_\alpha(U^*(\theta_1+\pi,\ldots)M^D U^\dagger(\theta_1+\pi,\ldots))^{\alpha\beta}\nu_\beta \\ &= \nu_\alpha P_1(U^*(\theta_1,\ldots)P_2 M^D P_2 U^\dagger(\theta_1,\ldots))^{\alpha\beta}P_1\nu_\beta \\ &= \nu'_\alpha y^{\alpha\beta}(\theta_1,\ldots)\nu'_\beta \end{aligned}$$

It implies that by applying the same procedure as before, $\theta_1+\pi$ is equivalent to θ_1, in the presence of Yukawa couplings.

Results

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Casas-Ibarra Parametrization-I

$$
\begin{aligned}
m_\nu &= \begin{pmatrix} 0_{3\times3} & yv \\ (yv)^t & M_{3\times3} \end{pmatrix} \\
U &= \begin{pmatrix} V & \Theta \\ -\Theta^\dagger V & 1_{3\times3} \end{pmatrix} = \begin{pmatrix} 1_{3\times3} & \Theta \\ -\Theta^\dagger & 1_{3\times3} \end{pmatrix} \begin{pmatrix} V & 0 \\ 0 & 1_{3\times3} \end{pmatrix} \\
(V^t yvM^{-1/2})(V^t yvM^{-1/2})^t &= \sqrt{\mathrm{diag}(m_1, m_2, m_3)} R^t R \sqrt{\mathrm{diag}(m_1, m_2, m_3)} \\
yv &= V^* \sqrt{\mathrm{diag}(m_1, m_2, m_3)} R^t M^{1/2} \\
\Theta &= V \sqrt{\mathrm{diag}(m_1, m_2, m_3)} R^\dagger M^{-1/2},
\end{aligned}
$$

The concept is to express yv and Θ in terms of V, R and masses.

$$M^D = U^\dagger m_\nu U$$

Casas-Ibarra Parametrization-II

$$U(\theta+\pi)$$

$$\begin{aligned}
&= \begin{pmatrix} V(\theta+\pi) & V(\theta+\pi)\sqrt{\mathrm{diag}(m_1,m_2,m_3)}R^\dagger M^{-1/2} \\ -M^{-1/2}R\sqrt{\mathrm{diag}(m_1,m_2,m_3)} & 1_{3\times3} \end{pmatrix} \\
&= \begin{pmatrix} P_1V(\theta)P_2 & P_1V(\theta)P_2\sqrt{\mathrm{diag}(m_1,m_2,m_3)}R^\dagger M^{-1/2} \\ -M^{-1/2}R\sqrt{\mathrm{diag}(m_1,m_2,m_3)} & 1_{3\times3} \end{pmatrix} \\
&= \begin{pmatrix} P_1 & 0 \\ 0 & 1_{3\times3} \end{pmatrix} \begin{pmatrix} V(\theta)P_2 & V(\theta)\sqrt{\mathrm{diag}(m_1,m_2,m_3)}P_2R^\dagger M^{-1/2} \\ -M^{-1/2}R\sqrt{\mathrm{diag}(m_1,m_2,m_3)} & 1_{3\times3} \end{pmatrix} \\
&= \begin{pmatrix} P_1 & 0 \\ 0 & 1_{3\times3} \end{pmatrix} \begin{pmatrix} V(\theta) & V(\theta)\sqrt{\mathrm{diag}(m_1,m_2,m_3)}P_2R^\dagger M^{-1/2} \\ -M^{-1/2}R\sqrt{\mathrm{diag}(m_1,m_2,m_3)}P_2 & 1_{3\times3} \end{pmatrix} \begin{pmatrix} P_2 & 0 \\ 0 & 1_{3\times3} \end{pmatrix} \\
&= \begin{pmatrix} P_1 & 0 \\ 0 & 1_{3\times3} \end{pmatrix} \begin{pmatrix} V(\theta) & V(\theta)\sqrt{\mathrm{diag}(m_1,m_2,m_3)}P_2R^\dagger M^{-1/2} \\ -M^{-1/2}RP_2\sqrt{\mathrm{diag}(m_1,m_2,m_3)} & 1_{3\times3} \end{pmatrix} \begin{pmatrix} P_2 & 0 \\ 0 & 1_{3\times3} \end{pmatrix}
\end{aligned} \qquad \text{(III.14)}$$

An Example on Leptogenesis

$$\epsilon_1 = \frac{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) + \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}$$

$$\simeq \frac{1}{8\pi} \frac{1}{(h_\nu h_\nu)_{11}} \sum_{i=2,3} \mathrm{Im}\left\{(h_\nu h_\nu^\dagger)^2_{1i}\right\} \cdot \left[f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right)\right]$$

An Example on Leptogenesis

$$V = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

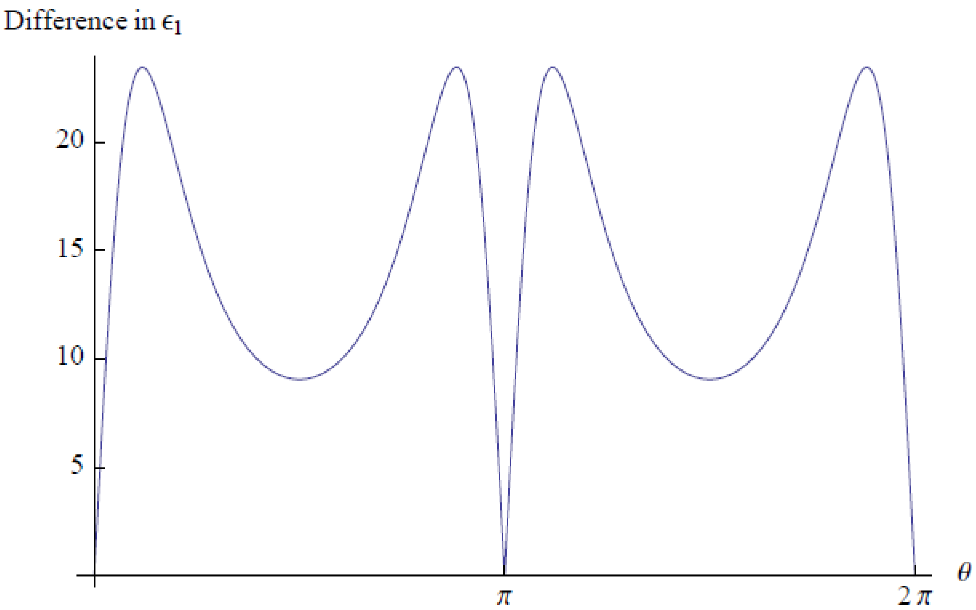
$$\sqrt{\mathrm{diag}(m_1, m_2)} = \begin{pmatrix} \sqrt{m_1} & 0 \\ 0 & \sqrt{m_2} \end{pmatrix}$$

$$R^t = \begin{pmatrix} \cos\beta & \sin\beta e^{i\theta} \\ -\sin\beta e^{-i\theta} & \cos\beta \end{pmatrix}$$

$$\bar{R}^t = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot R^t$$

$$M^{1/2} = \begin{pmatrix} \sqrt{M_1} & 0 \\ 0 & \sqrt{M_2} \end{pmatrix}$$

$$h_\nu v = V^* \sqrt{\mathrm{diag}(m_1, m_2)} R^t M^{1/2}.$$



Conclusions

- Introduction of Yukawa interactions has no effect on the physical range of parameters.
- In the Casas-Ibarra parametrization, DetR is physical, i.e. Majorana phases defined in the low-energy effective theory range from $-\pi$ to π if all mixing angles in V are to be constrained to the first quadrant.