Neutrino oscillation in the LHC era

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Outline

- Introduction
- Neutrino Oscillation in the Standard Model
- Neutrino oscillation beyond the SM
- Conclusions

1) Introduction

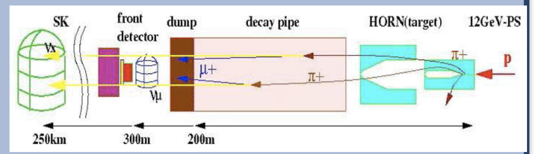
- In frame of the Standard Model (SM)
  - only left handed neutrinos (right-handed antineutrinos) are produce and detected,
  - the neutrino flavour states are well defined, they are orthogonal, phenomenon of neutrino oscillation is well described,
- At higher energies (e.g. at LHC), beyond the SM can appears and can be tested (e.g. supersymmetry), in such models, standard neutrino oscillation theory does not work,
- Generally there are many models with Non Standard neutrino Interaction (NSI) where the existing theory of oscillation can not be applied,
- Here we propose the general approach for neutrino oscillation, valid for any NSI where, neutrinos in positive and negative helicity states can be produced, and neutrino flavour states are only approximately defined,
- We found the necessary and sufficient conditions for the NSI, under which the standard neutrino oscillation theory is correct.

Different processes for neutrino production

✧Neutrino superbeams, Off axis neutrino beams

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$



✧Beta beams

$$n \rightarrow p + e^- + \bar{\nu}_e$$

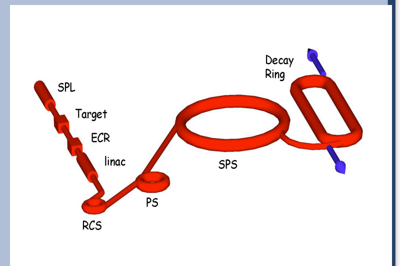
$$p \rightarrow n + e^+ + \nu_e$$

Average $E_{cms} = 1.937$ MeV

$${}^{6}_{2}He \rightarrow {}^{6}_{3}Li\; e^- \bar{\nu}_e$$

$${}^{18}_{10}Ne \rightarrow {}^{18}_{9}F\; e^+ \nu_e$$

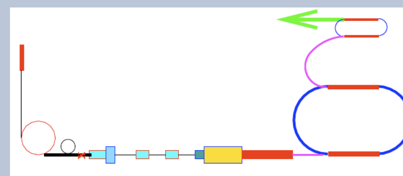
Average $E_{cms} = 1.86$ MeV



✧Neutrino factories

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$



Processes for neutrino detection

1) Water Cerenkov Detector
2) Liquid Argon Detector
3) Iron Calorimeter
4) Emulsion Detector

$$\nu_e + n \rightarrow p + e^-$$

$$\nu_e + {}^A J_Z \rightarrow {}^A J'_{Z+1} + e^-$$

$$\bar{\nu}_e + {}^A J_Z \rightarrow {}^A J'_{Z-1} + e^+$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

$${}^A J_Z = {}^{12}C_6, {}^{20}Ne_{10}, {}^{37}Cl_{17}, {}^{71}Ga_{31}, {}^{100}Mo_{42}, {}^{127}I_{53}$$

$${}^A J'_Z = {}^{37}Ar_{18}, {}^{71}Ge_{32}, {}^{100}Tc_{43}, {}^{115}Sn_{50}, {}^{127}Xe_{54}$$

$$\nu_\alpha + e^- \rightarrow \nu_\alpha + e^- \qquad \bar{\nu}_\alpha + e^- \rightarrow \bar{\nu}_\alpha + e^-$$

$$\nu_\alpha + d \rightarrow p + p + e^- \qquad \nu_\alpha + d \rightarrow p + n + \nu_\alpha$$

Neutrino oscillation in the SM

For production and detection processes - charge current

$$L_{CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha,\, i} l_\alpha \gamma^\mu (1-\gamma_5) \mathrm{U}_{\alpha i} \nu_i W_\mu^- + h.c$$

Relativistic (anti)neutrinos are produced in pure Quantum Mechanical flavour state

$$\left|\nu_\alpha \downarrow\right\rangle = \sum_i U^*_{\alpha\, i} \left|\nu_i \downarrow\right\rangle$$

Neutrinos always with negative helicity

$$\left|\bar{\nu}_\beta \uparrow\right\rangle = \sum_i U_{\beta\, i} \left|\bar{\nu}_i \uparrow\right\rangle$$

Antineutrinos always with positive helicity

Z. Maki, M. Nakagawa, S. Sakata, Prog.Theor.Phys. 28(1962)870

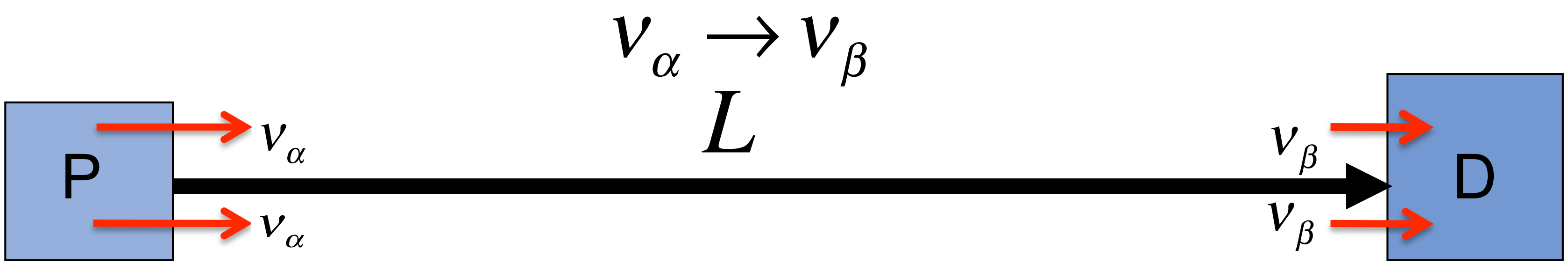
Neutrino propagation in the vacuum or in a matter – - neutral current

$$L_{NC} = \frac{e}{4\sin\theta_W \cos\theta_W} \sum_{i=1,2,3} \bar{\nu}_i \gamma^{\mu} (1-\gamma_5) \nu_i Z_{\mu}$$

$$\left|\nu_\alpha \downarrow\right\rangle = \sum_i U^*_{\alpha i} \left|\nu_i \downarrow\right\rangle \longrightarrow \text{No spin flip} \longrightarrow \left|\nu_\beta \downarrow\right\rangle = \sum_i U^*_{\beta i} \left|\nu_i \downarrow\right\rangle$$

$$\left|\bar{\nu}_\alpha \uparrow\right\rangle = \sum_i U_{\alpha i} \left|\bar{\nu}_i \uparrow\right\rangle \longrightarrow \text{No spin flip} \longrightarrow \left|\bar{\nu}_\beta \uparrow\right\rangle = \sum_i U_{\beta i} \left|\bar{\nu}_i \uparrow\right\rangle$$

Neutrinos oscillation rate in a detector is described by the factorized formula:



$$\Delta N_D(L,E) = j_\alpha(E)\, P_{\alpha\to\beta}(L,E)\, \sigma_\beta(E)\, N_D$$

- Number of the β neutrinos with energy E, which reach detector in a unit time
- Flux of the initial neutrinos of the α type
- Probability of the α to β neutrino conversion
- Detection cross section for β neutrinos
- Number of active scattering centres in a detector

Oscillation rate is the same for Dirac and Majorana neutrinos

There are models which predict

NonStandard neutrino Interaction (NSI)

in the weak scale range, which can modify

- **neutrino production process,**
- **oscillation inside matter, and**
- **detection process**

- Models which try to resolve problem of neutrino mass e.g. see-saw
- Charged Higgs,
- Right handed currents,
- Supersymmetric models.

LHC era

What we should change to describe future experiment with NSI of neutrinos??

Oscillation beyond the SM

For the neutrino production (detection - for low energy)

dimensional six operators -

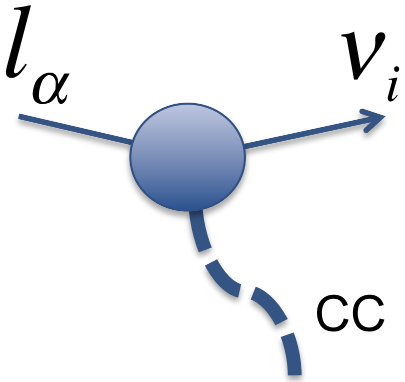
four-fermions effective Hamiltonian

e.g for muon decay (in neutrino factory)

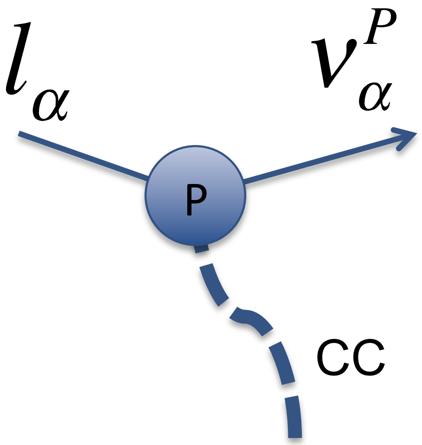
$$H = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\delta=S,V,T \\ \varepsilon,\eta=L,R}} \sum_{i,k=1}^{3} (g^{\delta}_{\varepsilon,\eta})_{i,k} (\bar{l}_\alpha \Gamma^\delta \nu_{i,\varepsilon})(\bar{\nu}_{k,\eta} \Gamma_\delta l_\beta) + h.c.$$

First suggestion that a result of neutrino oscillation depends on three ingredients, on the production process, on the propagation inside matter and on their detection, appeared in 1995

Y. Grossman,
Phys.Lett.B359,141(1995)

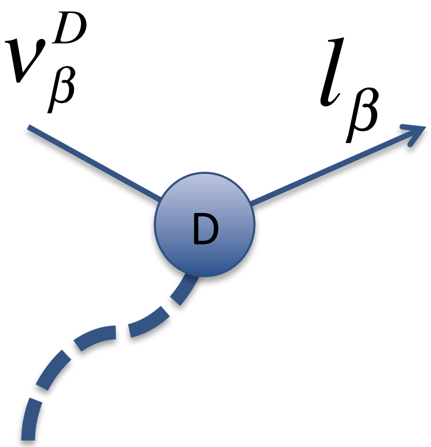


Neutrio masses are uknown- but are very small, experiments cannot observed neutrinos as mass eigenstates. But the mass basis $|\nu_i\rangle$ is well define. Such states are process independent.



Neutrinos are produced by charged current interaction. This process defines neutrino flavour. Such states are process dependent:

$$\left|\nu_\alpha^P\right\rangle = \sum_i U_{\alpha,i}^P \left|\nu_i\right\rangle$$



Detection process measures different state – the detection flavour neutrino states:

$$\left|\nu_\beta^D\right\rangle = \sum_i U_{\beta,i}^D \left|\nu_i\right\rangle$$

Production and detection flavour mixing matrices are constucted from the production and detection interaction Hamiltonians.

$$\left|U^{P}_{\alpha,\, i}\right|^2 \propto \left|\left\langle \nu_i ; f_P \middle| H^P \middle| l_\alpha ; i_P \right\rangle\right|^2$$

$$\left|U^{D}_{\beta,\, i}\right|^2 \propto \left|\left\langle l_\beta ; f_D \middle| H^D \middle| \nu_i ; i_D \right\rangle\right|^2$$

The probability of finding neutrinos in a $\left|\nu^D_\beta\right\rangle$ states in the original $\left|\nu^D_\beta\right\rangle$ beam at the time t is given by

$$P_{\alpha\to\beta}(t) = \left|\left\langle \nu^P_\beta \middle| e^{-iHt} \middle| \nu^D_\alpha \right\rangle\right|^2$$

Two types of such approaches can be found in the literature:

NSI and internal or external wave packets: Kayser(81),Giunti,Kim,Lee(91),Rich(93).

Field theory and of mass shell neutrino propagation: Grimus and Stckinger(96), MZ(98), Cardall(00), Giunti(02), Beuthe(03).

In these approaches, as in the Standard Model:

1) Production and detection states are pure Quantum Mechanical states

$$\left|\nu_\alpha^P\right\rangle \qquad \left|\nu_\beta^D\right\rangle$$

2) It is possible to define flavour change probability

$$P_{\alpha\to\beta}(t) = \left|\left\langle \nu_\beta^P \right| e^{-iHt} \left| \nu_\alpha^D \right\rangle\right|^2$$

which factorize:

$$\Delta N_D(L \approx t, E) = j_\alpha(E)\, P_{\alpha\to\beta}(t)\, \sigma_\beta(E)\, N_D$$

In the proper approach - neutrino states are calculated in the standard way

State of the neutrinos produced in the process

$$A \to B + \bar{l}_\alpha + \nu_i$$

is described by the density matrix (if initial particle **(A)** is not polarized and polarizations of the final particles **(B, l)** are not measured):

$$\rho^{\alpha}_{\lambda,\, i;\mu,\, k} = \frac{1}{N_\alpha} \sum \int f_i^{\alpha}(\lambda_A;\lambda_B,\lambda_l,\lambda) f_k^{\alpha *}(\lambda_A;\lambda_B,\lambda_l,\mu)$$

$Tr(\rho^{\alpha}) = 1$

where $f_i^{\alpha}(\lambda_A;\lambda_B,\lambda_l,\lambda)$ is the amplitude for the production process $A \to B + \bar{l}_\alpha + \nu_i$.

We need the density matrix in the laboratory (detector) frame = Lorentz boost + time evolution

$$\rho^{\alpha}_{CM}$$

Calculated in the CM of decaying particle:

$\pi^+ \to \mu^+ \nu_i$	$\mu^- \to e^- \bar{\nu}_e \nu_i$	$n \to p e^- \nu_i$
Accelerator neutrinos	**Neutrino factory**	**Beta beam**

Lorentz boost, Wigner spin rotation

$$\rho^{\alpha}_{LAB}(L=0) \cong \rho^{\alpha}_{CM}$$

M.Ochman,R.Szafron and MZ, **arXiv:0707.4089**

Neutrino propagation in the vacuum or in a matter

$$\rho^{\alpha}_{LAB}(L=T\neq 0)$$

$$\rho^{\alpha}(\mathrm{T{=}L}) = e^{-\mathrm{i\,H\,T}}\rho^{\alpha}(\mathrm{L{=}T}=0)e^{\mathrm{i\,H\,T}}$$

H – vacuum or matter Hamiltonian

Any detection process:

$$\nu_k + C \rightarrow l_\beta + D$$

A^β is the amplitude for the detection process

$$A^\beta(\Omega) \equiv A_k^\beta(\lambda_k, \lambda_C; \lambda_\beta, \lambda_D; \Omega)$$

$$\sigma_{\alpha\to\beta}(L,E) = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{2s_C+1}\sum_{spins}\int d\Omega\, A^\beta(\Omega)\rho^\alpha_{LAB}(L=T)A^{\beta*}(\Omega)$$

Generally the $\sigma_{\alpha\to\beta}$(E,L) cross section does not factorize

$$\sigma_{\alpha\to\beta}(E,L) \neq P_{\alpha\to\beta}(E,L)\ \sigma_\beta(E)$$

There is no factorization for the detection rate

$$N(E,L) = \rho_\alpha(E)\ \ \sigma_{\alpha\to\beta}(E,L)\ \ N_T$$

For Dirac neutrinos | **For muon decay**

In the SM

$$\bar{\nu}_e = \nu(\lambda = +1)$$

distinguishable

$$\nu_\mu = \nu(\lambda = -1)$$

$$|\bar{\nu}_e\rangle = \sum_{i=1}^{3} U_{e\,i} |\bar{\nu}_i\rangle$$

$$|\nu_\mu\rangle = \sum_{i=1}^{3} U^*_{\mu i} |\nu_i\rangle$$

Pure QM STATES

Beyond the SM

$$\bar{\nu}_e \rightarrow \nu(\lambda = +1)$$

$$\nu_\mu \rightarrow \nu(\lambda = -1)$$

→ Mixed QM STATES

↓

Density matrix required

↑

For Majorana neutrinos

In the SM or beyond

$$\nu_e \rightarrow \nu(\lambda = +1)$$

$$\nu_\mu \rightarrow \nu(\lambda = -1)$$

→ QM mixed STATE

$$\rho^{\alpha}_{-1,\,i;-1,\,k},\ \rho^{\alpha}_{+1,\,i;+1,\,k},\ \rho^{\alpha}_{-1,\,i;+1,\,k},\ \rho^{\alpha}_{+1,\,i;-1,\,k}$$

Non–standard description

If only positive(negative) helicity neutrinos(antineutrinos) are produced -- Theorem:

The necessary and sufficient condition for pure initial state of produced neutrinos with negative helicities is the factorization for spin and mass production amplitudes

$$f_i^{\alpha}(\lambda_A;\lambda_B,\lambda_l,\lambda=-1)=g^{\alpha}(\lambda_A;\lambda_B,\lambda_l,\lambda=-1)*h_i^{\alpha}\equiv g_{\mu}^{\alpha}h_i^{\alpha}$$

If we introduce the shortcut notation

$$\mu=(\lambda_A;\lambda_B,\lambda_l)$$

Then:

$$f_i^{\alpha}(\mu)=g^{\alpha}(\mu,\lambda=-1)*h_i^{\alpha}\equiv g_{\mu}^{\alpha}h_i^{\alpha}$$

Then the density matrix is given by:

$$\rho^{\alpha}(i,\lambda=-1;k,\mu=-1)\equiv\rho^{\alpha}_{i,k}=\frac{h_i^{\alpha}h_k^{\beta *}}{\sum_i\left|h_i^{\alpha}\right|^2}=\chi_i^{\alpha}\chi_k^{\alpha *}$$

where

$$\chi_i^{\alpha}=\frac{h_i^{\alpha}}{\sqrt{\sum_i\left|h_i^{\alpha}\right|^2}}$$

which is equivalent to the pure QM state:

$$\left|\nu_{\alpha}\right\rangle=\sum_i\chi_i^{\alpha}\left|\nu_i\right\rangle$$

$$\left|\nu_{\alpha}\right\rangle=\sum_i U^{*}_{\alpha\,i}\left|\nu_i\right\rangle$$

$$\left\langle\nu_{\alpha}\middle|\nu_{\beta}\right\rangle=\sum_i\chi_i^{\alpha}\chi_i^{\beta *}\neq\delta_{\alpha\beta}(=1\;for\;\;\alpha=\beta)$$

νSM

which are normalized but not necessarily orthogonal

$$\chi_i^{\alpha}=U^{*}_{\alpha\,i}$$

Factorization for the final oscillation rate

$$\sigma_{\alpha\to\beta}(L,E)=$$

$$\frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{i,k,\lambda,\lambda_C,\lambda_l,\lambda_D}\int d\Omega\; A_i^{\beta}(\lambda,\lambda_C,\lambda_l,\lambda_D;\Omega)\rho^{\alpha}(i,\lambda;k,\lambda;L,E)A_k^{\beta*}(\lambda,\lambda_C,\lambda_l,\lambda_D;\Omega)$$

The density matrix after oscillation

$$\rho^{\alpha}(i,\lambda=-1;k,\mu=-1;L,E)=\rho^{\alpha}_{i,k}(L,E)e^{-\frac{\delta m^2_{i,k}}{2E}L}$$

If the detection amplitudes factorize

$$A_i^{\beta}(\lambda=-1,\lambda_C,\lambda_l,\lambda_D;\Omega)=e_{\eta}^{\beta}(\theta,\varphi)*k_i^{\beta}$$

Then the final cross section factorize

$$\sigma_{\alpha\to\beta}(L,E)=$$

$$\frac{1}{32\pi s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{i,k,\eta}\int d\Omega\;(e_{\eta}^{\beta}*k_i^{\beta})(\rho^{\alpha}_{i,k}e^{-\frac{\delta m^2_{i,k}}{2E}L})(e_{\eta}^{\beta*}*k_k^{\beta*})=$$

$$=P_{\alpha\to\beta}(L,E)\;\sigma_{\beta}(E)$$

If there is factorization for the initial and final states?

$$\sigma_{\alpha\to\beta}(L,E)=\frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{2s_C+1}\sum_{spins}\int d\Omega\; A^{\beta}(\Omega)\rho^{\alpha}_{LAB}(L=T)A^{\beta *}(\Omega)$$

The density matrix after oscillation

$$\rho^{\alpha}(i,\lambda=-1;k,\mu=-1;L,E)=\chi_i^{\alpha}\chi_k^{\alpha *}e^{-\frac{\delta m^2_{i,k}}{2E}L}$$

If the detection amplitudes factorize

$$A_i^{\beta}(\lambda=-1,\lambda_C,\lambda_l,\lambda_D;\Omega)=e_{\eta}^{\beta}(\theta,\varphi)*k_i^{\beta}$$

Then the final cross section factorize

$$\sigma_{\alpha\to\beta}(L,E)=$$

$$\frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{i,k,\eta}\int d\Omega\;(e_{\eta}^{\beta}*k_i^{\beta})(\chi_i^{\alpha}\chi_k^{\alpha *}e^{-\frac{\delta m^2_{i,k}}{2E}L})(e_{\eta}^{\beta *}*k_k^{\beta *})=$$

$$=P_{\alpha\to\beta}(L,E)\;\sigma_{\beta}(E)$$

The oscillation probability is given by:

$$P_{\alpha\to\beta}(L,E)=\sum_{i,k}k_i^{\beta}\chi_i^{\alpha}\chi_k^{\alpha*}k_k^{\beta*}e^{-\frac{\delta m_{i,k}^2}{2E}L}$$

And the final detection cross section:

$$\sigma_{\beta}(E)=\frac{1}{64\pi^2 s}\frac{p_f}{p_i}\frac{1}{2s_f+1}\sum_{\eta}\int d\Omega\,\left|e_{\eta}^{\beta}(\theta,\varphi)\right|^2$$

The sum over all final flavours

$$\sum_{\beta=e,\mu,\tau}P_{\alpha\to\beta}(L,E)=\sum_{i,k}\sum_{\beta=e,\mu,\tau}(k_i^{\beta}k_k^{\beta*})\rho_{i,k}^{\alpha}e^{-\frac{\delta m_{i,k}^2}{2E}L}=\sum_{i}\rho_{i,i}^{\alpha}=1$$

if

$$\sum_{\beta=e,\mu,\tau}(k_i^{\beta}k_k^{\beta*})=\delta_{i,k}$$

The probability is conserved if the final states are orthogonal

☞ If dominant and subdominant neutrino helicity states are produced and detected then the description of neutrino oscillation is not standard.

☞ If only dominant neutrino helicity states are produced and detected the standard description of the neutrino oscillation is recovered then and only then

if

the production and detection amplitudes factorizes for spin and mass parts

Pure or mixed initial neutrino state ⟺ is $Tr(\rho^2) < 1$ or $Tr(\rho^2) = 1$

◎If the left-handed ν_L and right-handed ν_R chiral neutrino operators are present in NI – both type of neutrino helicities are produced -- **mixed states.**

◎If only one left handed operators ν_L describes the neutrino interaction then neutrino state depends on the number and structure of the helicity production amplitudes:

- If only one helicity amplitude describes production process, then, independently of the NI, neutrino **state is pure**, family lepton number can not be conserved,
- If there are more then one helicity amplitude, but all have the same structure -- **state is pure,**
- If there are more then one helicity amplitudes, and at least two of them have different structure -- **state is mixed.**

Dirac and Majorana neutrinos in oscillation experiments

- Production process (charge currents are responsible)
  - If only one neutrino appears in a production process (as in pion or in beta decays) – production process does not distinguish both types of neutrinos.
  - If two neutrinos (neutrinos + antineutrinos, as in the muon decay) are produced, the interference terms in the spin amplitudes, which are present for the Majorana neutrinos, and do not occur for Dirac neutrinos, can distinguish between two types of neutrinos in a production process.
- Propagation in matter distinguishes both types of neutrinos (neutral currents are crucial).

4) Conclusions

1) neutrino production states represented by density matrix can be
 # pure or mixed
 depending on the production mechanism,

2) final neutrino detection rates generally do not factorize,

4) Dirac and Majorana neutrinos oscillate in different way,

5) coherent and incoherent oscillation can be defined.

Present bounds on NI parameters give possibility that future precise experiments can see some effects