# Azimuthal correlation in decays of Higgs/massive-graviton to vector-boson pairs

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# Azimuthal correlations in $H \to ZZ \to 4f$

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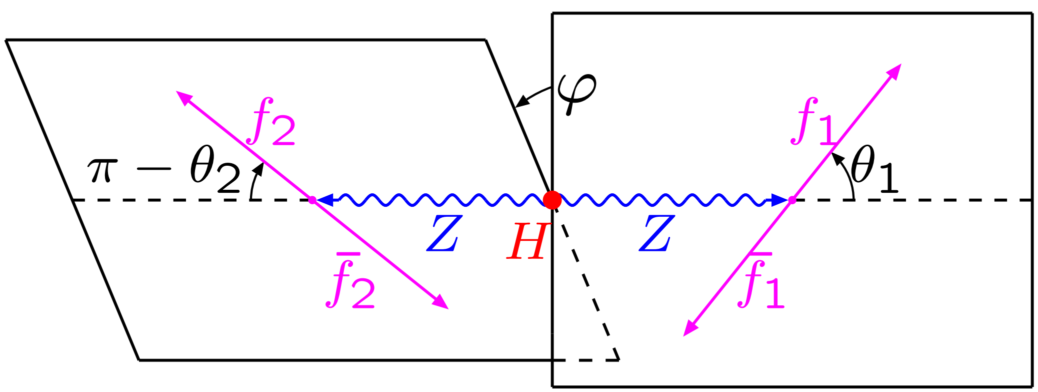
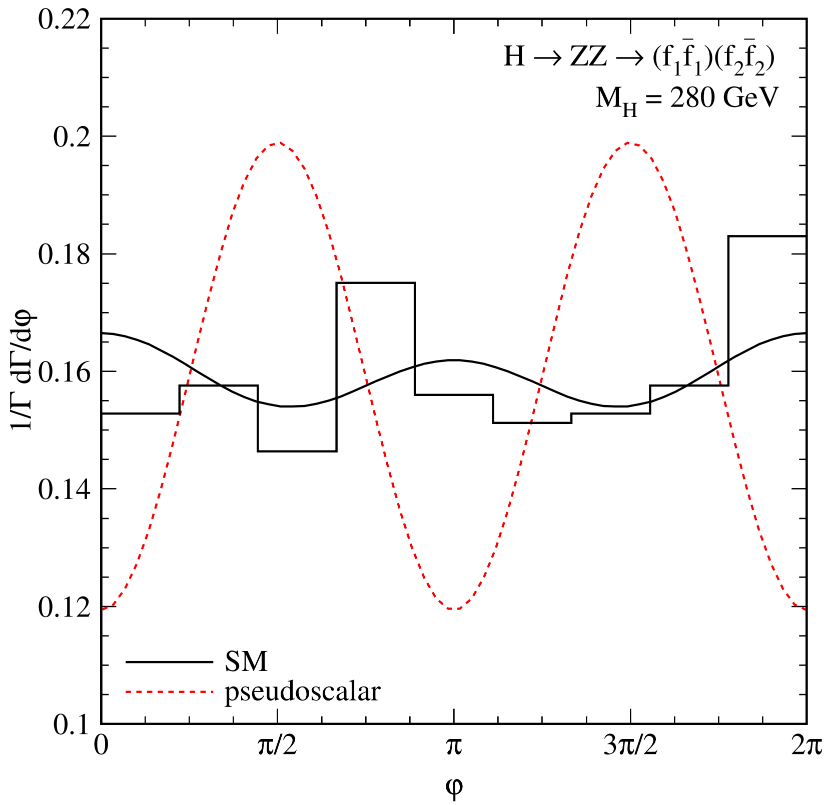



- Azimuthal correlations reflect the tensor structures of the $HVV$ coupling. Why does each tensor structure give such a distribution?
- How about the correlation for spin-2 massive gravitons?

# Contents

1. Introduction

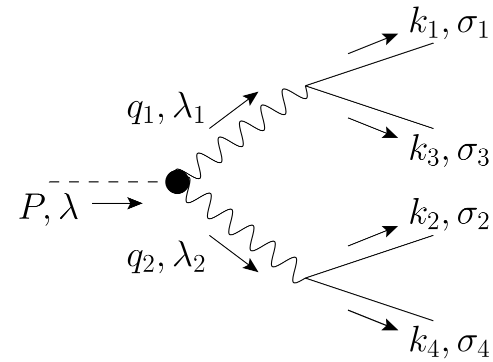
2. Helicity amplitude formalism and kinematics

3. Helicity amplitudes for $X \to VV \to 4f$

4. Azimuthal correlations between the decay planes
   - $CP$-even/odd Higgs boson ($J = 0$) decays
   - Randall-Sundrum (RS) massive graviton ($J = 2$) decays

5. Summary

# The helicity amplitude formalism




The helicity amplitudes for $X \to VV \to 4f$

$$\mathcal{M}^{\lambda}_{\sigma_1\sigma_3,\sigma_2\sigma_4} = \Gamma^{\mu_1\mu_2}_{XVV}(q_1, q_2; \lambda) \frac{-g_{\mu_1'\mu_1} + \frac{q_{1\mu_1'} q_{1\mu_1}}{m_V^2}}{q_1^2 - m_V^2} J^{\mu_1'}(k_1, k_3; \sigma_1, \sigma_3) \frac{-g_{\mu_2'\mu_2} + \frac{q_{2\mu_2'} q_{2\mu_2}}{m_V^2}}{q_2^2 - m_V^2} J^{\mu_2'}(k_2, k_4; \sigma_2, \sigma_4)$$

can be expressed by using

completeness relation $\qquad -g_{\mu'\mu} + \dfrac{q_{i\mu'} q_{i\mu}}{q_i^2} = \sum_{\lambda_i = \pm, 0} \epsilon_{\mu'}(q_i, \lambda_i)^* \, \epsilon_\mu(q_i, \lambda_i)$

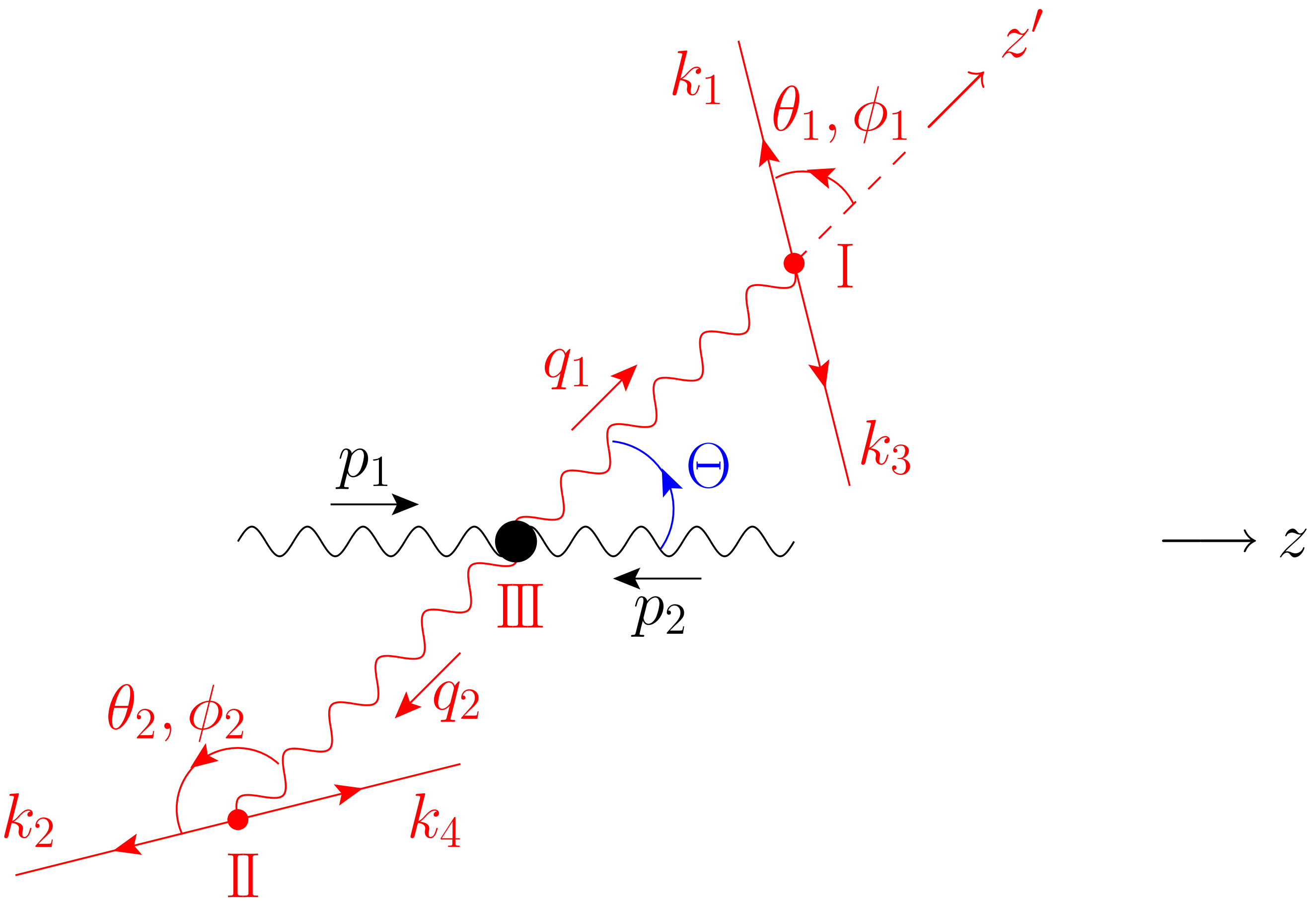
current conservation $\qquad q_{i\mu} J^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) = 0$

as the product of the three helicity amplitudes summed over the polarization of the intermediate vector-bosons:

$$\begin{aligned}
\mathcal{M}^{\lambda}_{\sigma_1\sigma_3,\sigma_2\sigma_4} &= \Gamma^{\mu_1\mu_2}_{XVV}(q_1, q_2; \lambda) \\
&\quad \times \frac{1}{q_1^2 - m_V^2} J^{\mu_1'}(k_1, k_3; \sigma_1, \sigma_3) \sum_{\lambda_1 = \pm, 0} \epsilon_{\mu_1'}(q_1, \lambda_1) \, \epsilon_{\mu_1}(q_1, \lambda_1)^* \\
&\quad \times \frac{1}{q_2^2 - m_V^2} J^{\mu_2'}(k_2, k_4; \sigma_2, \sigma_4) \sum_{\lambda_2 = \pm, 0} \epsilon_{\mu_2'}(q_2, \lambda_2) \, \epsilon_{\mu_2}(q_2, \lambda_2)^* \\
&= \frac{1}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)} \sum_{\lambda_1 = \pm, 0} \sum_{\lambda_2 = \pm, 0} \mathcal{M}_X{}^{\lambda}_{\lambda_1\lambda_2} \, \mathcal{J}_1{}^{\lambda_1}_{\sigma_1\sigma_3} \, \mathcal{J}_2{}^{\lambda_2}_{\sigma_2\sigma_4}
\end{aligned}$$

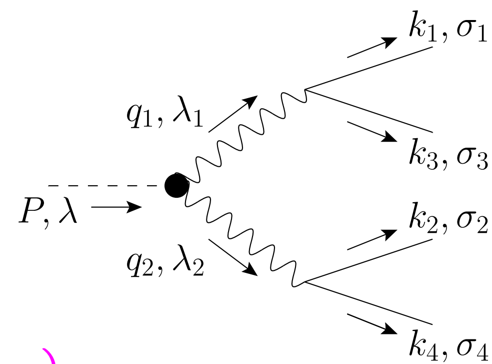
Note: The total amplitudes are generally the coherent sum of the 9 amplitudes which have the different helicity combinations of the decaying vector-bosons.

# Kinematics for $gg/q\bar{q} \to X \to VV \to (f\bar{f})(f\bar{f})$




Note: The azimuthal angles ($\phi_1$ and $\phi_2$) are measured individually from the $X$ production-decay ($gg/q\bar{q} \to X \to VV$) plane.

# Current amplitudes

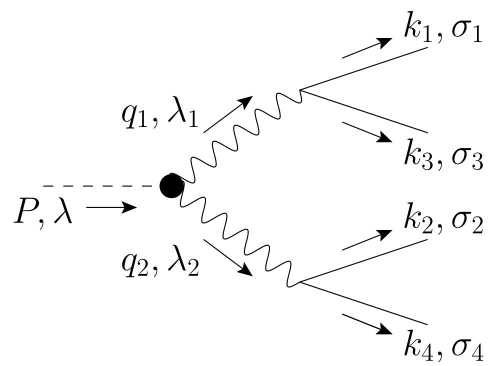



$$\mathcal{J}_{i\,\sigma_i\sigma_{i+2}}^{\lambda_i} = \epsilon_\mu(q_i,\lambda_i)\, J^\mu_{Vff'}(k_i,k_{i+2};\sigma_i,\sigma_{i+2})$$
$$= \epsilon_\mu(q_i,\lambda_i)\, g^{Vff'}_{\sigma_i}\, \bar{u}_{f'}(k_{i+2},\sigma_{i+2})\,\gamma^\mu\, u_f(k_i,\sigma_i)$$

| | $\mathcal{J}_{1\,\sigma_1\sigma_3}^{\lambda_1}$ | |
|---|---|---|
| • | $\mathcal{J}_{1\,+-}^{+} = -\left(\mathcal{J}_{1\,-+}^{-}\right)^*$ | $\frac{1}{2}(1+\cos\theta_1)\, e^{i\phi_1}$ |
| • | $\mathcal{J}_{1\,+-}^{0} = \mathcal{J}_{1\,-+}^{0}$ | $\frac{1}{\sqrt{2}}\sin\theta_1$ |
| • | $\mathcal{J}_{1\,+-}^{-} = -\left(\mathcal{J}_{1\,-+}^{+}\right)^*$ | $\frac{1}{2}(1-\cos\theta_1)\, e^{-i\phi_1}$ |
| | $\mathcal{J}_{1\,++}^{\lambda_1} = \mathcal{J}_{1\,--}^{\lambda_1}$ | 0 |

Note: The amplitudes for transversely polarized vector-bosons have a phase, $e^{+i\phi_1}$ or $e^{-i\phi_1}$, while those for londitudinal ones do not.

# $XVV$ vertex




- $X \to VV$ decay amplitudes:

$$\mathcal{M}_{X^{\lambda}_{\lambda_1\lambda_2}} = \Gamma^{\mu\nu}_{XVV}(q_1, q_2; \lambda)\, \epsilon_\mu(q_1, \lambda_1)^* \, \epsilon_\nu(q_2, \lambda_2)^*$$

- XVV vertex:

| $X$ | $(\lambda)$ | $\Gamma^{\mu\nu}_{XVV}/g_{XVV}$ |
|---|---|---|
| CP-even Higgs ($H$) | $(0)$ | $g^{\mu\nu}$ |
| CP-odd Higgs ($A$) | $(0)$ | $\epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta}$ |
| RS massive graviton ($G$) | $(\pm 2, \pm 1, 0)$ | $\epsilon_{\alpha\beta}\, \hat{\Gamma}^{\alpha\beta\mu\nu}_{GVV}$ |

The polarization tensor for a spin-2 particle:

$$\epsilon^{\mu\nu}(p, \pm 2) = \epsilon^\mu(p, \pm)\, \epsilon^\nu(p, \pm)$$

$$\epsilon^{\mu\nu}(p, \pm 1) = \frac{1}{\sqrt{2}}\Big[\epsilon^\mu(p, \pm)\, \epsilon^\nu(p, 0) + \epsilon^\mu(p, 0)\, \epsilon^\nu(p, \pm)\Big]$$

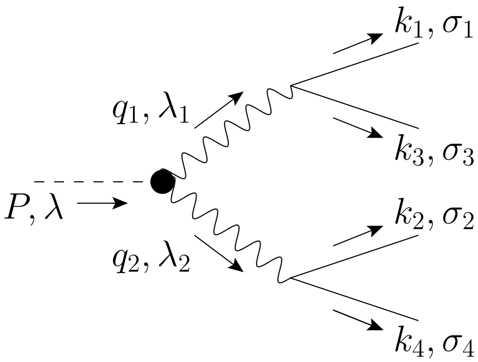
$$\epsilon^{\mu\nu}(p, 0) = \frac{1}{\sqrt{6}}\Big[\epsilon^\mu(p, +)\, \epsilon^\nu(p, -) + \epsilon^\mu(p, -)\, \epsilon^\nu(p, +) + 2\, \epsilon^\mu(p, 0)\, \epsilon^\nu(p, 0)\Big]$$

The $GVV$ vertex: [Giudice, Rattazzi, Wells ('99), T.Han, J.Lykken, R.Zhang ('99)]

$$\hat{\Gamma}^{\mu\nu,\rho\sigma}_{GVV}(q_1, q_2) = (m_V^2 + q_1 \cdot q_2)\, C^{\mu\nu,\rho\sigma} + D^{\mu\nu,\rho\sigma}(q_1, q_2)$$

$$C^{\mu\nu,\rho\sigma} = g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\nu} g^{\rho\sigma}$$

$$D^{\mu\nu,\rho\sigma}(q_1, q_2) = g^{\mu\nu} q_1^\sigma q_2^\rho - \Big[g^{\mu\sigma} q_1^\nu q_2^\rho + g^{\mu\rho} q_1^\sigma q_2^\nu - g^{\rho\sigma} q_1^\mu q_2^\nu + (\mu \leftrightarrow \nu)\Big]$$




# $X \to VV$ decay amplitudes

in the $q_{1,2}^2 \to m_V^2$ limit ($\beta = \sqrt{1 - 4m_V^2/M^2}$)

| $\lambda$ | $(\lambda_1\lambda_2)$ | $H$ | $A$ | $G$ |
|---|---|---|---|---|
| $\pm 2$ | $(\pm\mp)$ | $\cdot$ | $\cdot$ | $-M^2$ |
| $\pm 1$ | $(\pm 0), (0\mp)$ | $\cdot$ | $\cdot$ | $\sqrt{\frac{1}{2}(1-\beta^2)}M^2$ |
| 0 | $(\pm\pm)$ | $-1$ | $\mp\frac{i}{2}\beta M^2$ | $-\frac{1}{\sqrt{6}}(1-\beta^2)M^2$ |
| 0 | $(00)$ | $(1+\beta^2)/(1-\beta^2)$ | 0 | $-\frac{1}{\sqrt{6}}(2-\beta^2)M^2$ |

- light $H\,(\beta \to 0; M \sim 2m_V)$: decay into both longitudinally- and transversely-polarized VBs.
- heavy $H\,(\beta \to 1; M \gg m_V)$: decay into the longitudinally-polarized VBs.
- $A$: decay only into transversely-polarized VBs.
- $G\,(\beta \to 1)$: decay into both longitudinally- and $(\lambda_1\lambda_2) = (\pm\mp)$ transversely-polarized VBs.

# Azimuthal correlations for Higgs bosons

The $J = 0$ total amplitudes are the sum of the three amplitudes:

$$\mathcal{M}^{\lambda=0}_{\sigma_1\sigma_3,\sigma_2\sigma_4} = \frac{1}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)} \sum_{\lambda_1=\pm,0} \sum_{\lambda_2=\pm,0} \mathcal{M}_X{}^{\lambda=0}_{\lambda_1\lambda_2} \mathcal{J}1^{\lambda_1}_{\sigma_1\sigma_3} \mathcal{J}2^{\lambda_2}_{\sigma_2\sigma_4}$$

$$\sim \quad \mathcal{M}_X{}^0_{++} \mathcal{J}1^{+}_{\sigma_1\sigma_3} \mathcal{J}2^{+}_{\sigma_2\sigma_4} e^{-i(\phi_1-\phi_2)} + \mathcal{M}_X{}^0_{00} \mathcal{J}1^{0}_{\sigma_1\sigma_3} \mathcal{J}2^{0}_{\sigma_2\sigma_4} + \mathcal{M}_X{}^0_{--} \mathcal{J}1^{-}_{\sigma_1\sigma_3} \mathcal{J}2^{-}_{\sigma_2\sigma_4} e^{i(\phi_1-\phi_2)}$$

Therefore, the squared amplitudes are generally given by

$$\sum_{\sigma_{1,\cdots,4}} \left|\mathcal{M}^{\lambda=0}_{\sigma_1\sigma_3,\sigma_2\sigma_4}\right|^2 = \Sigma_0 + \Sigma_1 \cos\Delta\phi + \Sigma_2 \cos 2\Delta\phi \quad (\Delta\phi \equiv \phi_1 - \phi_2)$$

The azimuthal correlation is manifestly expressed by the interference among different helicity states of the intermediate vector-bosons.

The different tensor structures of the $H/A$ coupling to a $Z$-pair give rise to the different azimuthal angle dependences:

$H_{\text{(heavy)}}$ : $\mathcal{M}_{00} \gg \mathcal{M}_{++} = \mathcal{M}_{--}$ $\Rightarrow d\Gamma/d\Delta\phi \sim$ constant

$H_{\text{(light)}}$ : $\mathcal{M}_{00} \sim \mathcal{M}_{++} = \mathcal{M}_{--}$ $(\Sigma_1 \ll 1)$ $\Rightarrow d\Gamma/d\Delta\phi \sim \Sigma_0 + |\Sigma_2| \cos 2\Delta\phi$

$A$ : $\mathcal{M}_{00} = 0,\ \mathcal{M}_{++} = -\mathcal{M}_{--}$ $\Rightarrow d\Gamma/d\Delta\phi \sim \Sigma_0 - |\Sigma_2| \cos 2\Delta\phi$

# Azimuthal correlations in $H \to ZZ \to 4f$

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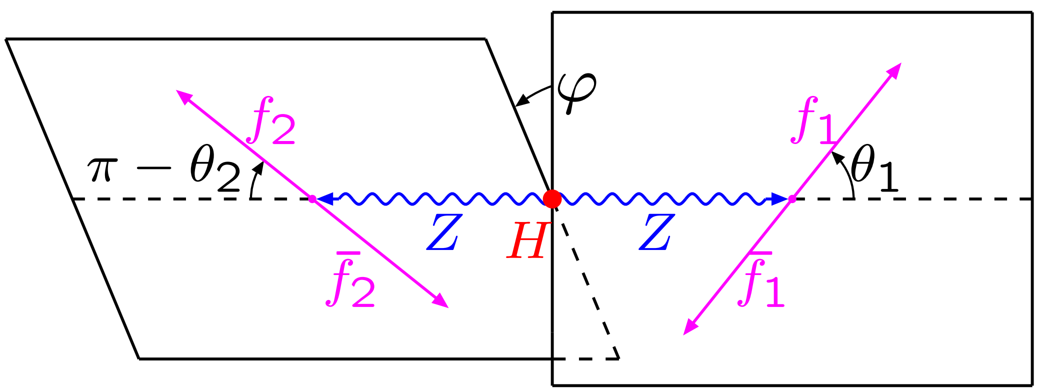
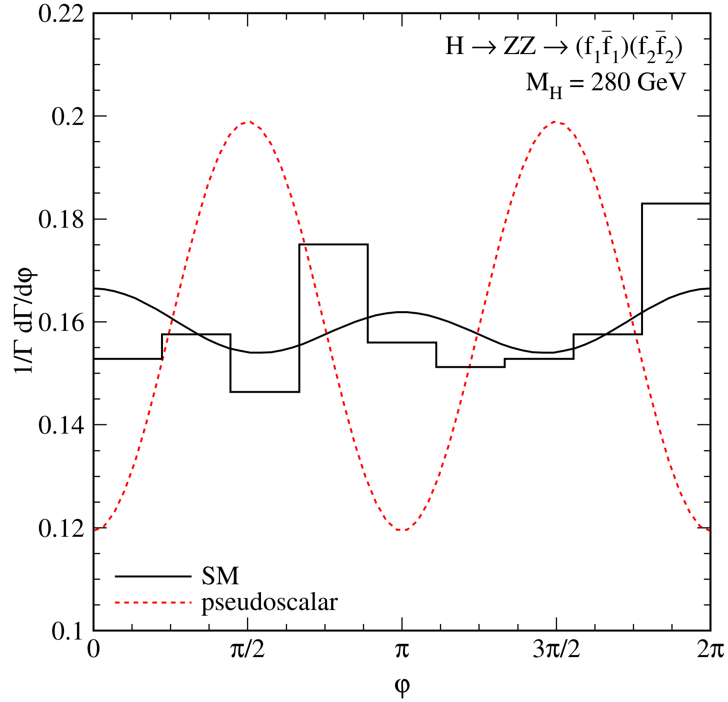


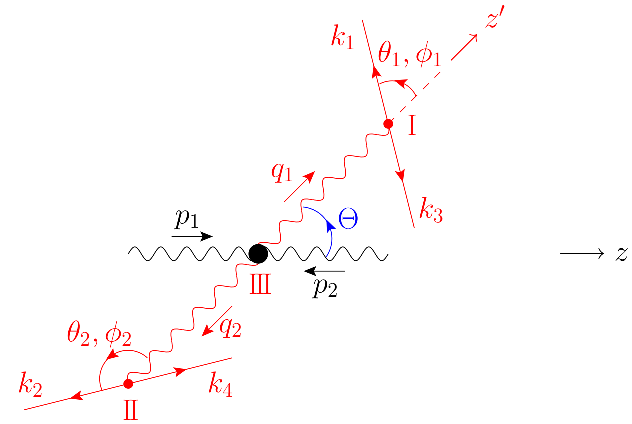




- Azimuthal correlations reflect the tensor structures of the $HVV$ coupling. Why does each tensor structure give such a distribution?

$$\text{SM (light) } H \Rightarrow d\Gamma/d\varphi \sim \Sigma_0 + |\Sigma_2| \cos 2\varphi$$
$$\text{pseudoscalar } H \Rightarrow d\Gamma/d\varphi \sim \Sigma'_0 - |\Sigma'_2| \cos 2\varphi$$

- How about the correlation for massive gravitons? $\Rightarrow$ Next slides!

# Azimuthal correlations for gravitons




The amplitudes for $gg/q\bar{q} \to G \to ZZ \to 4f$ are

$$\mathcal{M}_{\sigma^{\mathrm{I}}_{1,2};\sigma_{1,\cdots,4}} \propto \mathcal{M}_{\mathrm{I}\,\sigma^{\mathrm{I}}_1\sigma^{\mathrm{I}}_2}^{\lambda^{\mathrm{I}}=\sigma^{\mathrm{I}}_1-\sigma^{\mathrm{I}}_2} \sum_{\lambda_1=\pm,0}\sum_{\lambda_2=\pm,0} \frac{d^2_{\lambda^{\mathrm{I}},\lambda}(\Theta)}{P^2-M^2+iM\Gamma} \mathcal{M}_{G\,\lambda_1\lambda_2}^{\lambda=\lambda_1-\lambda_2} \mathcal{J}1^{\lambda_1}_{\sigma_1\sigma_3}\mathcal{J}2^{\lambda_2}_{\sigma_2\sigma_4}$$

$$\xrightarrow{\beta=1}\ d^2_{\lambda^{\mathrm{I}},+2}(\Theta)\mathcal{M}_{G\,+-}^{+2}\,\mathcal{J}1^{+}_{\sigma_1\sigma_3}\mathcal{J}2^{-}_{\sigma_2\sigma_4}e^{-i(\phi_1+\phi_2)} + d^2_{\lambda^{\mathrm{I}},0}(\Theta)\mathcal{M}_{G\,00}^{0}\,\mathcal{J}1^{0}_{\sigma_1\sigma_3}\mathcal{J}2^{0}_{\sigma_2\sigma_4}$$
$$+\, d^2_{\lambda^{\mathrm{I}},-2}(\Theta)\mathcal{M}_{G\,-+}^{-2}\mathcal{J}1^{-}_{\sigma_1\sigma_3}\mathcal{J}2^{+}_{\sigma_2\sigma_4}e^{i(\phi_1+\phi_2)}$$

Therefore, the squared amplitudes in the $M \gg m_V$ $(\beta = 1)$ limit are

$$\sum_{\sigma_{1,\cdots,4}} \left|\mathcal{M}_{\sigma^{\mathrm{I}}_{1,2};\sigma_{1,\cdots,4}}\right|^2 = \Sigma_0 + \Sigma_1 \cos\Phi + \Sigma_2 \cos 2\Phi \quad (\Phi \equiv \phi_1 + \phi_2)$$

$\lambda^{\mathrm{I}} = \pm 2$ ($gg$ initial state)

$$\Sigma_0 = 3 + 10\cos^2\Theta + 3\cos^4\Theta$$
$$\Sigma_1 = \kappa_1\kappa_2 \frac{9\pi^2}{64}(1-\cos^4\Theta)$$
$$\Sigma_2 = \frac{1}{4}\sin^4\Theta$$

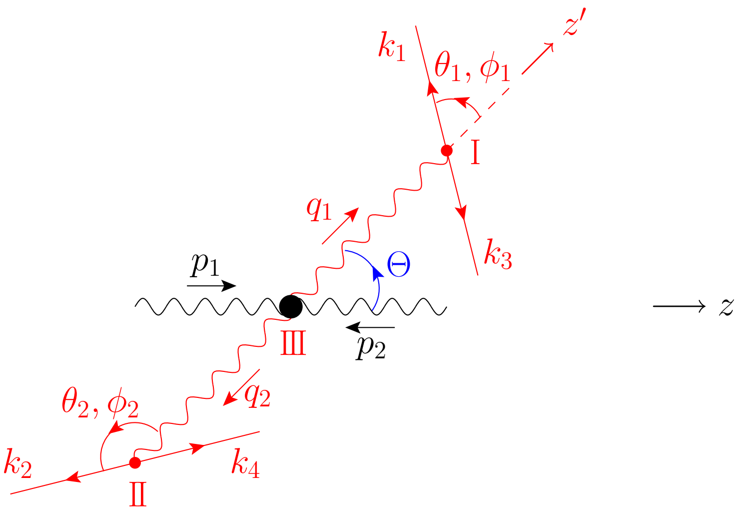
$\lambda^{\mathrm{I}} = \pm 1$ ($q\bar{q}$ initial state)

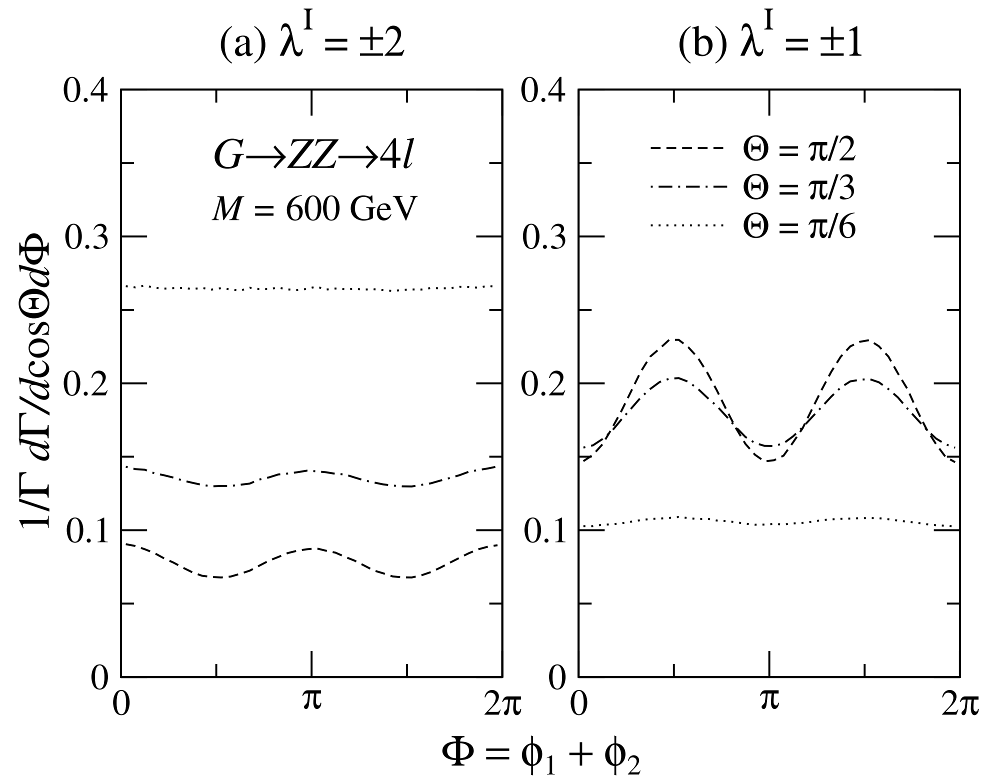
$$\Sigma_0 = 8 + 4\cos^2\Theta - 12\cos^4\Theta$$
$$\Sigma_1 = -\kappa_1\kappa_2 \frac{9\pi^2}{16}\sin^2\Theta\cos^2\Theta$$
$$\Sigma_2 = -\sin^4\Theta$$

$\Longrightarrow$ The azimuthal $\Phi$ correlations depend on $\Theta$ and $\lambda^{\mathrm{I}}$.

# Azimuthal correlations in $pp \to G \to ZZ \to 4\ell$

MG/ME with spin-2 particles [Hagiwara, Kanzaki, Q.Li, KM ('08)]




$$d\Gamma/d\cos\Theta d\Phi \sim \Sigma_0 + \Sigma_2 \cos 2\Phi$$

$\lambda^{\mathrm{I}} = \pm 2$ ($gg$ initial state; low $M$)

$$\Sigma_0 = 3 + 10\cos^2\Theta + 3\cos^4\Theta$$

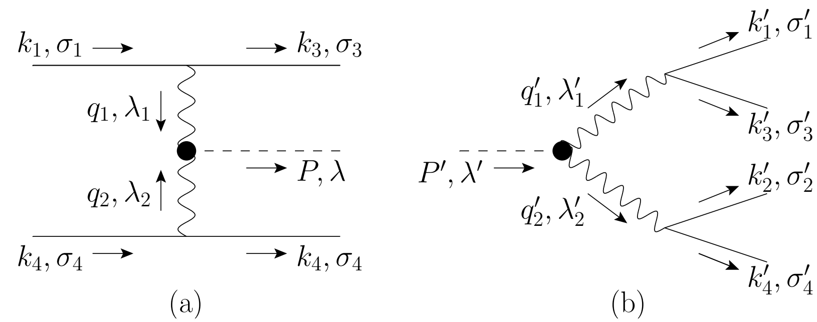
$$\Sigma_2 = \frac{1}{4}\sin^4\Theta$$

$\lambda^{\mathrm{I}} = \pm 1$ ($q\bar{q}$ initial state; high $M$)

$$\Sigma_0 = 8 + 4\cos^2\Theta - 12\cos^4\Theta$$

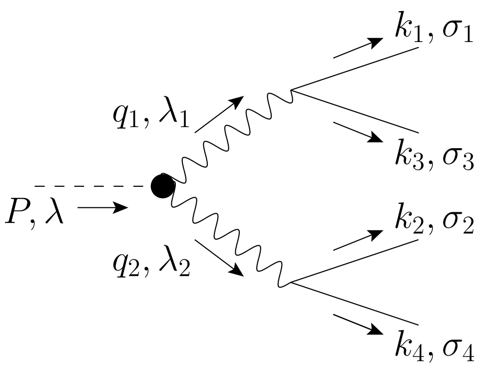
$$\Sigma_2 = -\sin^4\Theta$$

# Summary




- We have studied
  - heavy particle ($H/A$ and $G$) productions in association with two jets via VBF processes at the LHC; see my slides in Pheno09.
  - their decays into 4 leptons/jets via a vector-boson pair.
- We showed
  - the helicity amplitudes explicitly for $X \to VV \to 4f$.
  - non-trivial azimuthal correlations of the jets are manifestly expressed as the quantum interference among different helicity states of the intermediate vector-bosons.
- These correlations reflect the spin and $CP$ nature of the decaying heavy particles.

# Back up




# Current amplitudes

$$\mathcal{J}^{\lambda_i}_{i\,\sigma_i\sigma_{i+2}} = \epsilon_\mu(q_i, \lambda_i)\, J^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2})$$

- Fermion current vectors

$$J^\mu_{Vff'}(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) = g^{Vff'}_{\sigma_i}\, \bar{u}_{f'}(k_{i+2}, \sigma_{i+2})\, \gamma^\mu\, u_f(k_i, \sigma_i)$$

- Wavefunctions for the fermions

$$u(k_1, +) = \sqrt{2E_1} \begin{pmatrix} 0 \\ 0 \\ \cos(\theta_1/2) \\ \sin(\theta_1/2)\, e^{i\phi_1} \end{pmatrix} ;\ u(k_1, -) = \sqrt{2E_1} \begin{pmatrix} -\sin(\theta_1/2)\, e^{-i\phi_1} \\ \cos(\theta_1/2) \\ 0 \\ 0 \end{pmatrix}$$

- Wavefunctions for the decaying vector-bosons

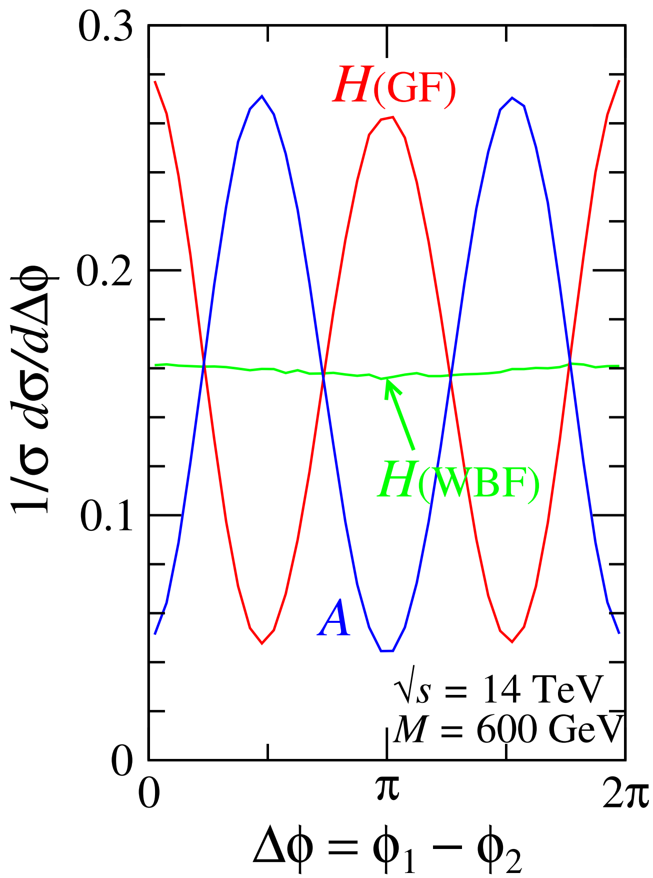
$$\epsilon^\mu(q_1, \pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$
$$\epsilon^\mu(q_1, 0) = (0, 0, 0, 1)$$

# Angular distributions for $G \to VV \to (f\bar{f})(f\bar{f})$

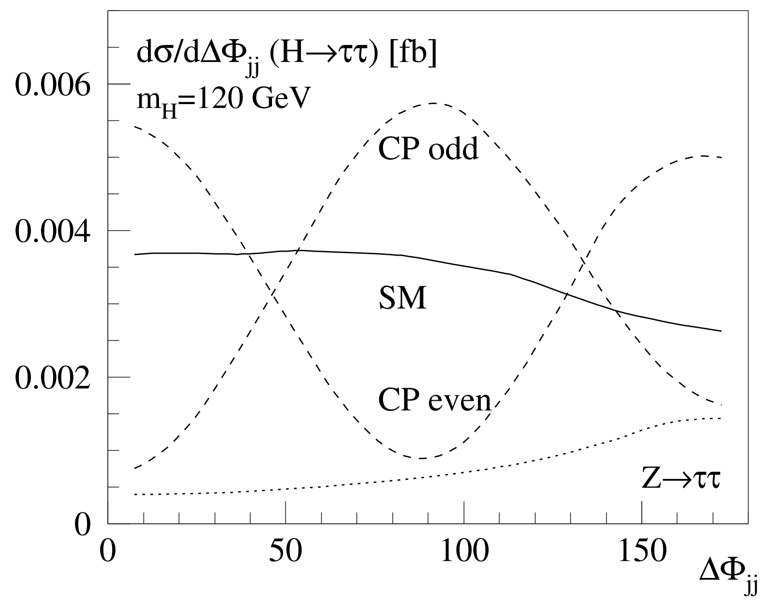
$$\frac{d\Gamma_G^{\lambda^{\mathrm{I}}=\pm 2}}{d\cos\Theta\, d\cos\theta_1\, d\cos\theta_2\, d\Phi}$$
$$\begin{aligned}
&\sim (1+6c^2+c^4)\{(1+c_1^2)(1+c_2^2)+4\kappa_1\kappa_2 c_1 c_2\}+2s^4 s_1^2 s_2^2 \\
&\quad \mp 8c(1+c^2)\{\kappa_1 c_1(1+c_2^2)+\kappa_2(1+c_1^2)c_2\} \\
&\quad + 4s^2 s_1 s_2\{(1+c^2)(\kappa_1\kappa_2+c_1c_2)\mp 2c(\kappa_2 c_1+\kappa_1 c_2)\}\cos\Phi \\
&\quad + s^4 s_1^2 s_2^2 \cos 2\Phi
\end{aligned}$$

$$\frac{d\Gamma_G^{\lambda^{\mathrm{I}}=\pm 1}}{d\cos\Theta\, d\cos\theta_1\, d\cos\theta_2\, d\Phi}$$
$$\begin{aligned}
&\sim 4(1-c^4)\{(1+c_1^2)(1+c_2^2)+4\kappa_1\kappa_2 c_1 c_2\}+8s^2c^2 s_1^2 s_2^2 \\
&\quad \mp 16c(1-c^2)\{\kappa_1 c_1(1+c_2^2)+\kappa_2(1+c_1^2)c_2\} \\
&\quad - 16s^2 c s_1 s_2[c(\kappa_1\kappa_2+c_1c_2)\mp(\kappa_2 c_1+\kappa_1 c_2)]\cos\Phi \\
&\quad - 4s^4 s_1^2 s_2^2 \cos 2\Phi
\end{aligned}$$

# $\Delta\phi\,(=\phi_1-\phi_2)$ distributions for Higgs bosons




Plehn, Rainwater, Zeppenfeld ('02)

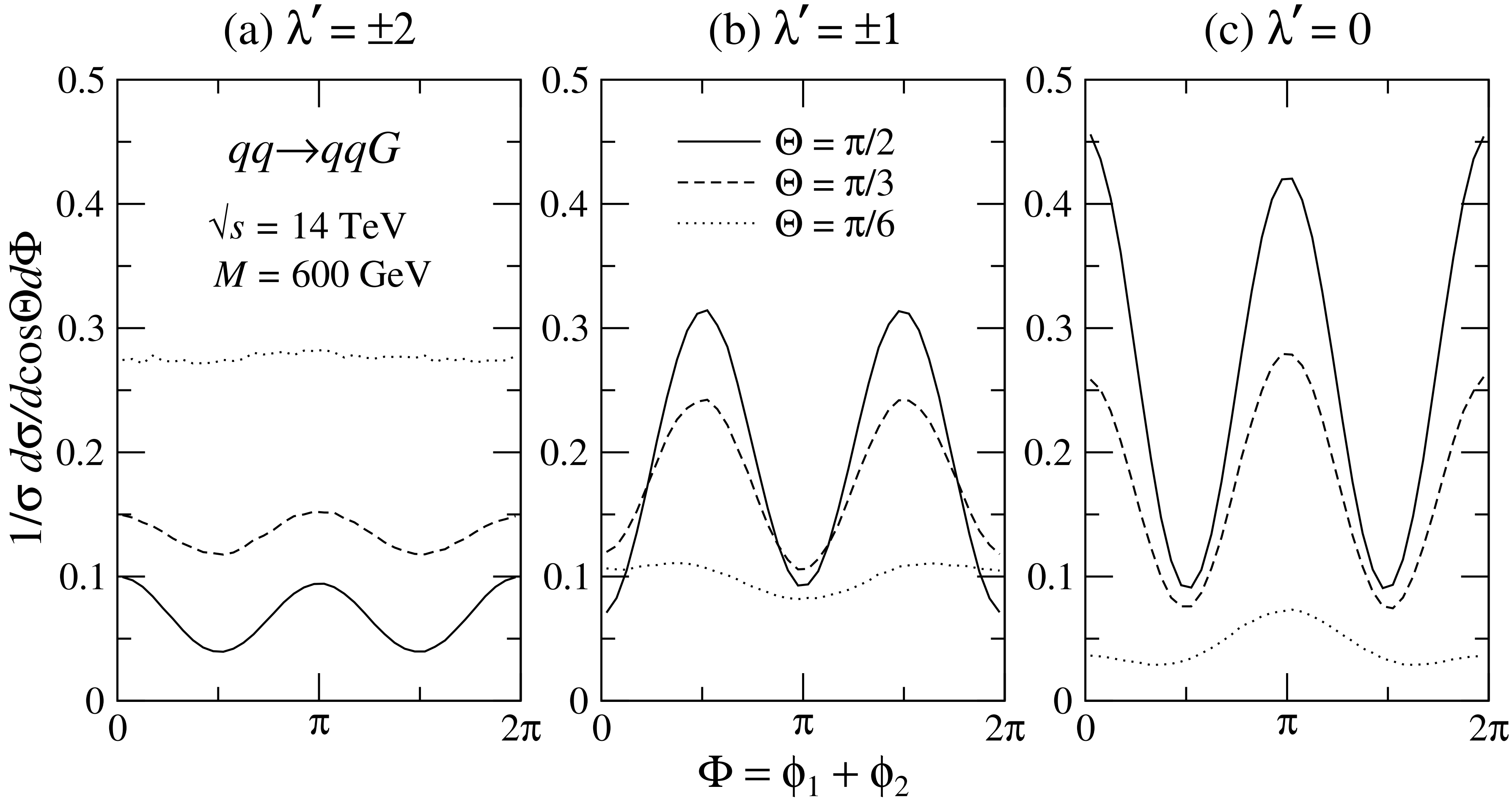



$H$(WBF) $\Rightarrow d\sigma/d\Delta\phi \sim$ constant

$H$(GF) $\Rightarrow d\sigma/d\Delta\phi \sim \Sigma_0 + |\Sigma_2|\cos 2\Delta\phi$

$A \Rightarrow d\sigma/d\Delta\phi \sim \Sigma_0 - |\Sigma_2|\cos 2\Delta\phi$

# $\Phi\,(=\phi_1+\phi_2)$ distributions for gravitons




$$d\sigma/d\cos\Theta d\Phi \sim \Sigma_0 + \Sigma_2 \cos 2\Phi; \quad \Sigma_2 \propto \begin{cases} +\frac{1}{8}\sin^4\Theta & \text{for } \lambda' = \pm 2 \\ -\frac{1}{2}\sin^4\Theta & \text{for } \lambda' = \pm 1 \\ +\frac{3}{4}\sin^4\Theta & \text{for } \lambda' = 0 \end{cases}$$

The $\Theta$ and $\lambda'$ dependent azimuthal $\Phi$ correlations !