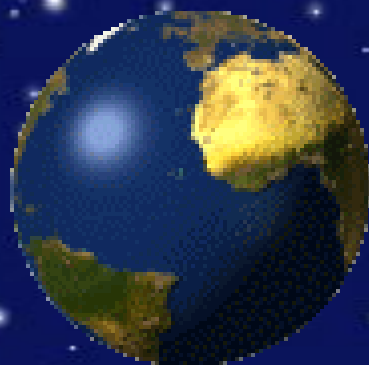


# Global 3 parameter Lorentz Violation model for neutrino oscillation with MiniBooNE

PRD74(2006)105009



Teppei Katori, V. Alan Kostelecký, and Rex Tayloe  
Indiana University

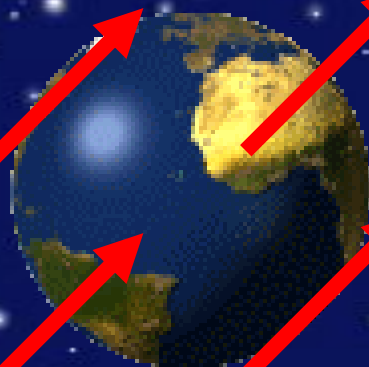
Pheno'08, Madison, Apr. 28, 08

Teppei Katori, Indiana University



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1. Neutrino oscillation - A natural interferometer
2. Tandem model
3. Tandem model oscillation signals
  - 3.1 Solar neutrinos
  - 3.2 Atmospheric neutrinos
  - 3.3 Reactor neutrinos
  - 3.4 LSND neutrinos and prediction for MiniBooNE
4. Global fit with MiniBooNE data
5. Conclusion



# 1. Neutrino oscillation - A natural interferometer

## 2. Tandem model

## 3. Tandem model oscillation signals

### 3.1 Solar neutrino

### 3.2 Atmospheric neutrino

### 3.3 Reactor neutrino

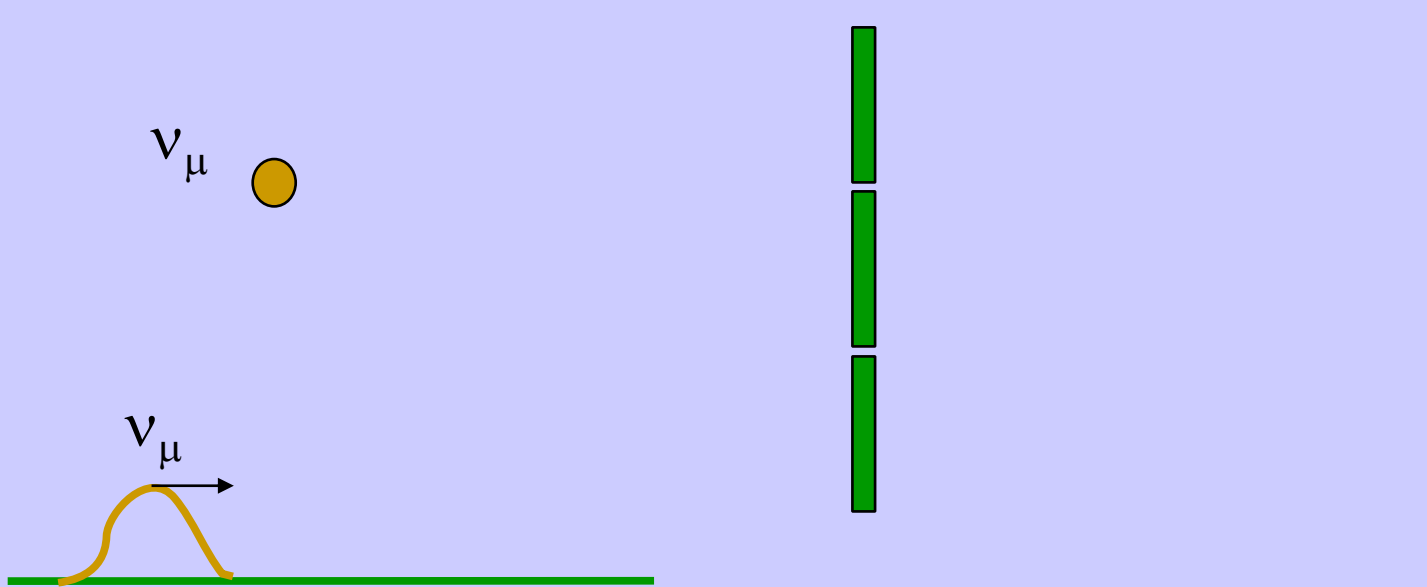
### 3.4 LSND neutrinos and prediction for MiniBooNE

## 4. Global fit with MiniBooNE data

## 5. Conclusion

# 1. Neutrino oscillation - A natural interferometer

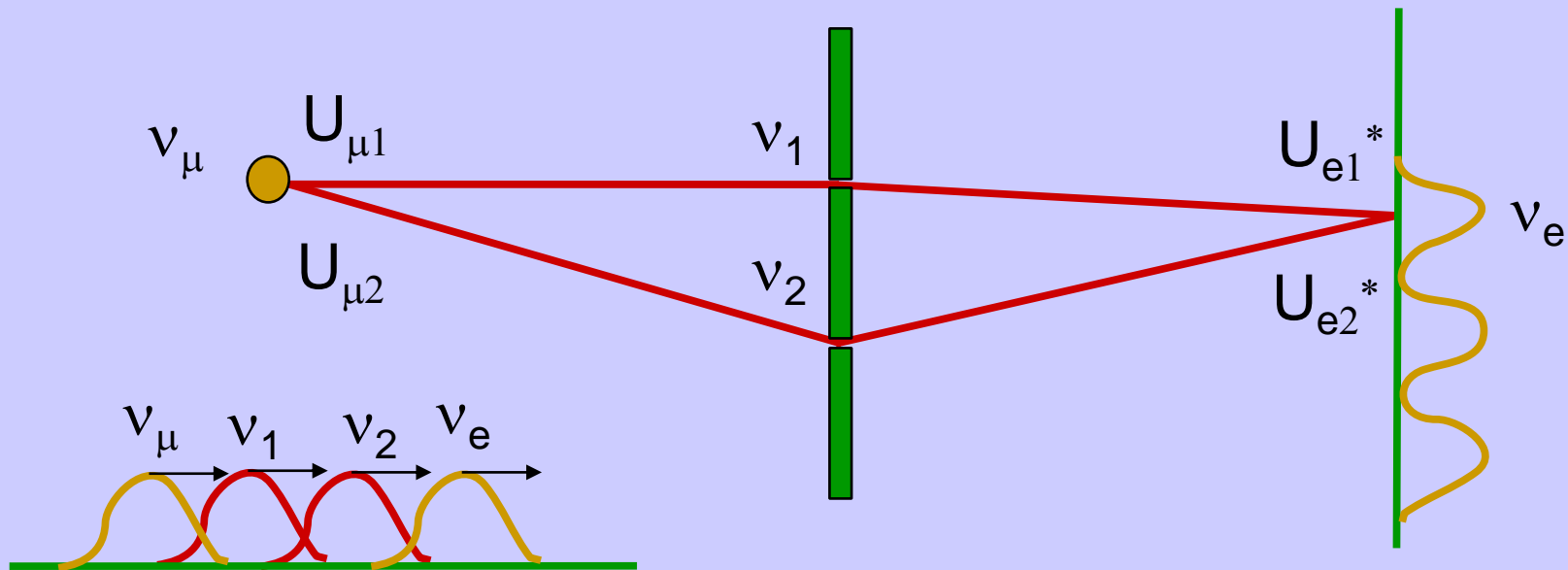
Neutrino oscillation is an interference experiment (cf. double slit experiment)



- If 2 neutrino Hamiltonian eigenstates,  $\nu_1$  and  $\nu_2$ , have different phase rotation, they cause quantum interference.

# 1. Neutrino oscillation - A natural interferometer

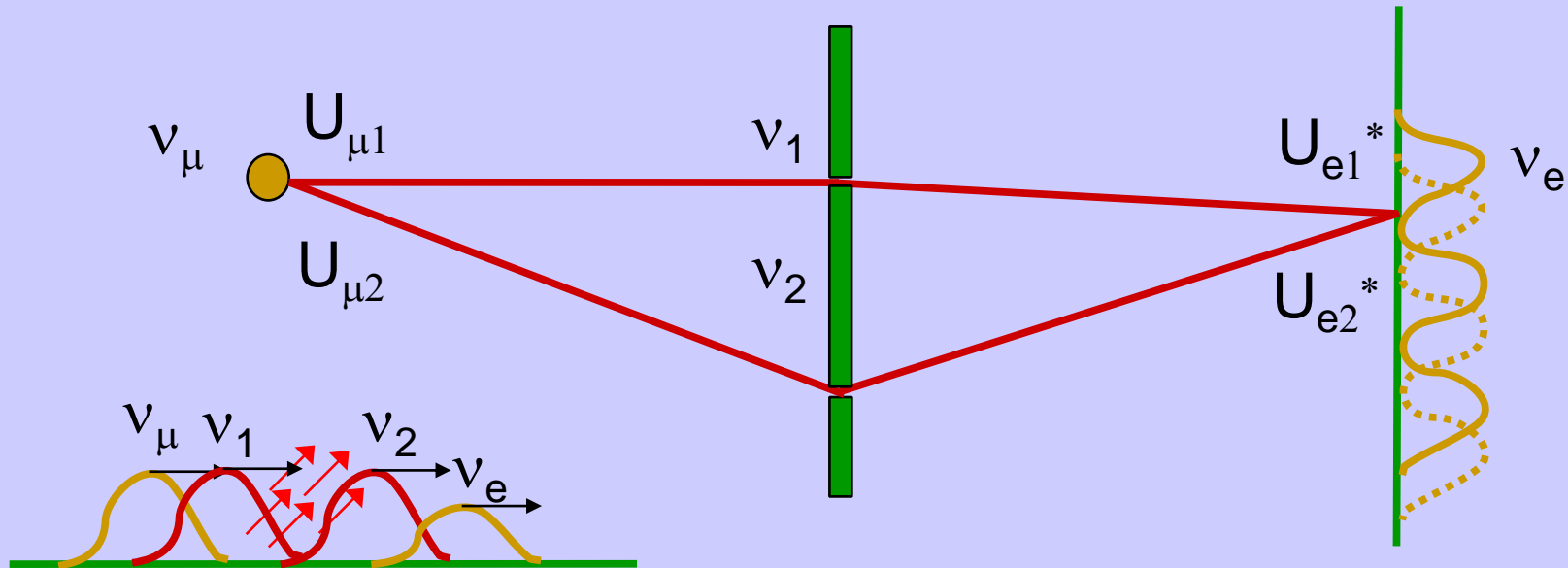
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- If 2 neutrino Hamiltonian eigenstates,  $\nu_1$  and  $\nu_2$ , have different phase rotation, they cause quantum interference.

# 1. Neutrino oscillation - A natural interferometer

Neutrino oscillation is an interference experiment (cf. double slit experiment)



- If 2 neutrino Hamiltonian eigenstates,  $\nu_1$  and  $\nu_2$ , have different phase rotation, they cause quantum interference.
- If  $\nu_1$  and  $\nu_2$ , have different coupling with **space-time properties (i.e. Lorentz violating field)**, interference fringe (oscillation pattern) depend on their couplings
- The measured scale of neutrino eigenvalue difference is comparable the target scale of Lorentz violation ( $<10^{-19}\text{GeV}$ ).

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## 2. Standard massive neutrino model

Standard Hamiltonian (7 free parameters)

$$(h_{eff})_{ab} \rightarrow \begin{pmatrix} \frac{m_{ee}^2}{2E} & \frac{m_{e\mu}^2}{2E} & \frac{m_{e\tau}^2}{2E} \\ \frac{m_{e\mu}^2}{2E} & \frac{m_{\mu\mu}^2}{2E} & \frac{m_{\mu\tau}^2}{2E} \\ \frac{m_{e\tau}^2}{2E} & \frac{m_{\mu\tau}^2}{2E} & \frac{m_{\tau\tau}^2}{2E} \end{pmatrix}$$

## 2. Standard massive neutrino model

Standard Hamiltonian (4 free parameters)

$$(h_{eff})_{ab} \rightarrow \begin{pmatrix} \frac{m_{ee}^2}{2E} & \frac{m_{e\mu}^2}{2E} & \frac{m_{e\tau}^2}{2E} \\ \frac{m_{e\mu}^2}{2E} & \frac{m_{\mu\mu}^2}{2E} & \frac{m_{\mu\tau}^2}{2E} \\ \frac{m_{e\tau}^2}{2E} & \frac{m_{\mu\tau}^2}{2E} & \frac{m_{\tau\tau}^2}{2E} \end{pmatrix}$$

$$\Delta m_{atm}^2 = 2.5 \times 10^{-3} eV^2$$

$$\Delta m_{sol}^2 = 8.0 \times 10^{-5} eV^2$$

$$\theta_{12} = 34^\circ$$

$$\theta_{23} = 45^\circ$$

2 mass differences and 2 mixing angles are sufficient to describe all known phenomena of neutrino oscillation.

## 2. Standard Model Extension (SME) for neutrino oscillations

Kostelecky and Mewes,  
PRD,69(2004)016005;70(2004)076002

Lorentz and CPT violating Hamiltonian  
(18 free parameters for real rotation invariant model)

$$(h_{eff})_{ab} \rightarrow \begin{pmatrix} c_{ee}E + a_{ee} + \frac{m_{ee}^2}{2E} & c_{e\mu}E + a_{e\mu} + \frac{m_{e\mu}^2}{2E} & c_{e\tau}E + a_{e\tau} + \frac{m_{e\tau}^2}{2E} \\ c_{e\mu}E + a_{e\mu} + \frac{m_{e\mu}^2}{2E} & c_{\mu\mu}E + a_{\mu\mu} + \frac{m_{\mu\mu}^2}{2E} & c_{\mu\tau}E + a_{\mu\tau} + \frac{m_{\mu\tau}^2}{2E} \\ c_{e\tau}E + a_{e\tau} + \frac{m_{e\tau}^2}{2E} & c_{\mu\tau}E + a_{\mu\tau} + \frac{m_{\mu\tau}^2}{2E} & c_{\tau\tau}E + a_{\tau\tau} + \frac{m_{\tau\tau}^2}{2E} \end{pmatrix}$$

Standard Model Extension (SME) is the minimum extension of Standard Model to include Particle Lorentz and CPT violation. It has all possible type of Lorentz and CPT violating tensors in the Lagrangian.

## 2. Standard Model Extension (SME) for neutrino oscillations

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$$(h_{eff})_{ab} \rightarrow \begin{pmatrix}
 \left[ c_{ee} E + a_{ee} + \frac{m_{ee}^2}{2E} \right] & \left[ c_{e\mu} E + a_{e\mu} + \frac{m_{e\mu}^2}{2E} \right] & \left[ c_{e\tau} E + a_{e\tau} + \frac{m_{e\tau}^2}{2E} \right] \\
 \left[ c_{e\mu} E + a_{e\mu} + \frac{m_{e\mu}^2}{2E} \right] & \left[ c_{\mu\mu} E + a_{\mu\mu} + \frac{m_{\mu\mu}^2}{2E} \right] & \left[ c_{\mu\tau} E + a_{\mu\tau} + \frac{m_{\mu\tau}^2}{2E} \right] \\
 \left[ c_{e\tau} E + a_{e\tau} + \frac{m_{e\tau}^2}{2E} \right] & \left[ c_{\mu\tau} E + a_{\mu\tau} + \frac{m_{\mu\tau}^2}{2E} \right] & \left[ c_{\tau\tau} E + a_{\tau\tau} + \frac{m_{\tau\tau}^2}{2E} \right]
 \end{pmatrix}$$

Lorentz violating coefficients (CPT-even)
Lorentz and CPT violating coefficients (CPT-odd)

## 2. Tandem model

TK, Kostelecky and Tayloe,  
PRD74(2006)105009

Lorentz and CPT violating Hamiltonian  
(3 free parameters for real rotation invariant model)

$$(h_{eff})_{ab} \rightarrow \begin{pmatrix} H_{ee}(a, c, m) & H_{e\mu}(a, c, m) & H_{e\tau}(a, c, m) \\ H_{\mu e}(a, c, m) & H_{\mu\mu}(a, c, m) & H_{\mu\tau}(a, c, m) \\ H_{\tau e}(a, c, m) & H_{\tau\mu}(a, c, m) & H_{\tau\tau}(a, c, m) \end{pmatrix}$$

Numerical parameter search shows 3 parameters model reasonably works to describe all 4 classes of known neutrino oscillation data (solar, atmospheric, reactor, and LSND signals).

1. Neutrino oscillation - A natural interferometer

2. Tandem model

**3. Tandem model oscillation signals**

**3.1 Solar neutrinos**

3.2 Atmospheric neutrinos

3.3 Reactor neutrinos

3.4 LSND neutrinos and prediction for MiniBooNE

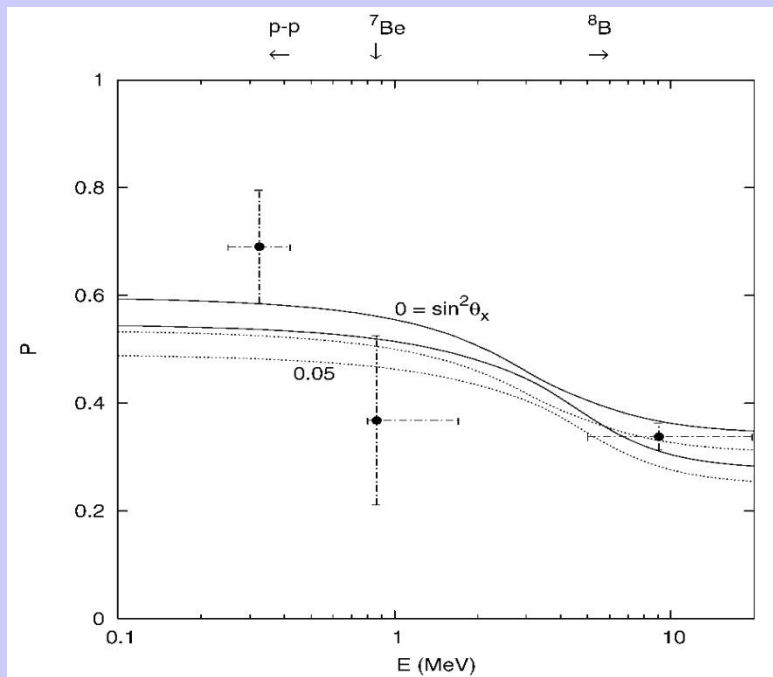
4. Global fit with MiniBooNE data

5. Conclusions

# 3.1 Solar neutrinos

Solar neutrino suppression is created by the energy dependences of mixing angles. So even long base line limit has energy dependence for neutrino oscillation.

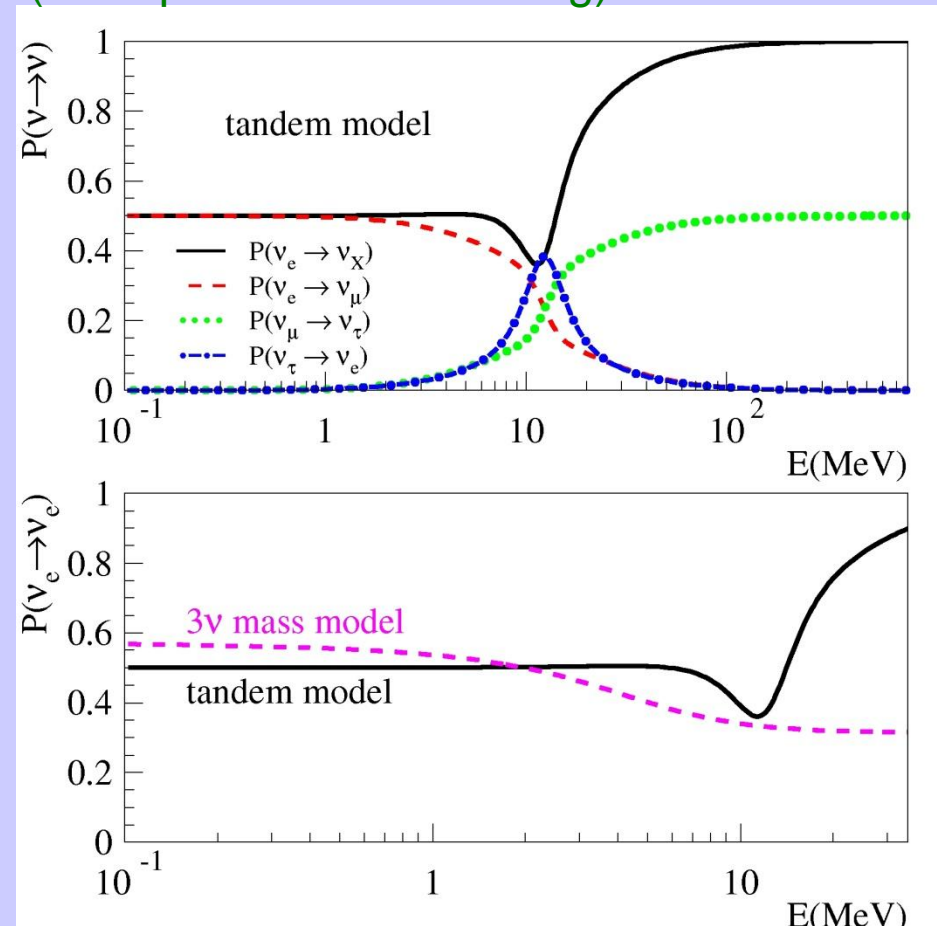
## Solar neutrino data



Barger et al., PLB,617(2005)78

Since Lorentz violating terms are bigger than solar matter potential, this model doesn't use MSW effect.

## Theoretical signals (no experimental smearing)



1. Neutrino oscillation - A natural interferometer

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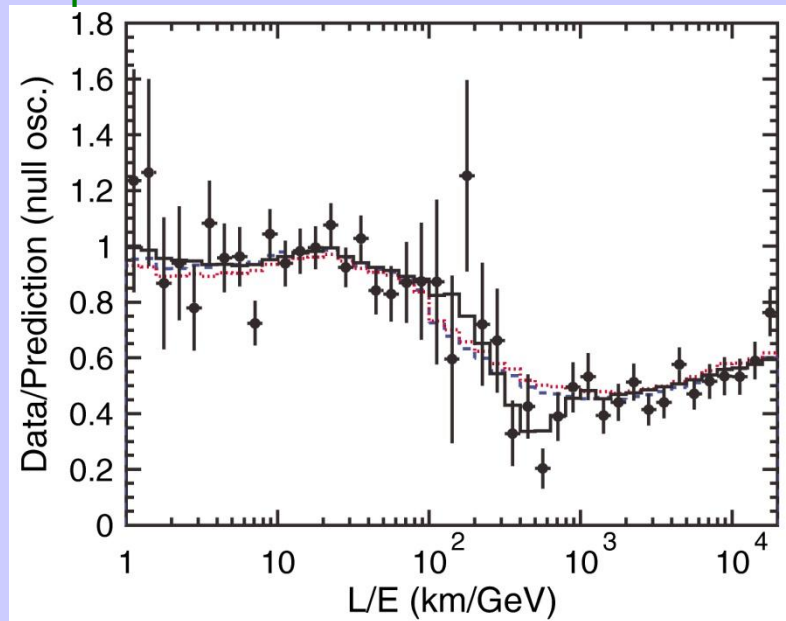
5. Conclusions



## 3.2 Atmospheric neutrinos

High energy neutrino oscillation is identical with standard maximum  $\nu_\mu$ - $\nu_\tau$  oscillation.

### Super-Kamiokande data



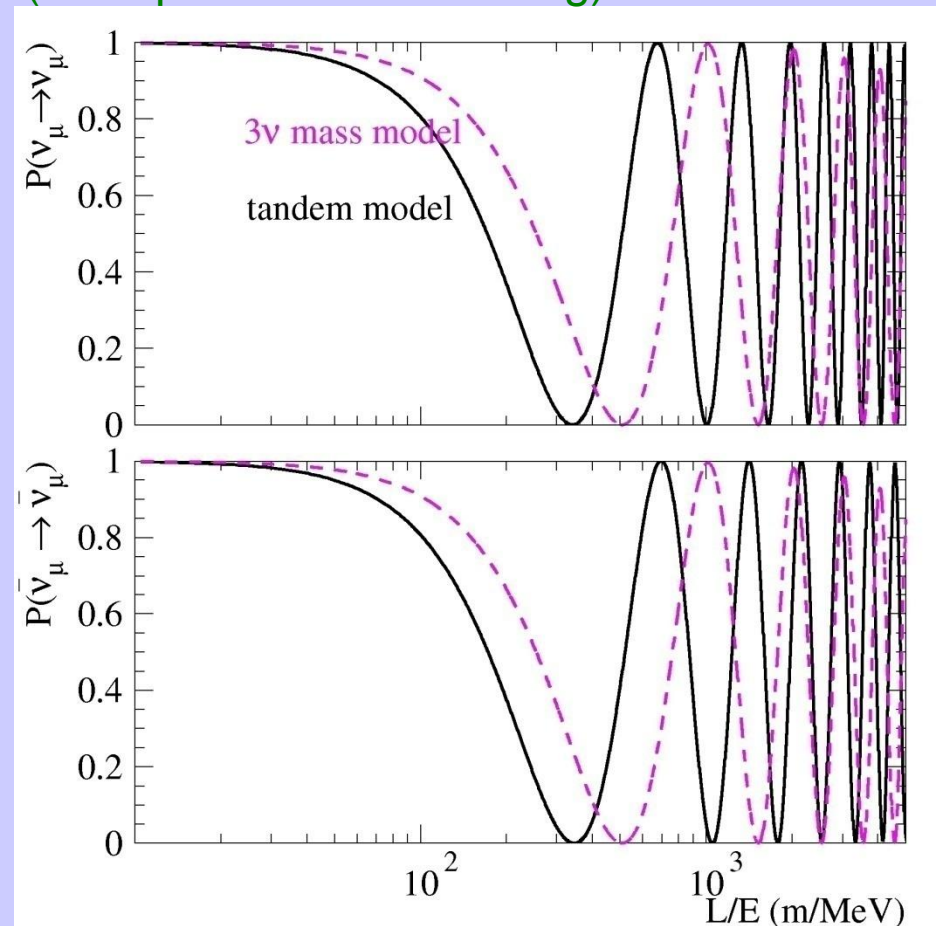
PRL,94(2005)101801

Although model has CPT-odd term, there is no difference for neutrinos and anti-neutrinos oscillation at high energy region (consistent with MINOS).

PRD,73(2006)072002

### Theoretical signals

(no experimental smearing)



1. Neutrino oscillation - A natural interferometer

2. Tandem model

**3. Tandem model oscillation signals**

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3.4 LSND neutrinos and prediction for MiniBooNE

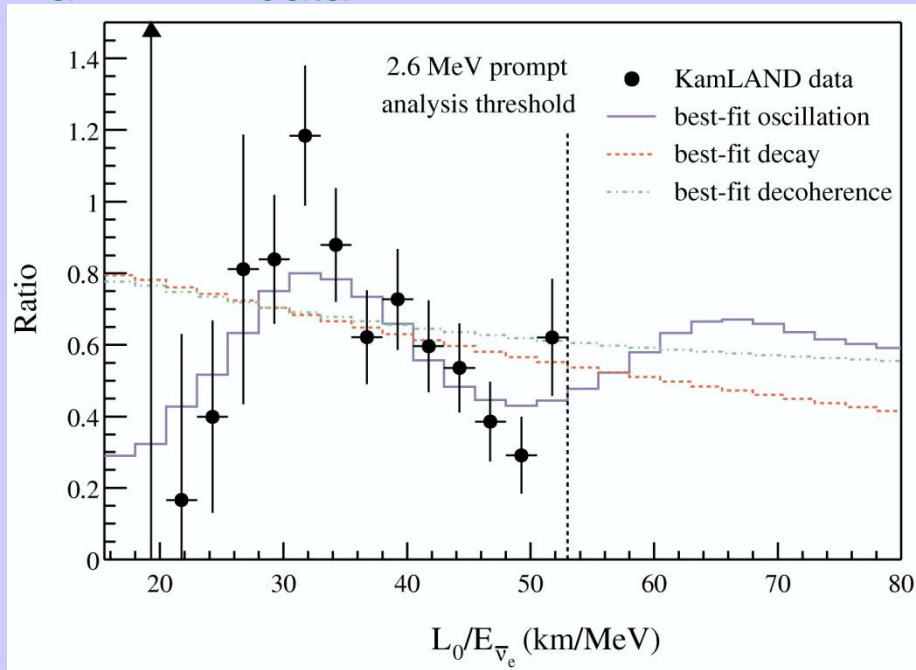
4. Global fit with MiniBooNE data

5. Conclusions

# 3.3 Reactor neutrinos

KamLAND neutrino oscillation is created by the combination of all oscillation channels.

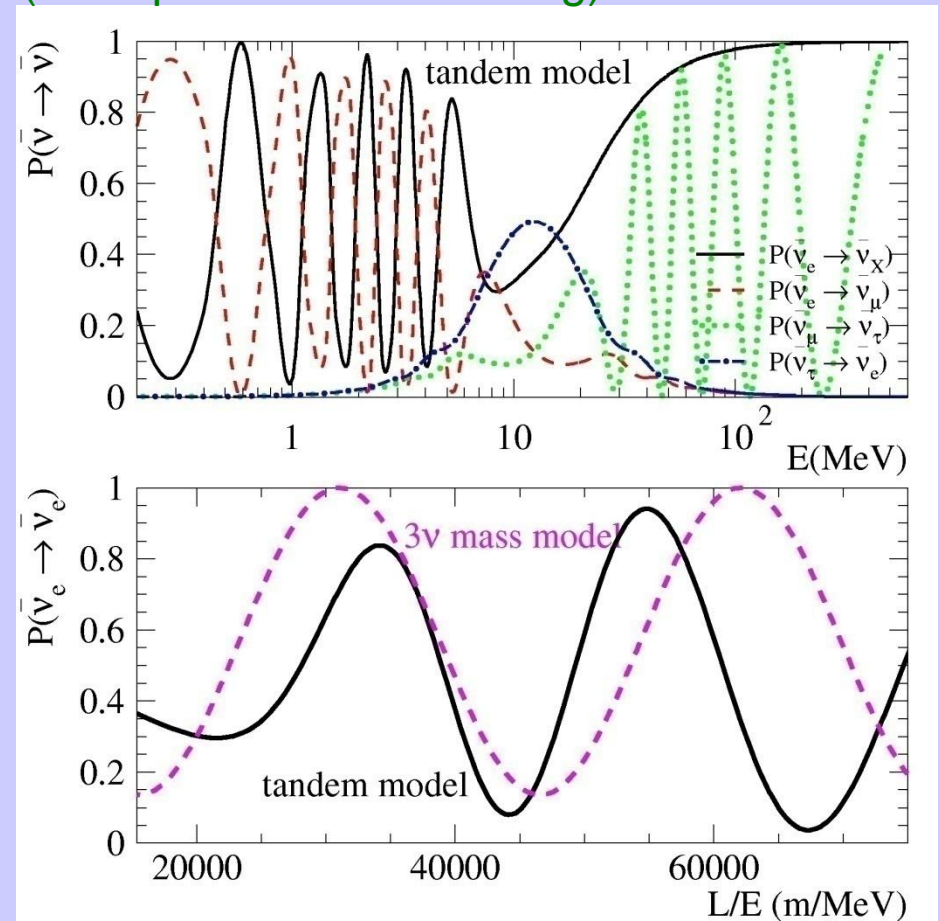
## KamLAND data



PRL,94(2005)081801

All  $\nu_e$  oscillation amplitudes go to zero at high energy limit ( $>100\text{MeV}$ ), so this model predicts null signal for NOvA and T2K.

## Theoretical signals (no experimental smearing)



1. Neutrino oscillation - A natural interferometer

2. Tandem model

**3. Tandem model oscillation signals**

3.1 Solar neutrinos

3.2 Atmospheric neutrinos

3.3 Reactor neutrinos

**3.4 LSND neutrinos and prediction for MiniBooNE**

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5. Conclusions

## 3.4 LSND neutrinos and MiniBooNE prediction

LSND signal is created by small, yet non zero amplitudes around 10-100 MeV region.

The tandem model predicts;

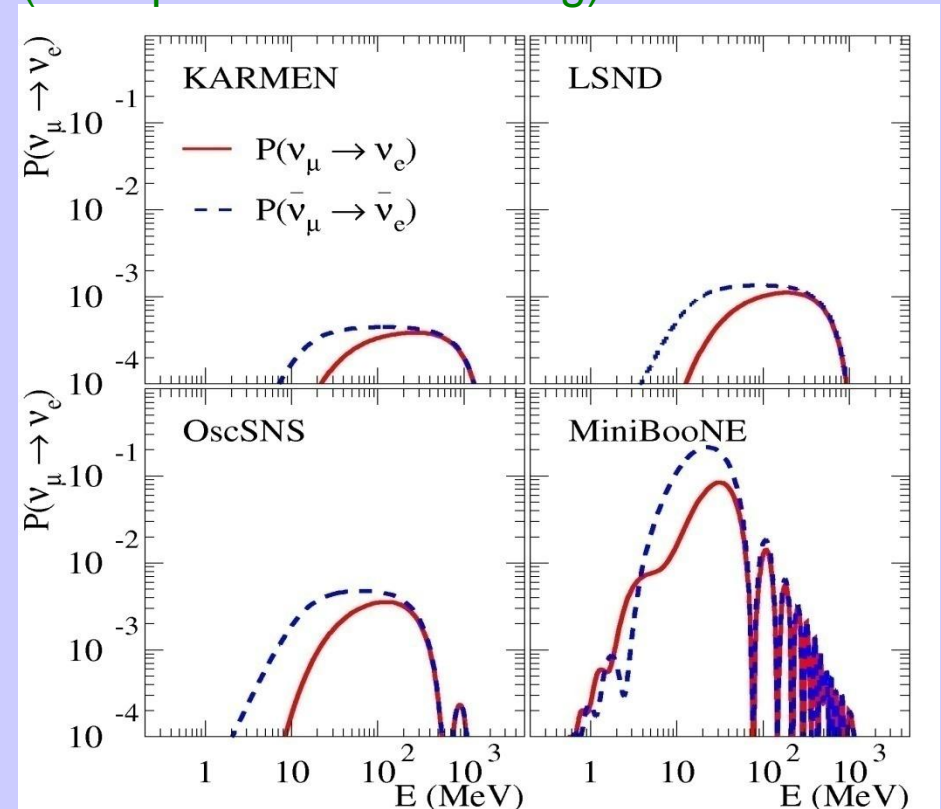
(1) factor 3 smaller appearance signal for KARMEN than LSND

(2) small ( $\sim 0.1\%$ ), but non zero appearance signal for LSND

(3) factor 3 large appearance signal for OscSNS than LSND

(4) Large signal at MiniBooNE low energy region with energy dependence.

Theoretical signals  
(no experimental smearing)



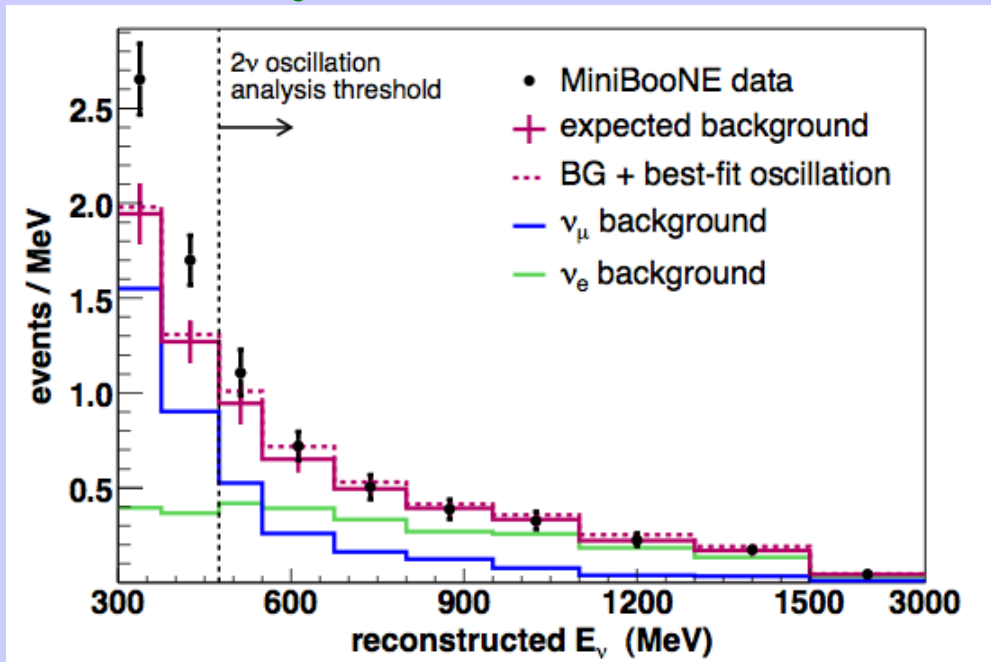
1. Neutrino oscillation - A natural interferometer
2. Tandem model
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  - 3.1 Solar neutrinos
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  - 3.3 Reactor neutrinos
  - 3.4 LSND neutrinos and prediction for MiniBooNE
4. **Global fit with MiniBooNE data**
5. Conclusions

# 4 MiniBooNE low energy excess

MiniBooNE didn't see the oscillation signal in the region where signal is expected from LSND motivated massive neutrino model.

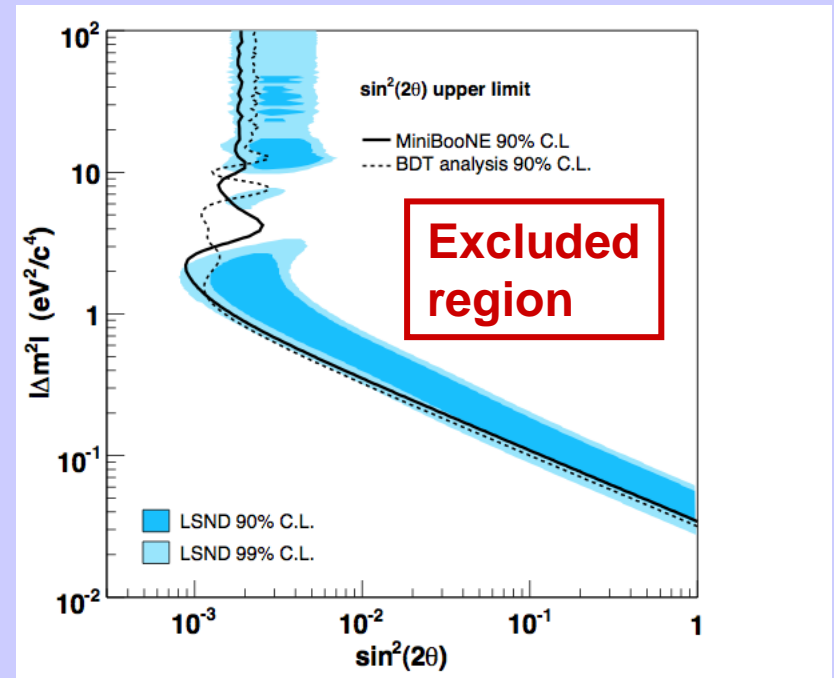
However MiniBooNE observed interesting excess of events at low energy region.

## MiniBooNE $\nu_e$ candidate data



PRL,98(2007)231801

Teppei Katori, Indiana University



Any 2 neutrino massive models fit with this excess.

Is this signal predicted by Tandem model?

## 4 MiniBooNE low energy excess

We used MiniBooNE public database

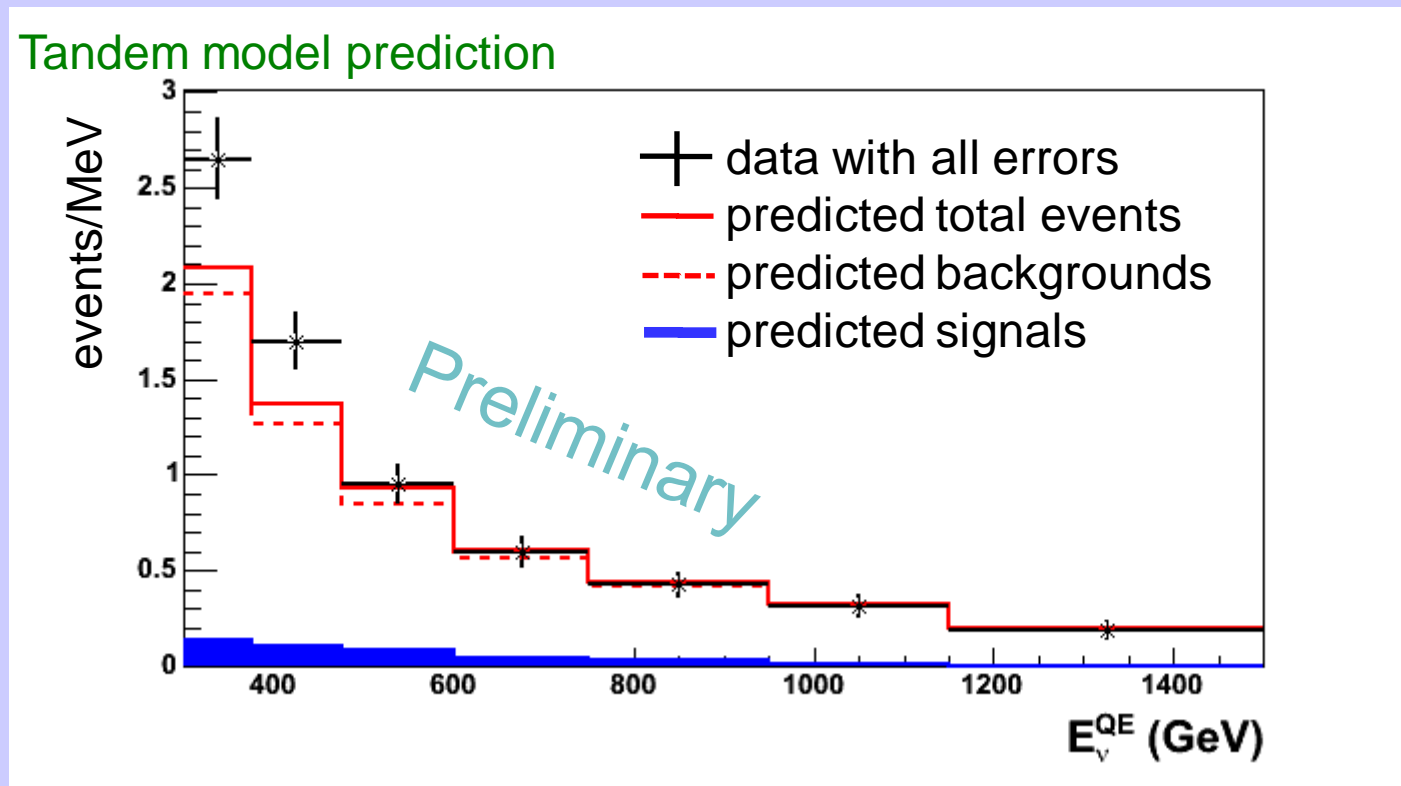
[http://www-boone.fnal.gov/for\\_physicists/april07datarelease/index.html](http://www-boone.fnal.gov/for_physicists/april07datarelease/index.html)

We did a quick survey in the parameter space to maximize MiniBooNE signal without breaking other oscillation signals.

Ongoing...but...

It is difficult to enhance MiniBooNE low E signal without breaking other oscillation signal.

The extension of the model is required?





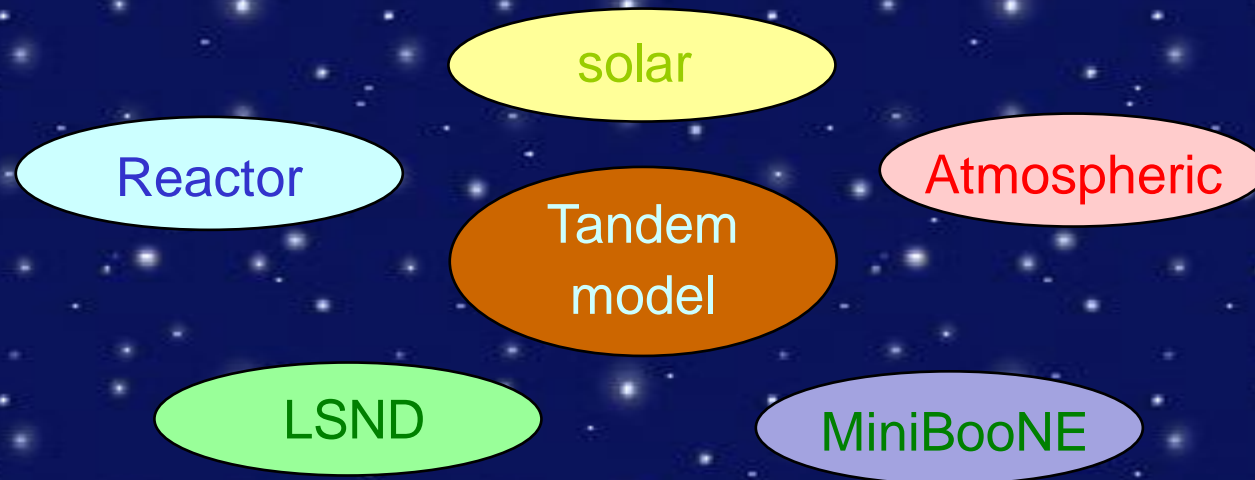
## 5. Conclusions

Lorentz and CPT violation is a predicted phenomenon from Planck scale physics.

The tandem model reasonably describes the existing all 4 classes of neutrino oscillation data (solar, atmospheric, KamLAND, and LSND) using only 3 parameters.

Tandem model predicts MiniBooNE low E excess, and fitting is ongoing

Quantitative predictions are offered for OscSNS, NOvA, and T2K.



Thank you for your attention!

# Backup

# 1. Neutrino oscillation - A natural interferometer

The neutrino weak eigenstate is described by neutrino Hamiltonian eigenstates,  $\nu_1$ ,  $\nu_2$ , and  $\nu_3$  and Hamiltonian mixing matrix elements.

$$|\nu_e\rangle = \sum_{i=1}^3 U_{ei} |\nu_i\rangle$$

The time evolution of neutrino weak eigenstate is written by Hamiltonian mixing matrix elements and eigenvalues of  $\nu_1$ ,  $\nu_2$ , and  $\nu_3$ .

$$|\nu_e(t)\rangle = \sum_{i=1}^3 U_{ei} e^{-i\lambda_i t} |\nu_i\rangle$$

Then the transition probability from weak eigenstate  $\nu_\mu$  to  $\nu_e$  is (assuming everything is real)

$$P_{\mu \rightarrow e}(t) = \left| \langle \nu_e(t) | \nu_\mu \rangle \right|^2 = -4 \sum_{i>j} (U_{\mu i} U_{\mu j} U_{ei} U_{ej}) \sin^2 \left( \frac{\Delta_{ij}}{2} L \right)$$

This formula is model **independent**

## 2. Tandem model

We used 2 independent methods to diagonalize our Hamiltonian. The solutions come from same Hamiltonian but different method should be same.



### (1) Analytical solution of cubic equation (Ferro-Cardano solution)

$$\lambda_1 = -2\sqrt{Q} \cos\left(\frac{\theta}{3}\right) - \frac{1}{3}A$$

$$\lambda_2 = -2\sqrt{Q} \cos\left(\frac{\theta + 2\pi}{3}\right) - \frac{1}{3}A$$

$$\lambda_3 = -2\sqrt{Q} \cos\left(\frac{\theta - 2\pi}{3}\right) - \frac{1}{3}A$$

$$A = -cE - m^2 / 2E$$

$$b = cm^2 / 2 - 3a^2$$

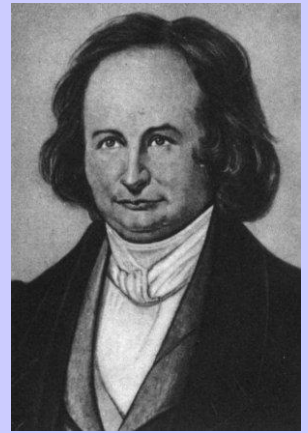
$$c = a^2 (cE \mp 2a + m^2 / 2E)$$

$$Q = (A^2 - 3b) / 9$$

$$R = (2A^3 - 9ab + 27c) / 54$$

$$\theta = \arccos(R / \sqrt{Q^3})$$

## 2. Tandem model



### (2) Numerical matrix diagonalization (Jacobi method)

$$\begin{aligned}
 \begin{pmatrix} h_{ee} & h_{e\mu} & h_{te} \\ h_{e\mu} & h_{\mu\mu} & h_{\mu\tau} \\ h_{te} & h_{\mu\tau} & h_{\tau\tau} \end{pmatrix} &= O_1^T \begin{pmatrix} h'_{ee} & 0 & h'_{te} \\ 0 & h'_{\mu\mu} & h'_{\mu\tau} \\ h'_{te} & h'_{\mu\tau} & h'_{\tau\tau} \end{pmatrix} O_1 = O_1^T O_2^T \begin{pmatrix} h''_{ee} & \delta & 0 \\ \delta & h''_{\mu\mu} & h''_{\mu\tau} \\ 0 & h'_{\mu\tau} & h''_{\tau\tau} \end{pmatrix} O_2 O_1 \dots \\
 &= \underbrace{\dots O^T O^T O^T}_{U^T} \begin{pmatrix} \sim \lambda_1 & \sim 0 & \sim 0 \\ \sim 0 & \sim \lambda_2 & \sim 0 \\ \sim 0 & \sim 0 & \sim \lambda_3 \end{pmatrix} \underbrace{OOO\dots}_U
 \end{aligned}$$

These 2 methods are independent algorithms, and important check for the diagonalization of the Hamiltonian.

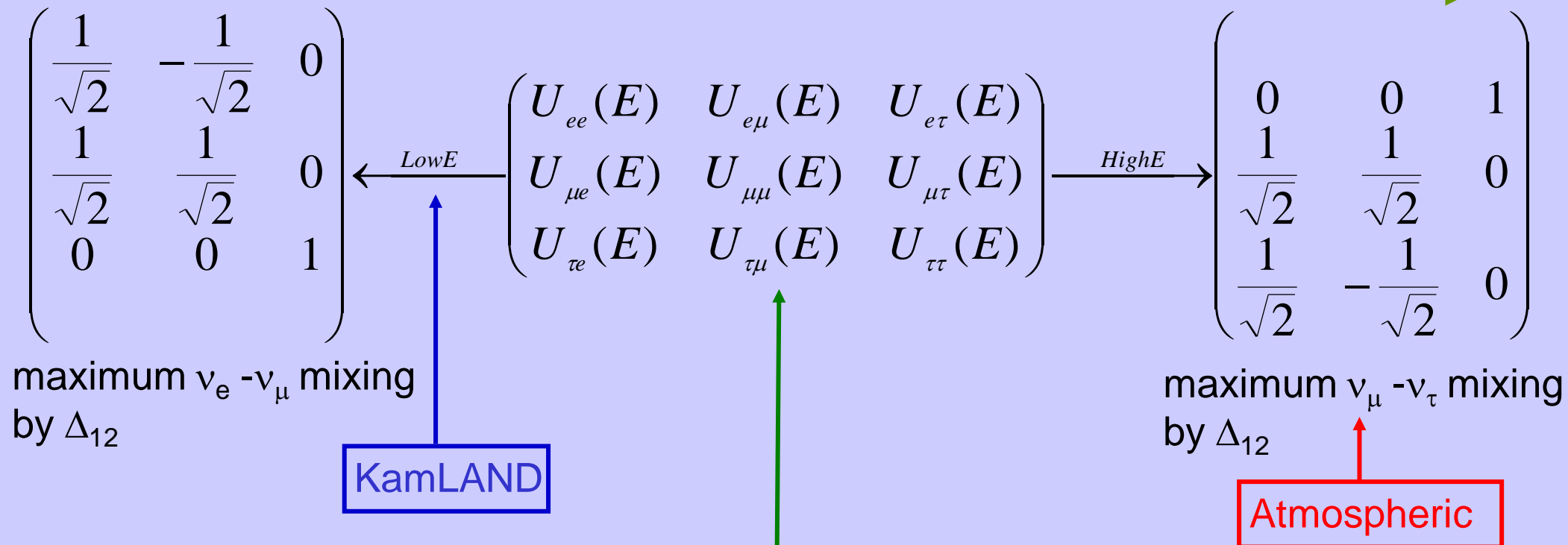
# 3. Tandem model

## Mixing angles of Hamiltonian

Mixing angles are energy dependent now, and the model creates maximum  $\nu_e - \nu_\mu$  mixing at low energy and maximum  $\nu_\mu - \nu_\tau$  mixing at high energy regions.

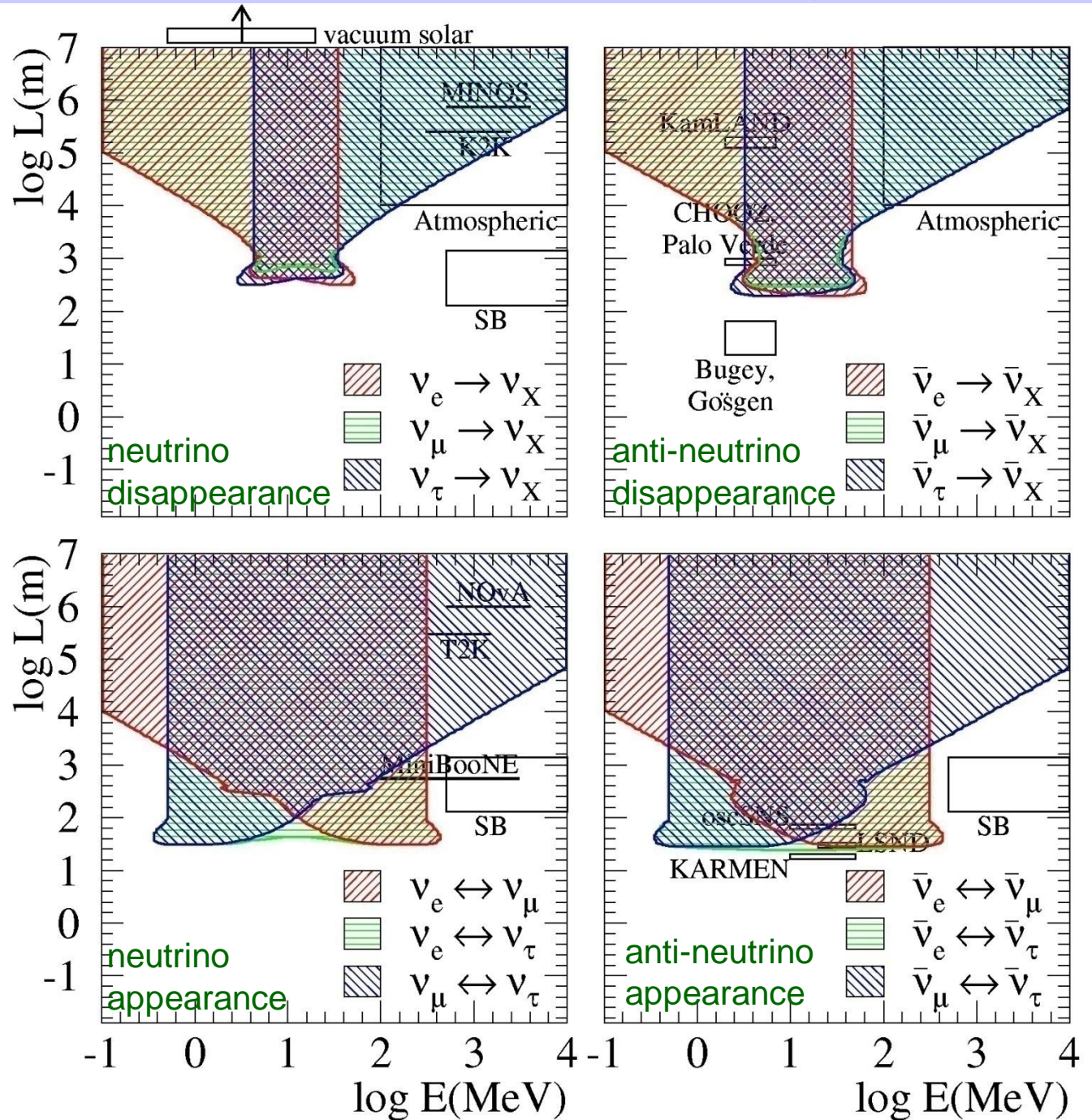
Low E

High E



Middle energy region is responsible for **LSND** and **solar**. But it has a complicated energy dependences, and only understandable by numerical calculation (see next).

# 4-5. Global signal predictions



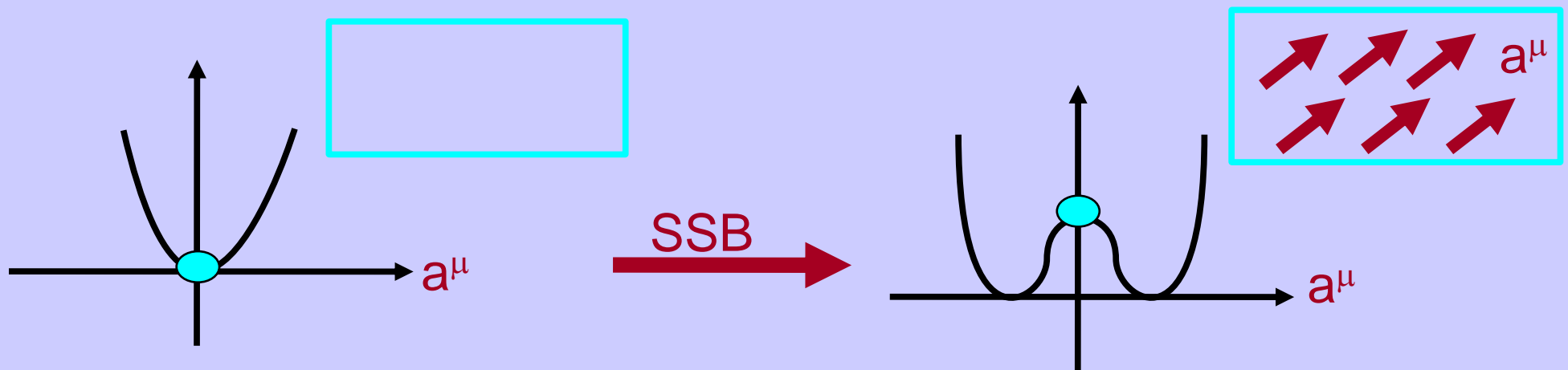
disappearance signals  
 $P > 10\%$

appearance signals  
 $P > 0.1\%$

# 1-1. Motivation

PRD,39(1989)683

In a field theory, spontaneous symmetry breaking (SSB) occurs when symmetries of the Lagrangian are not respected by the ground state of the theory



In string theory, there is a possibility that some Lorentz tensor fields acquire a nonzero vacuum expectation value  
(spontaneous Lorentz symmetry breaking)

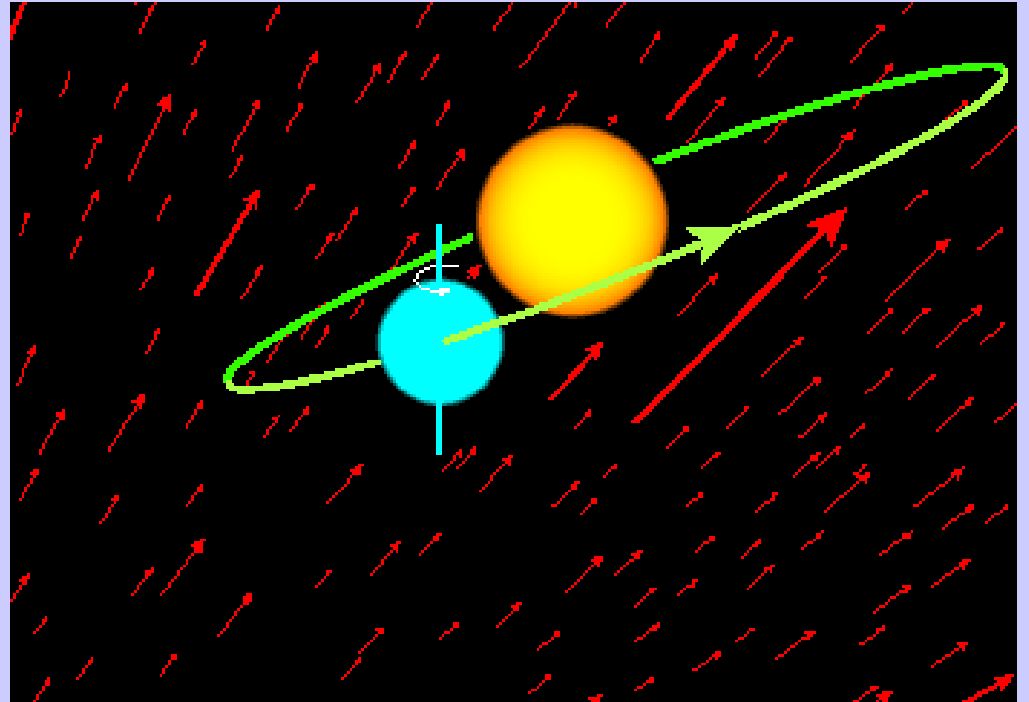


# 1-1. Motivation

Universe might be polarized!

The polarization of vacuum is the source of Particle Lorentz violation.

The signature of Lorentz violation is sidereal time variation of physics observables in a fixed coordinate system.



(Today's talk is nothing about sidereal variation).

# 1-4. Standard Model Extension (SME)

PRD,55(1997)6760;58(1998)116002

SME is the minimum extension of Standard Model to include Particle Lorentz and CPT violation. It has all possible type of Lorentz and CPT violating tensors in the Lagrangian.

## Modified Dirac Equation

$$i(\Gamma^\mu_{AB} \partial_\mu - M_{AB}) \psi_B = 0$$

## SME parameters

$$\Gamma^\mu_{AB} = \gamma^\mu \delta_{AB} + c_{AB}^{\mu\nu} \gamma_\nu + d_{AB}^{\mu\nu} \gamma_\nu \gamma_5 + e_{AB}^\mu + i f_{AB}^\mu \gamma_5 + \frac{1}{2} g_{AB}^{\mu\nu\lambda} \sigma_{\nu\lambda}$$

$$M_{AB} = m_{AB} + i m_{5AB} \gamma_5 + a_{AB}^\mu \gamma_\mu + b_{AB}^\mu \gamma_\mu \gamma_5 + \frac{1}{2} H_{AB}^{\mu\nu} \sigma_{\mu\nu}$$

# 1-4. Standard Model Extension (SME)

PRD,55(1997)6760;58(1998)116002

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$$M_{AB} = m_{AB} + im_{5AB} \gamma_5 + \boxed{a_{AB}^\mu} \gamma_\mu + \boxed{b_{AB}^\mu} \gamma_\mu \gamma_5 + \frac{1}{2} \boxed{H_{AB}^{\mu\nu}} \sigma_{\mu\nu}$$

Lorentz and CPT violating coefficients (CPT-odd)

Lorentz violating coefficients (CPT-even)

# 1-5. SME for neutrino oscillation

PRD,69(2004)016005;70(2004)076002

Effective Hamiltonian (minimal SME)

$$(a_L)_{ab}^\mu = (a+b)_{ab}^\mu \quad (c_L)_{ab}^{\mu\nu} = (c+d)_{ab}^{\mu\nu}$$

$$(h_{eff})_{ab} = |\vec{p}| \delta_{ab} + \frac{1}{2|\vec{p}|} (m^2)_{ab} + \frac{1}{|\vec{p}|} \left[ \boxed{(a_L)^\mu} p_\mu - \boxed{(c_L)^{\mu\nu}} p_\mu p_\nu \right]_{ab}$$

CPT-odd                      CPT-even

Rotation invariant form (no sidereal dependence)

$$(h_{eff})_{ab} \approx E \delta_{ab} + \frac{1}{2E} (m^2)_{ab} + [(a_L) - \frac{4}{3} (c_L) E]_{ab}$$

Real hermitian matrix expression (18 free parameters)

$$(h_{eff})_{ab} \rightarrow \begin{pmatrix} \check{c}_{ee} E + \check{a}_{ee} + \frac{\check{m}_{ee}^2}{2E} & \check{c}_{e\mu} E + \check{a}_{e\mu} + \frac{\check{m}_{e\mu}^2}{2E} & \check{c}_{e\tau} E + \check{a}_{e\tau} + \frac{\check{m}_{e\tau}^2}{2E} \\ \check{c}_{e\mu} E + \check{a}_{e\mu} + \frac{\check{m}_{e\mu}^2}{2E} & \check{c}_{\mu\mu} E + \check{a}_{\mu\mu} + \frac{\check{m}_{\mu\mu}^2}{2E} & \check{c}_{\mu\tau} E + \check{a}_{\mu\tau} + \frac{\check{m}_{\mu\tau}^2}{2E} \\ \check{c}_{e\tau} E + \check{a}_{e\tau} + \frac{\check{m}_{e\tau}^2}{2E} & \check{c}_{\mu\tau} E + \check{a}_{\mu\tau} + \frac{\check{m}_{\mu\tau}^2}{2E} & \check{c}_{\tau\tau} E + \check{a}_{\tau\tau} + \frac{\check{m}_{\tau\tau}^2}{2E} \end{pmatrix}$$

## 2-2. L-E plane

Criteria for alternative model may be;

- (1) based on quantum field theory
- (2) renormalisable
- (3) acceptable description of data
- (4) mass is  $< 0.1 \text{ eV}$  for seesaw compatibility
- (5) less than equal 4 parameters
- (6) Lorentz violation is  $< 10^{-17} \sim (M_Z / M_{\text{Planck}})$
- (7) has LSND signal

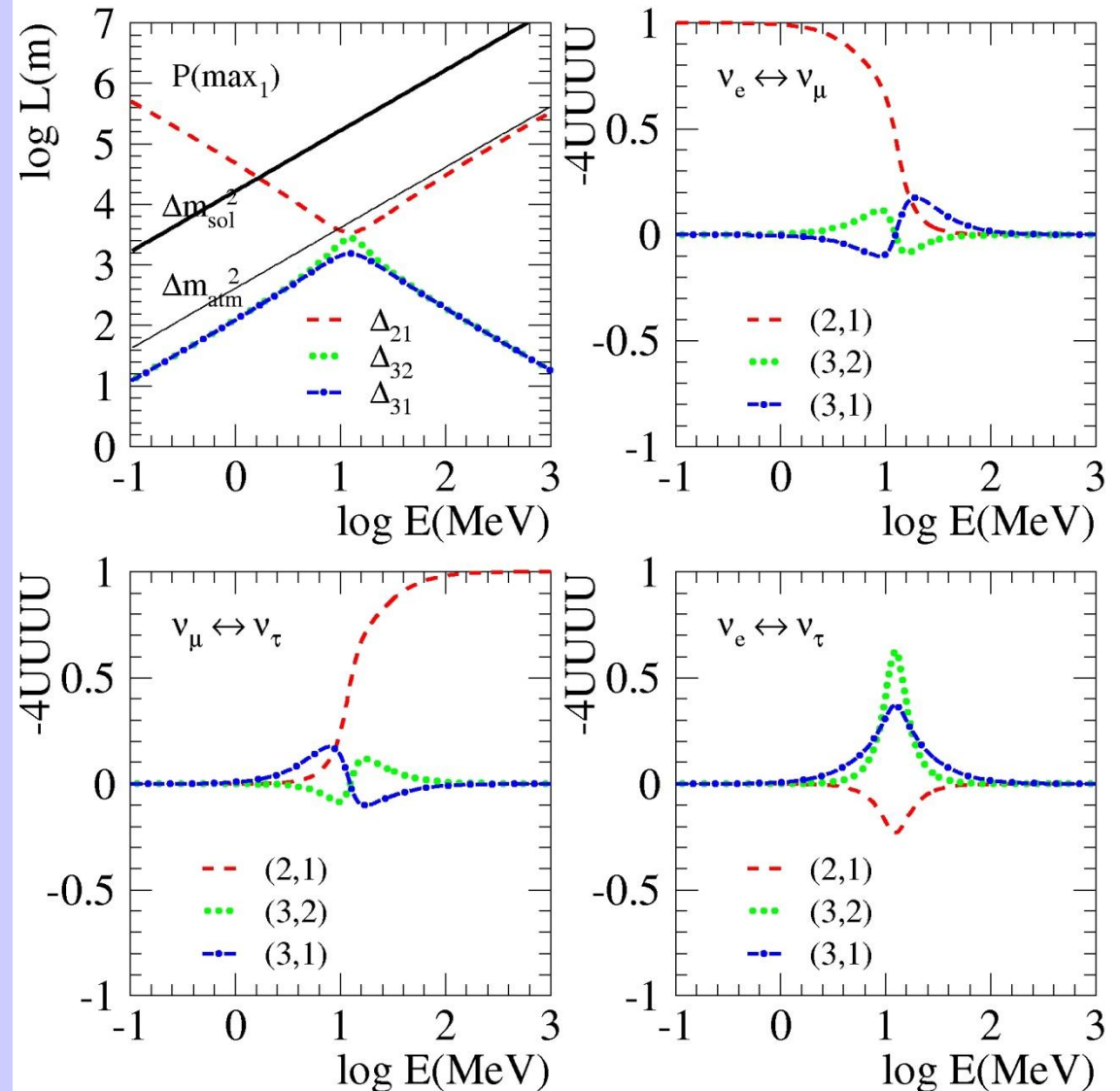
# 2-2. L-E plane

## Neutrino behavior in the tandem model

The solution of global 3-parameter model using Lorentz violation (tandem model).

In fact, energy dependence of the amplitudes are also important.

This model can reproduce all data with acceptable values.



# 2-2. L-E plane

## Anti-neutrino behavior in the tandem model

Solution is different for neutrino and anti-neutrino due to CPT-odd terms.

Although there is CPT violation, in tandem model, sometimes there is no difference between neutrinos and anti-neutrinos.

