

LHC as Time Machine

(Adventures in Extra-Dimensions)

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Syllabus:

- * Closed Timelike Curves (CTCs)
- * Goedel, van Stockum, Tipler- planes
- * Spacetime metrics, warped space
- * From 5D to 6D (un-compactified)
- * and back to 5D (compactified)
- * Energy distribution, energy conditions (if time)
- * Conclude

CTCs defined

Superluminal travel means signaling faster than speed of light,

- e.g. LSND/MiniBooNE and sterile neutrinos taking shortcuts through Xdim bulk
 - (Paes, Pakvasa, Weiler, hep-ph/0504096)
 - (Hollenberg, Micu, Paes, Weiler, 0906.0150)

More challenging is to receive a signal BEFORE it leaves!

“Closed Timelike Curve” or CTC

- e.g. (Paes, Pakvasa, Dent, Weiler, gr-qc/0603045)

Hawking’s Chronology Protection Theorem simply asserts that this is too pathological to be permissible.

Open strings = SM brane particles,
Closed strings = singlet bulk/brane particles

Our context:

- I. Stringy Model where gauge charges live on the ends of open strings,
and stick these SM “particles” to our brane;
gauge-singlets are then closed strings,
free to explore the bulk
 - (e.g., graviton, sterile neutrinos)
 - (higgs singlets produced/detected at the LHC?)

- II. Einstein’s GR, where geometry is destiny.

Gödel-Tipler-vonStockum spacetimes

Gödel metric describes a pressure-free perfect fluid with negative cosmological constant and rotating matter, and the Tipler-van-Stockum (TvS) spacetime is being generated by a rapidly rotating infinite cylinder. In both cases the metric can be written as

$$ds^2 = +g_{tt}(r) dt^2 + 2g_{t\phi}(r) dt d\phi - g_{\phi\phi}(r) d\phi^2 - g_{rr} dr^2 - g_{zz} dz^2. \quad (1)$$

Distortions of the “lengths” ϕ and t in the radial direction is an example of warping.

And an example of time-warping is the Robertson-Walker big-bang metric.

Negative time:

A dynamical approach to GTvS causality examines the purely azimuthal null-curve with $ds^2 = 0$. One gets

$$\dot{\phi}_{\pm} = \frac{g_{t\phi} \pm \sqrt{g_{t\phi}^2 + g_{tt} g_{\phi\phi}}}{g_{\phi\phi}}, \quad (3)$$

where the \pm refers to co-rotating and counter-rotating lightlike signals. The coordinate time for a co-rotating path is

$$\Delta T_+ = \Delta\phi \left(\frac{g_{\phi\phi}}{g_{t\phi} + \sqrt{g_{t\phi}^2 + g_{\phi\phi} g_{tt}}} \right). \quad (4)$$

As $g_{\phi\phi}$ goes from positive to negative, the light-cone tips such that the azimuthal closed path is traversed in negative time

$$\Delta T_+ = \frac{-2\pi |g_{\phi\phi}|}{g_{t\phi} + \sqrt{g_{t\phi}^2 + g_{\phi\phi} g_{tt}}} \quad [g_{\phi\phi} < 0]. \quad (5)$$

The quantum returns to its origin before it left, marking the existence of a CTC.

Lightcone slopes:

The clear discriminator of the arrows of time are the slopes of the local light-cone,

$$s_{\pm}(r) = (r\dot{\phi}_{\pm})^{-1} = \frac{1}{r} \frac{g_{\phi\phi}}{g_{t\phi} \pm \sqrt{-g_4}} = -\frac{1}{r} \frac{g_{t\phi} \mp \sqrt{-g_4}}{g_{tt}}. \quad (7)$$

Notice that if $g_{\phi\phi}$ and g_{tt} are positive, then regardless of the sign of $g_{t\phi}$, the light-cones (worldlines) remain in the first and second quadrants of the (t, ϕ) plane (as is the case of the Minkowski light-cone). Thus, for a backward flow of time, $g_{\phi\phi}$ (or, g_{tt}) must go through zero and become negative.

GTVS - the good, bad, and ugly

It is instructive to mention the visceral arguments against the relevance of the Gödel and TvS metrics. First of all, they are not asymptotically flat, and so presumably cannot occur within our Universe; rather, they must be our Universe, which contradicts observation. Secondly, the initial conditions from which they can evolve are either non-existent (Gödel) or sick (TvS). Furthermore, the TvS metric assumes an infinitely-long cylinder of matter, which is unphysical. On the positive side, literally, the Einstein equation endows $\rho = T^0_0 = (R^0_0 - \frac{1}{2}R)/8\pi G_N$ (with the geometric RHS determined by the metric) with a positive value everywhere; there is no need for “exotic” $\rho < 0$ matter. A further positive feature is the simplicity of finding the CTC by travel along the periodic variable ϕ .

CTCs (PPDW)

Closed timelike curves in asymmetrically warped brane universes

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A linear path off the brane

We may replace the periodic coordinate of GTvS with the unbounded x coordinate, and omit the y and z coordinates for brevity. Then one obtains

$$ds^2 = g_{tt}(u, v) dt^2 + 2g_{tx}(u, v) dx dt - g_{xx}(u, v) dx^2 - du^2 - dv^2. \quad (12)$$

(Notice in particular the sign convention on the coefficient of dx^2 .)

The speed of light at any point will depend on (u, v) through the metric elements. The restriction to Lorentzian signature implies that

$$-g_6 \equiv -\text{Det}(g_{\mu\nu}) = g_{tt}(u, v) g_{xx}(u, v) + g_{tx}^2(u, v) > 0. \quad (13)$$

World lines for lightlike travel (null lines) satisfy

$$0 = g_{tt}(u, v) + 2g_{tx}(u, v) \dot{x} - g_{xx}(u, v) \dot{x}^2 - \dot{u}^2 - \dot{v}^2. \quad (14)$$

The solutions to (14) for the analogs of co-rotating and counter-rotating light speed at fixed (u, v) are

$$\dot{x}_{\pm} = \frac{g_{tx}(u, v) \pm \sqrt{-g_6}}{g_{xx}(u, v)}. \quad (15)$$

On the brane, \dot{x} must equal $c = 1$, so we again choose $g_{tt}(0, 0) = g_{xx}(0, 0) = 1$ and $g_{tx}(0, 0) = 0$.

Causal properties (continued) :

On the other hand, if g_{xx} is negative, then Eq. (17) shows that one light-cone slope has changed sign. The small g_{xx} limit of the slopes

$$s_{\pm}(\text{leading order in } g_{xx}) = \begin{cases} \frac{g_{xx}}{2g_{tx}} \\ -\frac{2g_{tx}}{g_{tt}} \end{cases} \quad (18)$$

reveals that it is the positive slope which has passed through zero to become negative, signifying a world line moving from the first quadrant, through the x -axis, into the fourth quadrant where times flows backwards for increasing x . With both slopes negative, one has that $\dot{x}_{\pm}(g_{xx} < 0) < 0$. Thus, travel with increasing time is in the negative x direction, while travel with decreasing time is in the positive x direction. We summarize the causal properties of the metric (12) in Table 1.

	$g_{xx} > 0$	$g_{xx} < 0$
$\Delta T > 0$	$\dot{x}_+ > 0$ ($\Delta x > 0$)	
	$\dot{x}_- < 0$ ($\Delta x < 0$)	$\dot{x}_- < 0$ ($\Delta x < 0$)
$\Delta T < 0$		$\dot{x}_+ < 0$ ($\Delta x > 0$)

Table 1: Solution types for metric (12), and their casual properties. In particular, no solution exists for motion backwards in time along the negative- x direction.

CTC condition(s) :

The necessary condition relating the outgoing and return paths of a CTC is that the sum $\Delta T_2 + \Delta T_1$ be less than zero. Equivalently, the CTC conditions are that

$$\frac{-g_{xx}(u_2, v_2)}{g_{tx}(u_2, v_2) - \sqrt{-g_{66}(u_2, v_2)}} + \frac{g_{xx}(u_1, v_1)}{g_{tx}(u_1, v_1) + \sqrt{-g_{66}(u_1, v_1)}} < 0, \quad (22a)$$

and that

$$g_{xx}(u_1, v_1) < 0, \quad (22b)$$

We show (Heinrich's perseverance) that a metric with two branes in the (u, v) dimensions, with relative motion (thereby hardwiring a boost), admits CTCs (see the pub for the somewhat complicated metric)

Fig: boosted branes

- (i) A signal leaves the brane and arrives at the $v > 0$ with $u = 0$ hyperslice. The transit time is negligible compared to the next step.
 - (ii) As viewed from the brane, the signal takes a spacelike shortcut at constant $v > 0$, $u = 0$, from the origin O to point $P1$ with $t_1 < 0$. The dt/dx slope $s1$ is outside of the light-cone.
 - (iii) The signal travels from $(v, u) = (> 0, 0)$ to $(v, u) = (0, > 0)$ in negligible time compared to step (ii).
 - (iv) As viewed from the brane, the signal then travels back along a path of constant $u > 0$, with $v = 0$, from $P1$ to $P2 = (t, 0)$. The dt/dx slope $s2$ is more outside of the light-cone, such that elapsed time t remains negative.
 - (v) The signal returns to the brane at $(v, u) = (0, 0)$ in negligible time compared to (ii).
- A Lorentz boost transforms $P1$ and $P2$ along their respective brane-space hyperbolas to $P1'$, and to $P2' = (t', x')$. It can be shown that time t' is again negative, and $|x'| < |t'|$ so that a world-line inside the light-cone closes the curve. The Lorentz-transformed paths are related to shortcuts via $u > 0$ with $v = 0$ first, and $v > 0$ with $u = 0$ second.

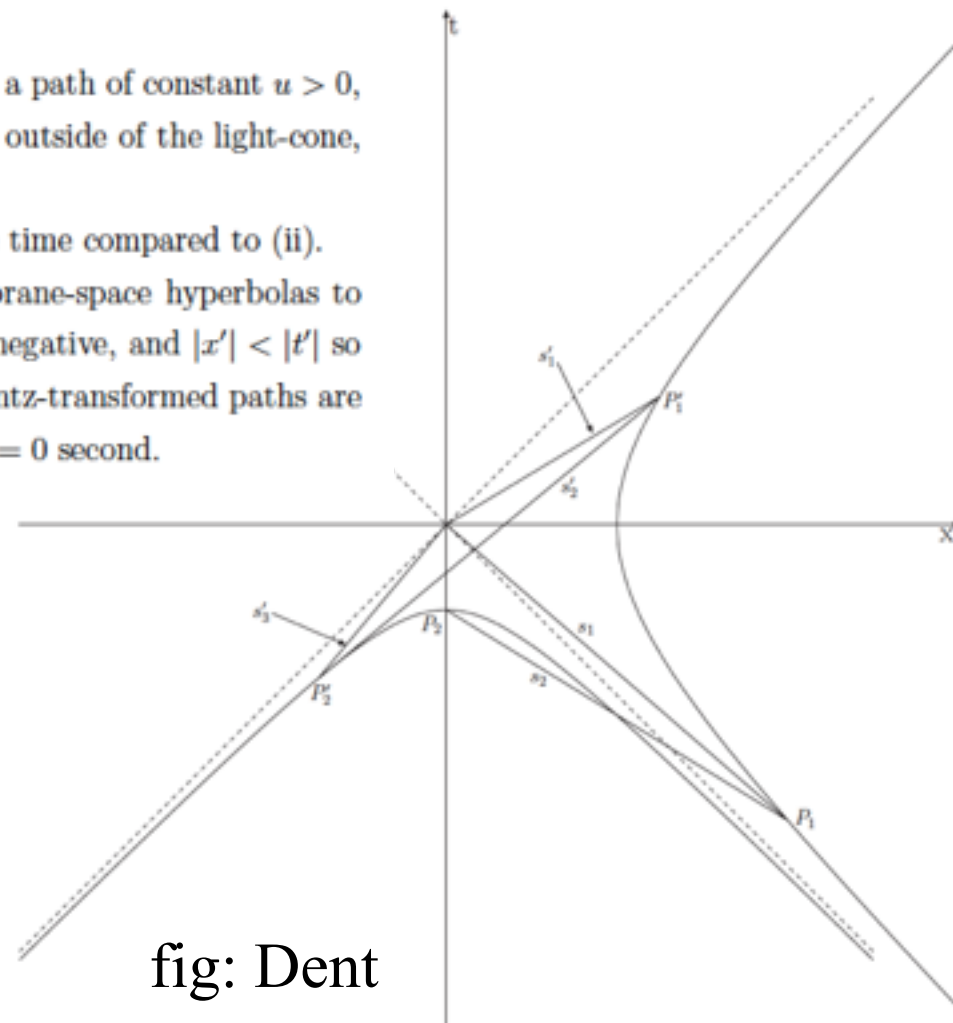


fig: Dent

Energy (philosophic discussion)

The negative energy density that afflicts many wormhole and CTC solutions in four dimensions is avoided on the brane in the example for an extra-dimensional CTC presented here. However, ρ becomes negative as one moves away from the brane into the bulk, so that the WEC and DEC are violated off the brane, while the NEC remains conserved. We have successfully constructed a metric exhibiting CTCs in an extra-dimensional spacetime by "moving" the negative energy density from the brane to the bulk. One might even speculate that the negative energy density in the bulk is related to the compactification of the extra dimensions, or possibly to the repulsion of Standard matter from the bulk.

One also sees in Fig. (2) that $G^y_y = G^z_z = G^v_v$ are equal to G^0_0 on the $v = 0$ slice. This equality amounts to a dark energy or cosmological constant equation of state for the y -, t -, and v -directed pressures, namely, $w^j \equiv p^j / \rho = -1$. There may be some intriguing physics underlying this result.

It is also possible that an anthropic argument applies here: Life may evolve only where energy density is positive. Then lifeless bulk regions of negative energy density can communicate their existence to living beings only via geometry, perhaps mediated by the exchange of gravitons or appropriately named, "sterile" neutrinos.

CTCs in $N+1$, $N=2,3,\dots$

We have seen GTvS CTCs in $N=2$ (ϕ, r).

Therefore, we expect metrics in $N=3,4,\dots$ with CTCs.

Eqn (22) shows that this is so.

The **mathematical** recipe that emerges from Eqn (22) is simply:

- (i) allow g_{xx} to change sign as a function of another spatial variable;
- (ii) take g_{tx} nonzero;
- (iii) arrange a suitably “fast” return path.

AND, there is an aesthetic input:

choose a metric that is **physically motivated**.

Compactified 5th Dim and Higgs Singlets (at the LHC?)

Causality Violating Scalar Singlets at the LHC

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This time actually solving for the geodesic!

Consider metric

$$ds^2 = \eta_{ij} dx^i dx^j + dt^2 + 2g(u) dt du - h(u) du^2$$

The metric (2.1) is completely Minkowskian on the brane, and its determinant is $\text{Det}[g_{\mu\nu}] = g^2 + h$. The geodesic equations for x, y and z directions are trivial while the geodesic equations for t and u are given by

$$\ddot{t}(\tau) = \frac{1}{2} \frac{-2g'h + gh'}{g^2 + h} \dot{u}^2, \quad (2.2)$$

$$\ddot{u}(\tau) = -\frac{1}{2} \frac{2gg' + h'}{g^2 + h} \dot{u}^2 = -\frac{1}{2} \frac{(g^2 + h)'}{g^2 + h} \dot{u}^2, \quad (2.3)$$

where τ is the proper time, with the superscripts “prime” and “dot” denoting differentiations with respect to u and τ respectively. Apparently, there seems to be infinite ways that we can choose for $g(u)$ and $h(u)$. But a systematic way to proceed would be to fix $g^2 + h$ such that at $u = 0$, it is unity, as required by the condition that the speed of light being unity in the 4D Minkowskian spacetime. It proves to be simple to impose

$$g^2 + h = \left(1 + \frac{u}{L_0}\right)^\alpha, \quad [\text{CM Ho ansatz}] \quad (2.4)$$

geodesic solutions:

$$ds^2 = \eta_{ij} dx^i dx^j + dt^2 + 2g_0 dt du + \left[g_0^2 - \left(1 + \frac{u}{L_0} \right)^\alpha \right] du^2,$$

where g_0 is a constant, and α can be any real number such that $-2 < \alpha < 0$.

$$u(\tau) = L_0 \left[\left(\frac{\alpha + 2}{2L_0} \dot{u}_0 \tau + 1 \right)^{\frac{2}{\alpha+2}} - 1 \right], \quad (\text{mod } L)$$

$$t(\tau) = -g_0 L_0 \left[\left(\frac{\alpha + 2}{2L_0} \dot{u}_0 \tau + 1 \right)^{\frac{2}{\alpha+2}} - 1 \right] + (g_0 \dot{u}_0 + \gamma_0) \tau$$

Got and not-Gott

This new metric resembles Gott's 3+1 for spinning cosmic strings and associated CTCs
(but without his infinite-energy, infinite red/blue shifting pathologies).

E.g., a well-known Thm says low-dimensional metrics are conformal to Minkowski space, i.e. locally Minkowskian but topologically complicated.

For the Gott metric $ds^2 = (dt + 4GJ d\theta)^2 - dr^2 - (1 - 4Gm)^2 r^2 d\theta^2 - dz^2$

the identifications $\tilde{t} = t + 4GJ\theta$ $\varphi = (1 - 4Gm)\theta$,

bring the form to locally Minkowski space away from the strings, but subject to the global identifications

$$\varphi \sim \varphi + 2\pi - 8\pi Gm \text{ and } \tilde{t} \sim \tilde{t} + 8\pi GJ.$$

But, Gott's metric violates all energy conditions but W(eak)EC.

Local Minko, Global Time Machine

With the redefinition $\bar{t} = t + g_0 u$,

our metric becomes locally Minkowskian too.

but subject to the global boundary condition $u \sim u + L$

i.e. $\bar{t} \sim \bar{t} + g_0 L$.

i.e., compact $u \Rightarrow$ periodic time !

Being everywhere Minkowskian, on and off the brane (after redefining u),

the Einstein eqn gives $0 = T_{\mu\nu}$

and so all energy conditions are (trivially satisfied).

[Since matter fields must be added only to the brane,
we expect the geodesics for bulk travel to be little affected by matter.]

Implications for LHC

If higgses are made, double-singlet mixing may mean (i.e. “right” metric) we are all connected by one degree of separation, our past and future 5-volumes.

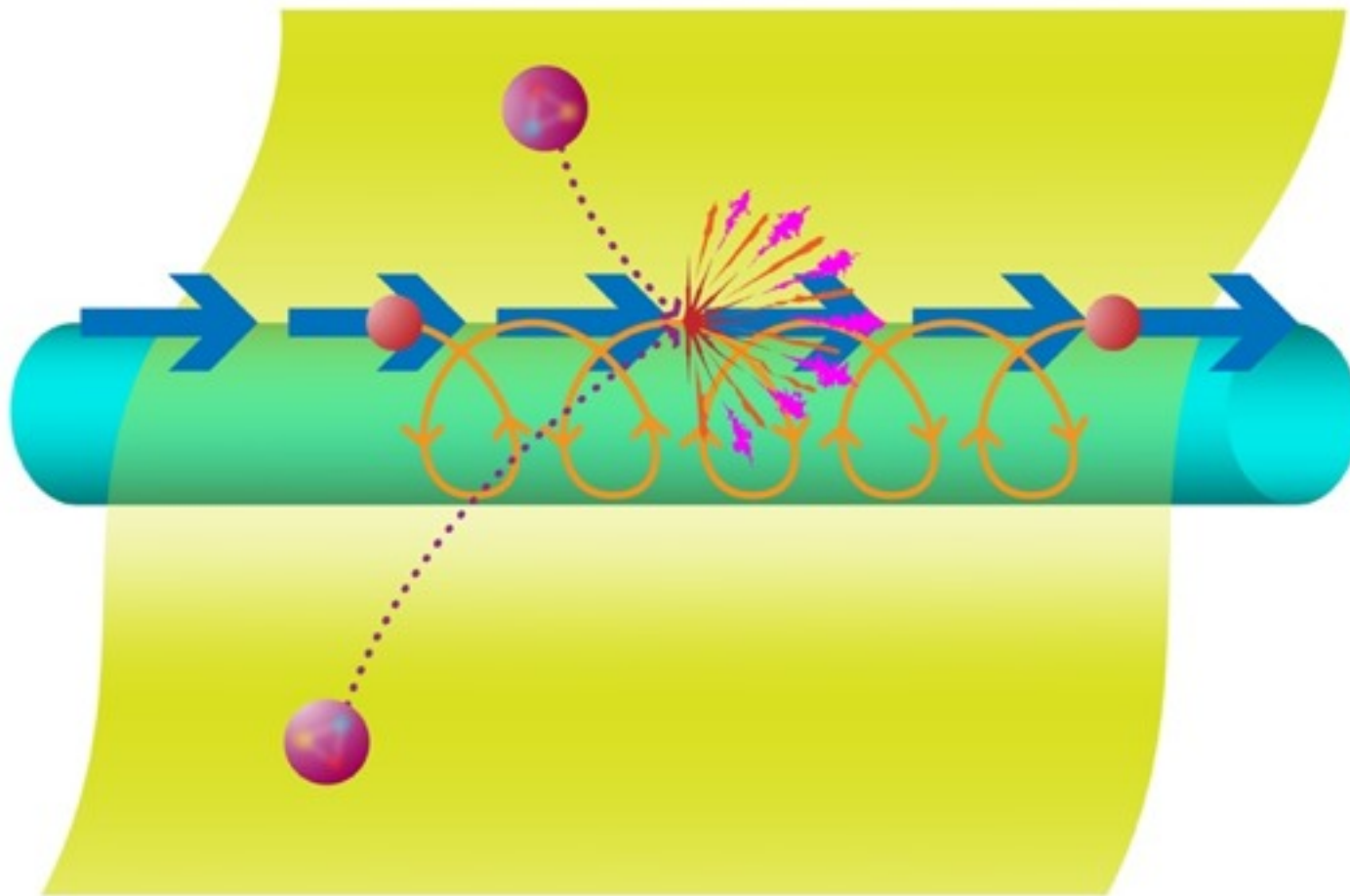
May find spontaneous appearance of KK modes of singlets, periodic in future and past times, stable in the bulk, but decaying (via higgs mixing) as they traverse the brane.

These singlets may have been made by us at the LHC NOW, or in the LHC FUTURE, OR, by OTHER CIVILIZATIONS signaling us NOW.

For the first time in human history, we have the machines and detectors to talk to extra-terrestrials --- OK, we need some more engineering and physics to sort it out.

[S. Hawking says this is too bizarre to happen; and if it does happen, run and hide!]

What it really, really looks like, in true colors!



Summary

- ^ GTvS CTCs easily generalize to more dimns (general CTC conditions on metric given)
- * There exist spatially warped metrics in infinite 6D (not 5D) exhibiting CTCs
- * There exist spatially warped metrics in compact 5D exhibiting CTCs
- * These CTCs challenge “chronology protection”, and may enable inter-temporal communication
- * Intriguing energetics,
 - zero or positive on brane,
 - zero or negative in bulk
- * More implications, more models to investigate

Extra Slides

Product of lightcone slopes:

It is useful to consider the product of slopes

$$s_+(r)s_-(r) = \frac{-1}{r^2} \frac{g_{\phi\phi}}{g_{tt}}. \quad (8)$$

For time to move backwards one of the world lines defining the light-cone must move into the lower half of the $t - \phi$ plane. From (8) one can see that (i) this happens smoothly if $g_{\phi\phi}$ goes through zero; (ii) happens discontinuously if g_{tt} goes through zero; (iii) that a smooth change in the sign of $g_{t\phi}$ cannot move either slope through zero to the domain of negative time.

Lightcone slopes

With the focus here on a smooth change of sign for $g_{\phi\phi}$, it is useful to examine the slopes at small $g_{\phi\phi}$. One finds

$$s_{\pm}(\text{leading order in } g_{\phi\phi}) = \begin{cases} \frac{1}{2r} \frac{g_{\phi\phi}}{g_{t\phi}} \\ -\frac{2}{r} \frac{g_{t\phi}}{g_{tt}} \end{cases} \quad (9)$$

It is clear that the slope s_+ goes through zero with $g_{\phi\phi}$, leaving the first quadrant and moving into the fourth quadrant. With increasing ϕ , time for the associated co-rotating world line runs backwards. On the other hand, the sign of s_- remains unchanged, and time for the associated counter-rotating world line continues to run forward. In the following we will apply similar arguments to different scenarios of asymmetrically warped spacetimes.

Stress-energy tensor and energy conditions

As a check on the consistency of the picture, we should diagnose the stress-energy tensor which sources the extra-dimensional metric, for any pathologies. In particular, we will be interested in the resulting matter distributions on and off the brane. Thus, our task is to calculate the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R, \quad (40)$$

from the spacetime metric of Eq. (34), and then to obtain the stress-energy tensor $T_{\mu\nu}$ via the Einstein equation

$$T_{\mu\nu} = \frac{1}{8\pi G_N} G_{\mu\nu}. \quad (41)$$

We note that in general, $T_{\mu\nu}$ contains contributions from matter, fields, and cosmological constant on and off the brane, and from brane tension on the brane.

Energy figure

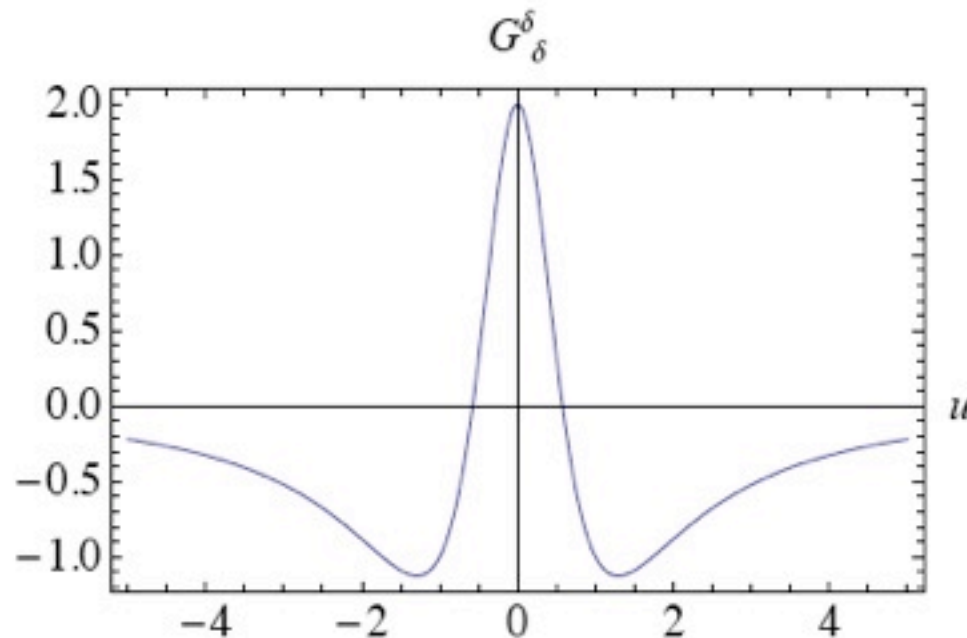


Figure 2: Nonzero elements of the Einstein tensor G^μ_ν (in arbitrary units): $G^\delta_\delta \equiv G^0_0 = G^y_y = G^z_z = G^v_v$, on the $v = 0$ slice, as a function of u . Assumed are warp factors $\alpha(u) = 1/(u^2 + c^2)$ and $\eta(v) = 1/(v^4 + c^2)$, with $c = 1$. We find that the weak and dominant energy conditions are violated in the bulk, while all energy conditions with the exception of the SEC are satisfied on the brane.

Energy conditions:

There is considerable theoretical prejudice that stable Einstein tensors should satisfy certain “energy conditions” relating energy density ρ and directional pressures p^j . The null, weak, strong and dominant energy conditions state that

$$\text{NEC} : \rho + p^j \geq 0, \quad \forall j. \quad (42)$$

$$\text{WEC} : \rho \geq 0; \quad \text{and} \quad \rho + p^j \geq 0, \quad \forall j. \quad (43)$$

$$\text{SEC} : \rho + p^j \geq 0, \quad \forall j; \quad \text{and} \quad \rho + \sum_j p^j \geq 0. \quad (44)$$

$$\text{DEC} : \rho \geq 0; \quad \text{and} \quad p^j \in [\rho, -\rho], \quad \forall j. \quad (45)$$