Multi-condensate low temperature finite density holographic systems

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Introduction

For sufficiently large charge, a charged planar black hole in Anti-de Sitter space can develops charged scalar hair. (Gubser)

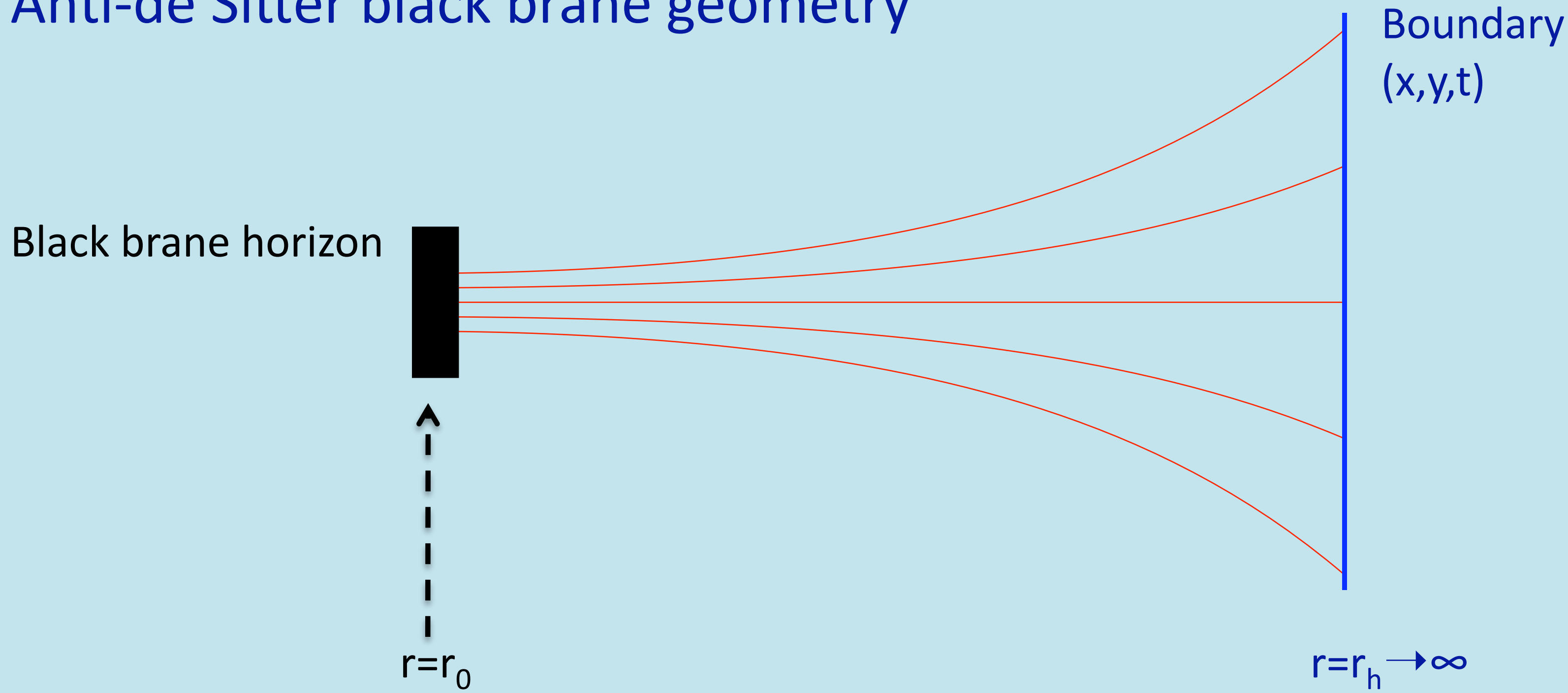
Through gauge-gravity duality, the mass of the black hole corresponds to temperature in the dual theory on the boundary, the U (1) gauge invariance yields a global U (1) symmetry on the boundary, and the scalar hair gives rise to a charged condensate that spontaneously breaks this global U (1) symmetry.

Such systems therefore therefore have been interpreted as holographic superconductors and superfluids. (Hartnoll, Herzog and Horowitz)

Inspired by this opportunity to study symmetry breaking in strongly interacting systems, we consider a theory with $U_A(1) \times U(1)_B$ gauge symmetry and two charged scalar fields in a 3+1 dimensional Anti-de Sitter black brane background in order to study how the existence of one type of condensate affects the formation of another.

We work in the probe limit, where the back-reaction of the scalar fields and gauge fields on the metric is negligible.

Anti-de Sitter black brane geometry



Metric:

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(dx^2 + dy^2),$$

$$f(r) = \frac{r^2}{R_{\rm AdS}^2} - \frac{r_0^3}{R_{\rm AdS}^2 r}.$$

Hawking Temperature:

$$T = \frac{3}{4\pi}\frac{r_0}{R_{\rm AdS}^2}.$$

The Model

The model has $U_A(1) \times U_B(1)$ gauge invariance. It contains two complex scalar fields ϕ_I and ϕ_{II} that carry charges (1,1) and (1,-1) respectively. The invariant action in the Anti-de Sitter black brane background is given by

$$S = \int d^4x \, \sqrt{g} \Big[D_\mu \bar{\phi}_I D^\mu \phi_I + D_\mu \bar{\phi}_{II} D^\mu \phi_{II} - m_I^2 \bar{\phi}_I \phi_I - m_{II}^2 \bar{\phi}_{II} \phi_{II} - \frac{1}{4} F_{A\mu\nu} F_A^{\mu\nu} - \frac{1}{4} F_{B\mu\nu} F_B^{\mu\nu} \Big].$$

The covariant derivatives of the two scalar fields are

$$\begin{aligned} D_\mu \phi_I &= (\partial_\mu - i g_A A_\mu - i g_B B_\mu) \phi_I, \\ D_\mu \phi_{II} &= (\partial_\mu - i g_A A_\mu + i g_B B_\mu) \phi_{II}, \end{aligned}$$

where g_A and g_B are the two U(1) gauge coupling constants.

The model can be cast into a different form by the field redefinitions

$$\begin{aligned} A_\mu &= \sin\theta\,(X_\mu + Y_\mu) \\ B_\mu &= \cos\theta\,(X_\mu - Y_\mu) \end{aligned}$$

$$\cos\theta \equiv \frac{g_A}{\sqrt{g_A^2 + g_B^2}} \qquad \sin\theta \equiv \frac{g_B}{\sqrt{g_A^2 + g_B^2}}$$

In terms of the gauge fields X_μ and Y_μ, the invariant action takes the form

$$S = \int d^4x\,\sqrt{g}\,\Big[D_\mu\bar{\phi}_I\,D^\mu\phi_I + D_\mu\bar{\phi}_{II}\,D^\mu\phi_{II} - m_I^2\bar{\phi}_I\phi_I - m_{II}^2\bar{\phi}_{II}\phi_{II} - \frac{1}{4}F_{X\mu\nu}F_X^{\mu\nu} - \frac{1}{4}F_{Y\mu\nu}F_Y^{\mu\nu} + \frac{1}{2}\cos 2\theta F_{X\mu\nu}F_Y^{\mu\nu}\Big].$$

Kinetic mixing

The covariant derivatives of the scalar fields are now

$$\begin{aligned} D_\mu\phi_I &= (\partial_\mu - ig_0\sin 2\theta\,X_\mu)\,\phi_I\,, \\ D_\mu\phi_{II} &= (\partial_\mu - ig_0\sin 2\theta\,Y_\mu)\,\phi_{II}\,. \end{aligned}$$

$$g_0 \equiv \sqrt{g_A^2 + g_B^2}.$$

Disentangled scalar kinetic sector

In terms of the $U_X(1) \times U_Y(1)$ gauge symmetry, the scalar fields ϕ_I and ϕ_{II} carry charges (1,0) and (0,1), so that each scalar field serves as an order parameter for the spontaneous breaking of only one U(1) symmetry. The price to pay for this simplification is that now mixing appears in the gauge kinetic terms.

The strength of the gauge kinetic mixing is proportional to

$$\cos 2\theta = \frac{g_A^2 - g_B^2}{g_A^2 + g_B^2},$$

so that no mixing occurs in case $g_A = g_B$.

Equations of motion

We are interested in solutions to the equations of motion where the scalar fields and the zeroth components of the gauge fields only depend on s, and all other components of the gauge fields vanish.

The reduced equations of motion for the scalar fields take the form:

$$\frac{d^2\phi_I}{ds^2} + \left[\frac{2}{s} + \frac{1}{u(s)}\frac{du}{ds}\right]\frac{d\phi_I}{ds} + \left[\frac{1}{u(s)^2}\underline{\left(g_A A_0 + g_B B_0\right)^2} - \frac{1}{u(s)}m_I^2\right]\phi_I = 0,$$

$$\frac{d^2\phi_{II}}{ds^2} + \left[\frac{2}{s} + \frac{1}{u(s)}\frac{du}{ds}\right]\frac{d\phi_{II}}{ds} + \left[\frac{1}{u(s)^2}\underline{\left(g_A A_0 - g_B B_0\right)^2} - \frac{1}{u(s)}m_{II}^2\right]\phi_{II} = 0,$$

Effective mass terms

The opposite sign in the two effective mass terms causes frustration. In some regions of parameter space one condensate will form, while the other will not.

The reduced equations of motion for the gauge fields are:

$$\frac{d^2A_0}{ds^2} + \frac{2}{s}\frac{dA_0}{ds} - \frac{2}{u(s)}g_A\left[g_A A_0\left(\phi_I^2 + \phi_{II}^2\right) + g_B B_0\left(\phi_I^2 - \phi_{II}^2\right)\right]\phi_I = 0,$$

$$\frac{d^2B_0}{ds^2} + \frac{2}{s}\frac{dB_0}{ds} - \frac{2}{u(s)}g_B\left[g_A A_0\left(\phi_I^2 - \phi_{II}^2\right) + g_B B_0\left(\phi_I^2 + \phi_{II}^2\right)\right]\phi_I = 0.$$

Near horizon solution

Near the horizon regular solutions to the equations of motion can be Taylor expanded as:

$$
\begin{aligned}
\phi_I &= c_0 + c_1(s - s_0) + c_2(s - s_0)^2 + \dots \\
\phi_{II} &= d_0 + d_1(s - s_0) + d_2(s - s_0)^2 + \dots \\
A_0 &= a_1(s - s_0) + a_2(s - s_0)^2 + \dots \\
B_0 &= b_1(s - s_0) + b_2(s - s_0)^2 + \dots
\end{aligned}
$$

Here a1 , b1 , c0 and d0 are integration constants, and the other coefficients in the expansion are determined from the equations of motion as:

$$
\begin{aligned}
c_1 &= \frac{1}{3s_0} m_I^2 c_0, \\
d_1 &= \frac{1}{3s_0} m_{II}^2 d_0, \\
a_2 &= -\frac{1}{3s_0}\left[3 - g_A^2(c_0^2 + d_0^2)\right] a_1 + \frac{1}{3s_0} g_A g_B (c_0^2 - d_0^2) b_1, \\
b_2 &= -\frac{1}{3s_0}\left[3 - g_B^2(c_0^2 + d_0^2)\right] b_1 + \frac{1}{3s_0} g_A g_B (c_0^2 - d_0^2) a_1, \\
c_2 &= -\frac{1}{3s_0}\left[6m_I^2 - m_I^4 + (g_A a_1 + g_B b_1)^2\right] c_0, \\
d_2 &= -\frac{1}{3s_0}\left[6m_{II}^2 - m_{II}^4 + (g_A a_1 - g_B b_1)^2\right] d_0.
\end{aligned}
$$

Near boundary solution

The asymptotic form of the solutions for large s near the boundary is:

$$
\begin{aligned}
A_0 &= \mu_A - \frac{\rho_A}{s}, \\
B_0 &= \mu_B - \frac{\rho_B}{s}, \\
\phi_I &= \phi_{I1}\, s^{\Delta_{I+}} + \phi_{I2}\, s^{\Delta_{I-}}, \\
\phi_{II} &= \phi_{II1}\, s^{\Delta_{II+}} + \phi_{II2}\, s^{\Delta_{II-}},
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{I\pm} &= -\frac{3}{2} \pm \sqrt{\frac{9}{4} + m_I^2}, \\
\Delta_{II\pm} &= -\frac{3}{2} \pm \sqrt{\frac{9}{4} + m_{II}^2}.
\end{aligned}
$$

Here ρ_A and ρ_B are the two types of charge densities of the boundary theory, and μ_A and μ_B are the associated chemical potentials.

The sources ϕ_{I1} and ϕ_{II1} are set to zero, so that the charged condensates in the boundary theory are:

$$
\begin{aligned}
< 0_{I2} > &= \phi_{I2}, \\
< 0_{II2} > &= \phi_{II2}.
\end{aligned}
$$

In what follows we consider the specific choice of masses:

$$
m_I^2 = m_{II}^2 = -2: \qquad \Delta_{I+} = \Delta_{II+} = -1, \quad \Delta_{I-} = \Delta_{II-} = -2.
$$

Numerical integration in bulk

The near horizon series expansion solution provides boundary conditions close to the the horizon for the numerical integration of the equations of motion.

At large values of s, the numerical solution is matched to the asymptotic solution near the boundary.

The integration constants c_0 , d_0 , a_1 and b_1 are iteratively adjusted so as to obtain $\phi_{I0} = \phi_{II0} = 0$ for the sources and the desired values of ρ_A and ρ_B.

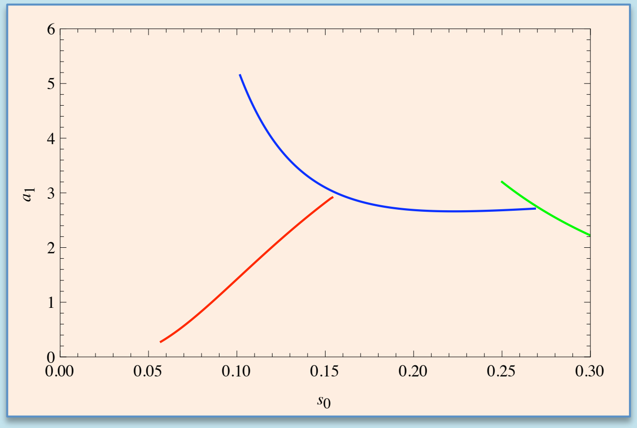
The values of the condensates $<O_{I2>}$ and $<O_{II2}>$ are thus determined as well as the values of the chemical potentials μ_A and μ_B .

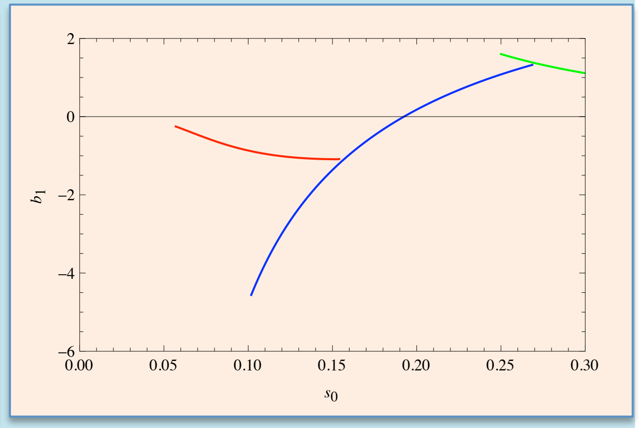
Results

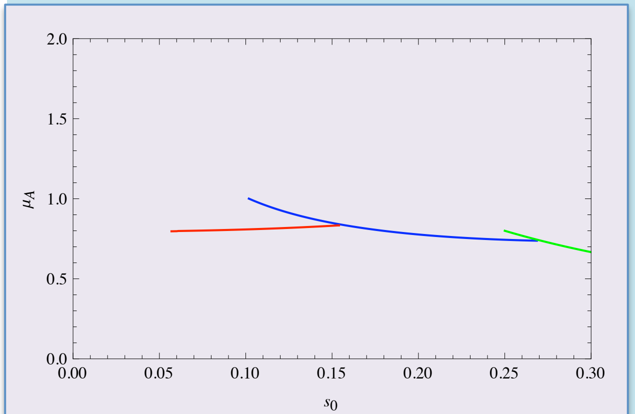
ρ_A=0.2, ρ_B=0.1

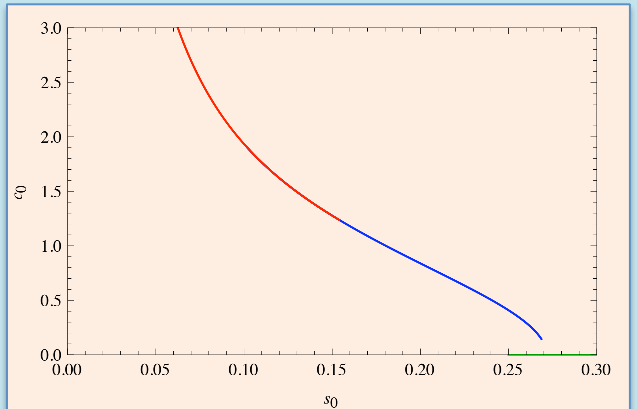
g_A = g_B =1.0

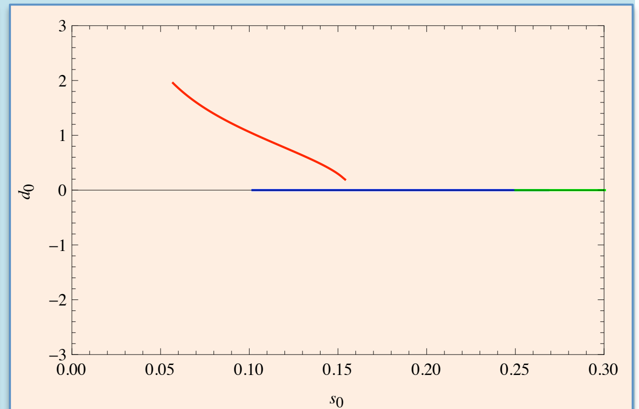
Varying temperature.

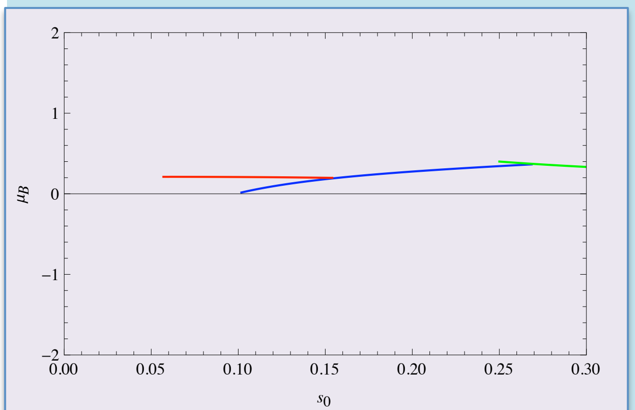


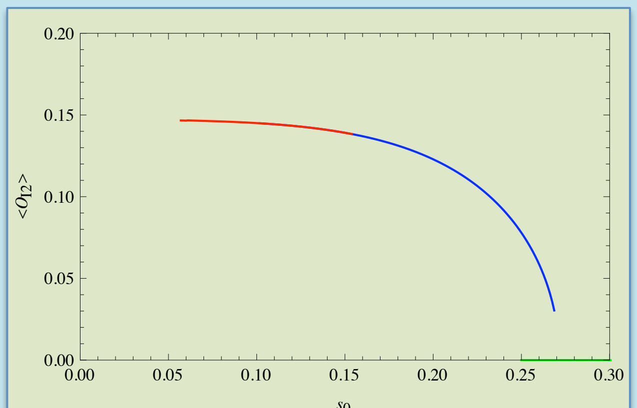


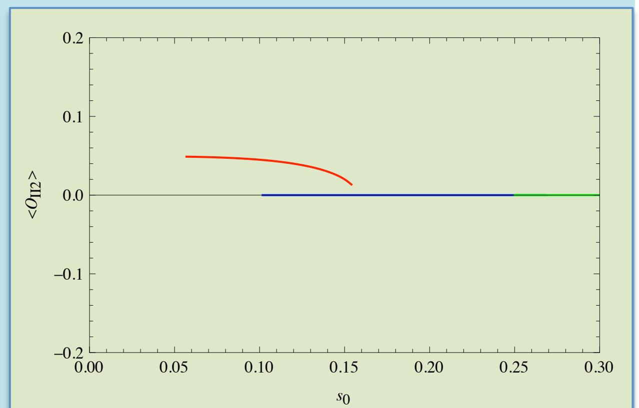








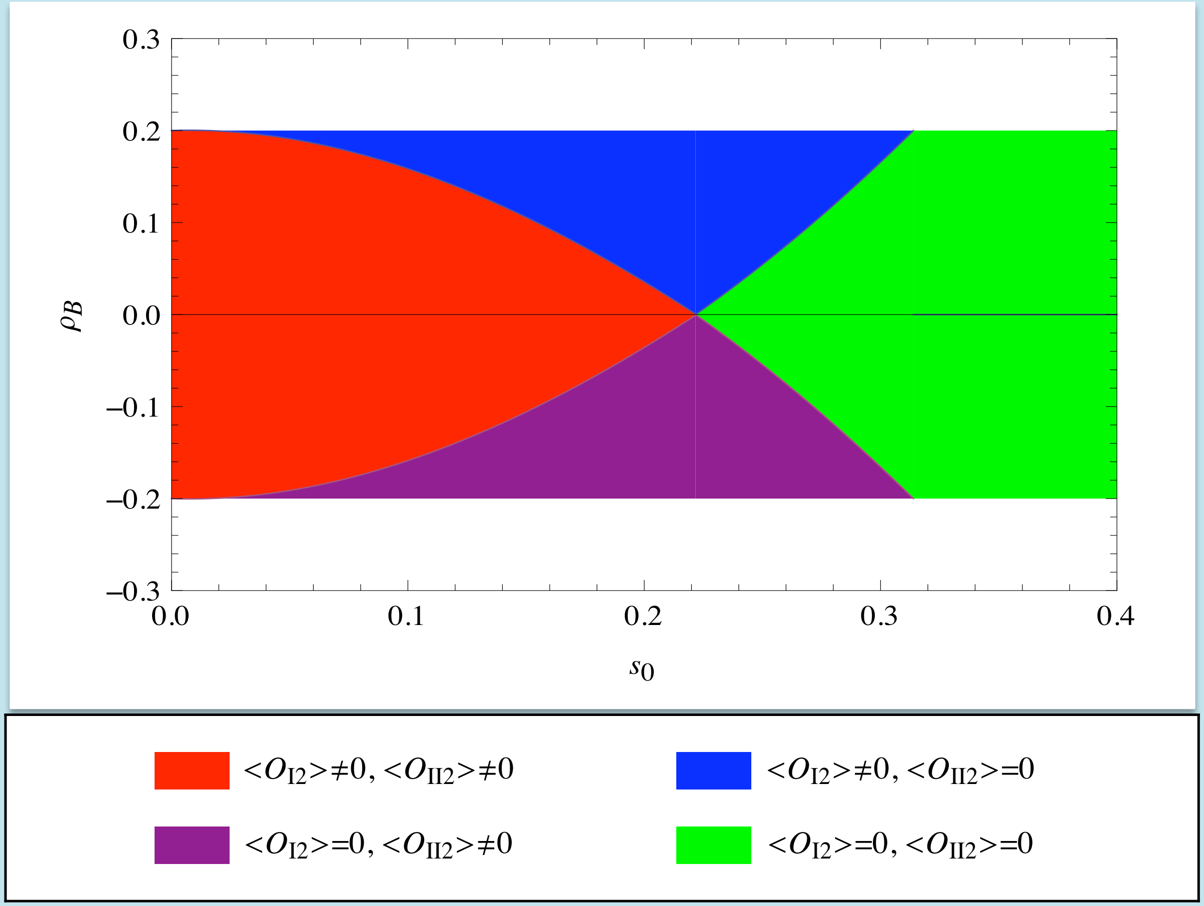




By scanning the parameter space, the following phase diagram is obtained:

ρ_A=0.2

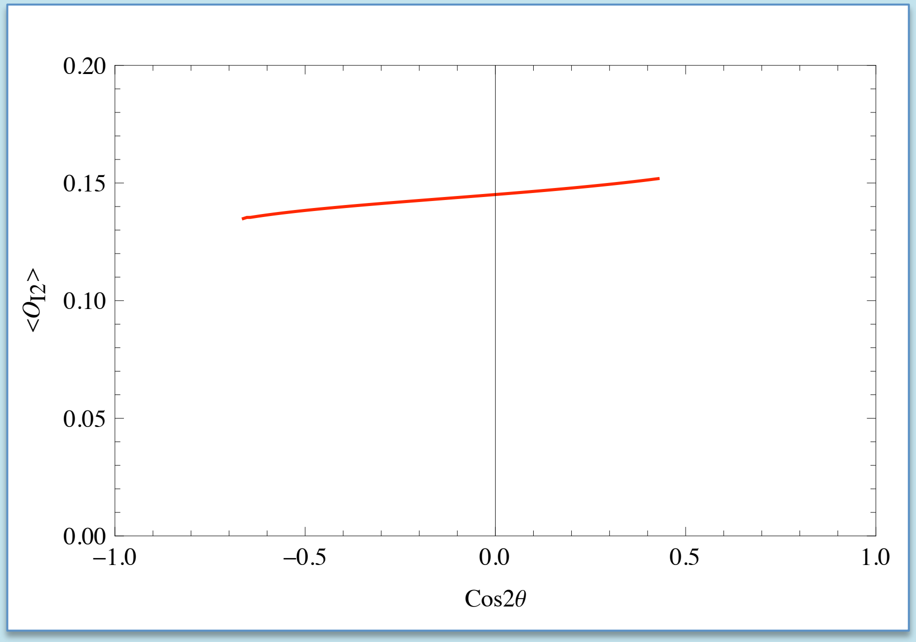
g_A = g_B =1.0

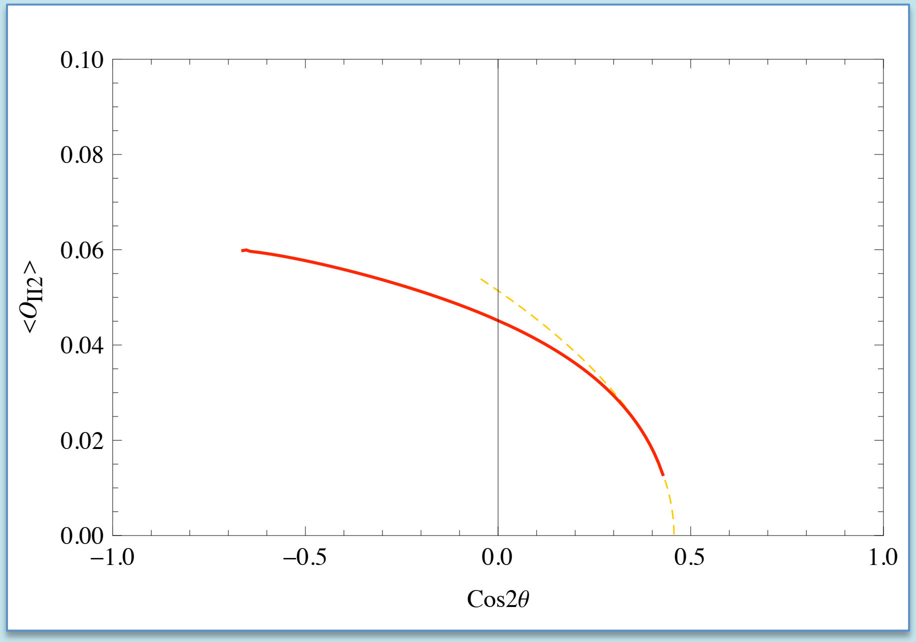


- At ρ_B=0, B_0=0, the two effective masses are equal, and therefore the condensates are identical.
- At ρ_B=ρ_A, B_0=A_0, one of the effective masses vanishes, and therefore only one condensate forms. Frustration is manifest.
- This phase diagram will remain identical even if a scalar potential in the bulk is switched on, since the scalar fields are very small near the phase transitions.

How does this phase diagram change when the gauge couplings are not equal?

Preliminary study of the case $g_A \neq g_B$:





ρ_A=0.2, ρ_B=0.1

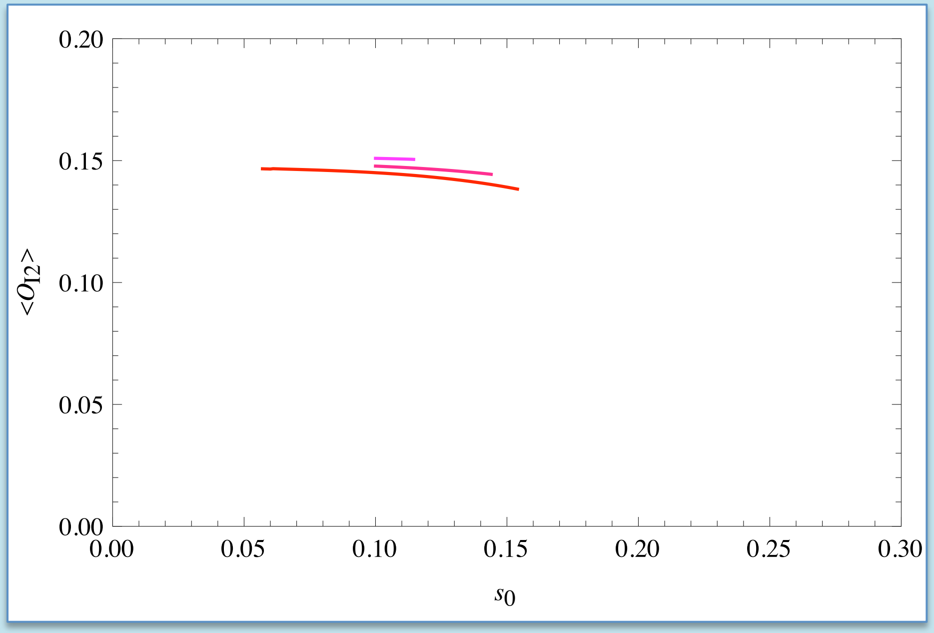
S_0=0.1

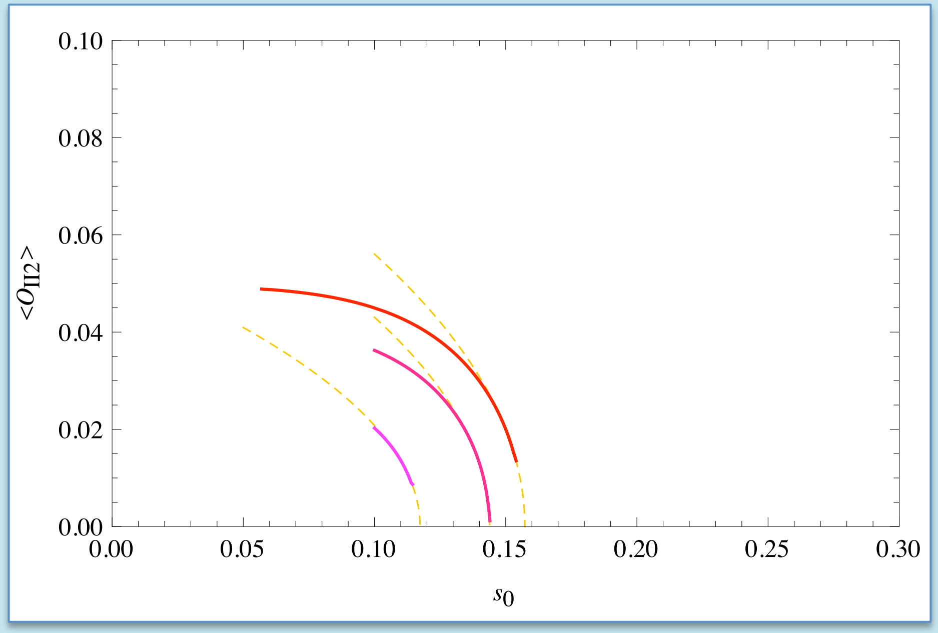
g_0=1.0
Varying ratio of gauge couplings

Recall that:

$$\cos 2\theta = \frac{g_A^2 - g_B^2}{g_A^2 + g_B^2},$$

Condensates as a function of temperature, various values of θ.





ρ_A=0.2, ρ_B=0.1

Θ=6π/32, 7π/32, 8π/32

g_0=1.0

Varying temperature

Critical exponent independent of Cos 2θ:

$$< O_{II2} > \sim C(\theta)\,(T - T_c(\theta))^{1/2}\,.$$

Conclusions

We considered a system with two charged scalar fields and two Abelian gauge fields in the background of a 3+1 dimensional planar Anti-de Sitter black hole, and analyzed the formation of charged condensates in the dual boundary theory.

The phase diagram of the boundary theory was mapped out in the case of two equal gauge coupling constants.

In some regions of parameter space only one condensate forms due to frustration.

A preliminary analysis of the system with unequal gauge couplings was also performed, and it was found that the critical exponent is universal and does not depend on the ratio of the gauge couplings.

A more complete analysis of the phase diagram in the case of unequal gauge coupling constants is in progress.