Mass determination with missing energy at the LHC

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arXiv: 0802.4290, H. Cheng, D. Engelhardt, J. Gunion, ZH, B. McElrath

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Outline

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Introduction: mass determination with missing particles

Kinematic Constraints

Conclusion

New physics and missing particles at the LHC

- Dark Matter+Electroweak constraints $\Rightarrow$ stable neutral particle.
  SUSY with R-parity, little Higgs with T-Parity, UED with KK parity...

At LHC,

- We will see large missing transverse momenta.
- Mass determination is hard – there is no peak in invariant mass distributions.

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- The edge/end-point method.
- Kinematic constraints.

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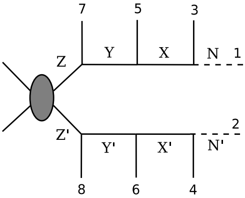
The event topology



Example: $\tilde{q} \rightarrow q\tilde{\chi}_2^0 \rightarrow q\tilde{l}l \rightarrow q\tilde{\chi}_1^0 ll$.

- This could come from a longer decay chain as long as there is no extra missing particle.
- Assume all intermediate particles on-shell.
- Assume $m_N = m_{N'}, m_X = m_{X'}, m_Y = m_{Y'}, m_Z = m_{Z'}$.

Count the constraints

One event, 6 constraints:

$$(M_N^2 \quad =) \qquad\qquad p_1^2 \quad = \quad p_2^2,$$

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$$\begin{array}{lrcl} (M_N^2 \;=) & p_1^2 & = & p_2^2, \\ (M_X^2 \;=) & (p_1+p_3)^2 & = & (p_2+p_4)^2, \end{array}$$

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(M_N^2 &=) & p_1^2 &= p_2^2, \\
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$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.$$

But 8 unknowns: p_1, p_2. Number of equations less than number of unkowns.

A pair of events.

Add one event,

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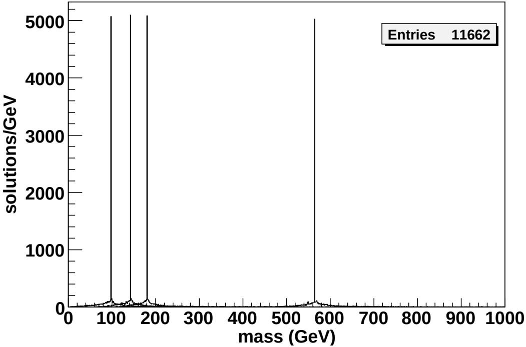
The system can be reduced to 13 linear equations+ 3 quadratic equations, so generally we have 8 complex solutions. Only keep real and positive ones for masses.

An ideal example

$\tilde{q}\tilde{q} \to q\tilde{\chi}_2^0 q\tilde{\chi}_2^0 \to q\tilde{l}lq\tilde{l}l \to q\tilde{\chi}_1^0 llq\tilde{\chi}_1^0 ll$
SPS1a, masses: (97.4, 142.5, 180.3, 564.8) GeV



2 solutions per pair on average.

Realistic case

- Wrong combinations. One event, 8 combinations for $2\mu 2e$ channel, 16 for 4μ or $4e$ channel. A pair of events, 64, 128 or 256 combinations.

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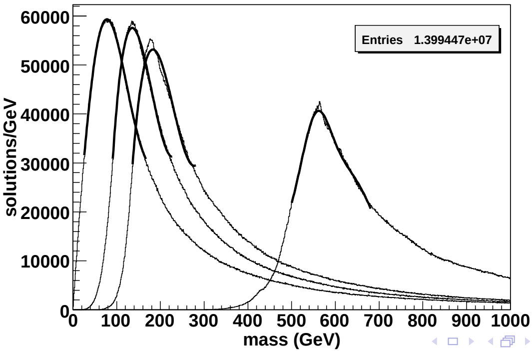
- Initial/final state radiation.
- Extra jets. $\tilde{g} \to q\tilde{q}_L$.
- Experimental resolutions. Simulated with ATLFAST.
- Background events.

Realistic solution distributions

Cuts:

1. 4 isolated leptons with $p_T > 10$ GeV, $|\eta| < 2.5$, consistent flavors and charges.
2. No b-jet, ≥ 2 jets with $p_T > 100$ GeV, $|\eta| < 2.5$. Take 2 highest-p_T jets as partiles 7 and 8.
3. $p_{Tmiss} > 50$ GeV.

About 1000 events ($\sim$ 700 signals) after cuts for 300 fb^{-1}.



Fit the masses

Fitting each curve using a sum of a Gaussian and a quadratic polynomial and take the peak positions as the estimated masses, we get $\{77.8, 135.6, 182.7, 562.0\}\ \mathrm{GeV}$.
Averaging over 10 different data sets:

$$m_N = 76.7 \pm 1.4\ \mathrm{GeV}, \quad m_X = 135.4 \pm 1.5\ \mathrm{GeV},$$
$$m_Y = 182.2 \pm 1.8\ \mathrm{GeV}, \quad m_Z = 564.4 \pm 2.5\ \mathrm{GeV}.$$

The statistical errors are very small, but the masses are biased.
Inputs: (97.4, 142.5, 180.3, 564.8) GeV

Some model-independent techniques
I. Cut off “bad” combinations

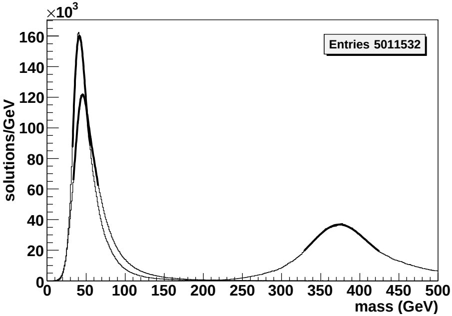
- For the ideal case, the correct combination of one event can always pair with any other event and yield solutions. So the number of events that pair with this combination is maximized as $N_{evt} - 1$.
- After smearing, this is no longer true, but the correct combinations still have statistically larger number of events to pair.
- We cut on this number so that we have about 4 combinations per event left (originally 11).

II. Number of solutions weighting.

A pair with many solutions enters with a large weight, although at most one of the solutions can be the true masses.
$\rightarrow$ Treat each pair equally, weight the solutions by $1/n$, n=number of solutions for the pair.

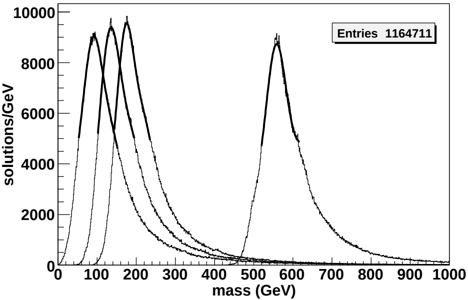
III. Cut on mass difference

Some solutions may have one or more, but not all four masses to be close to the true masses. Remove these solutions by a mass window cut.



Require all three mass differences to be within the mass window defined by $0.7\times$ peak height.

Mass peaks with smaller biases
SPS1a

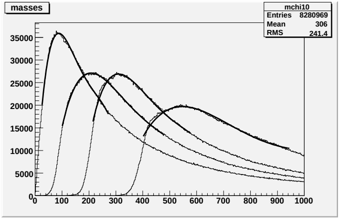


10 sets:

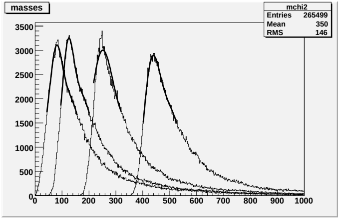
$$m_N = 94.1 \pm 2.8\ \mathrm{GeV}, \quad m_X = 138.8 \pm 2.8\ \mathrm{GeV},$$
$$m_Y = 179.0 \pm 3.0\ \mathrm{GeV}, \quad m_Z = 561.5 \pm 4.1\ \mathrm{GeV}.$$

Compare: { 97.4, 142.5, 180.3, 564.8 } GeV

Another mass point



Inputs $\{85.3, 128.4, 246.6, 431.1/438.6\}$ GeV.



10 sets:

$m_N = 85 \pm 4$ GeV, $m_X = 131 \pm 4$ GeV, $m_Y = 251 \pm 4$ GeV, $m_Z = 444 \pm 5$ GeV.

With this precision, we know more about the “model” and thus know how to Monte Carlo. The remaining biases can be removed by comparing with simulation.

Outline

Conclusion

For mass determination with 2 missing particles,

- Use as much as kinematic information as possible, esp. missing transverse momentum.
- Mass-shell constraints.
- Less events needed than the endpoint/edge method.
- A good start point for a more complicated method, ex, a full likelyhood fit with matrix element, PDF...