Dynamically Solving the μ/B_μ Problem in Gauge Mediated SUSY Breaking

Tao Liu

Enrico Fermi Institute
University of Chicago

In collaboration with **Carlos Wagner**, arXiv: 0803.2895

Pheno08, 04/28/2007

I. Outlines

- μ/B_μ Problem in Gauge-mediated SUSY Breaking (GMSB)
- A Simple Model to Solve μ/B_μ Problem
- Some General Comments on Numerical Results
- Gauge Coupling Unification
- Conclusions

II. μ/B_μ Problem in GMSB

A. μ/B_μ Problem and EW Symmetry Breaking

- In the MSSM, we need a term

$$\Delta\mathcal{L} = \int d^2\theta \mu \mathbf{H_d H_u} + h.c.$$

  to give the Higgsinos a mass.
- To stabilize the EW scale, μ and B_μ parameters need to satisfy the minimization conditions of the Higgs potential

$$\begin{aligned} \mu^2 &= -\frac{M_Z^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}, \qquad \tan\beta = \frac{v_u}{v_d} \\ B_\mu &= (m_{H_d}^2 + m_{H_u}^2)\frac{\sin 2\beta}{2} \end{aligned}$$

- μ/B_μ problem: is there a dynamical way to induce the EW scale values for μ and B_μ? Particularly in the GMSB framework.

II. μ/B_μ Problem in GMSB

B. Light Singlet Mechanism

- Assume exact global symmetries (NMSSM, nMSSM) or gauge symmetry (UMSSM) to forbid μ term in MSSM
- Introduce a singlet chiral superfield **N** in the observable sector which has the coupling

$$\Delta\mathcal{L} = \int d^2\theta\lambda \mathbf{N}\mathbf{H_d}\mathbf{H_u} + h.c.$$

- The effective μ and B_μ parameters arise as

$$\mu = \lambda v_N, \qquad B_\mu = \lambda\langle F_N\rangle + A_\lambda\mu.$$

- As long as N and F_N are stabilized at the EW scale, we will have the correct relationship

$$\frac{B_\mu}{\mu} \sim \frac{\langle F_N\rangle}{v_N} \sim 10^2 - 10^3\mathrm{GeV}.$$

II. μ/B_μ Problem in GMSB

- Main difficulties: (1) how to generate a negative enough $m_N^2(\Lambda_{EW})$ term in the Higgs potential, to stabilize N to Λ_{EW}; (2) how to avoid the light $U(1)_R$ axion problem? (M. Dine et. al. '93)
- In the minimal GMSB

$$W = \lambda \mathbf{S}\bar{\mathbf{q}}\mathbf{q} + \gamma \mathbf{S}\bar{\mathbf{l}}\mathbf{l}$$

  where $(q + l, \bar{q} + \bar{l}) = (3 + 2, \bar{3} + \bar{2})$ are vector-like messengers, and $\mathbf{S} = S + \theta^2 F_S$ is the SUSY breaking spurion, one never gets a negative enough $m_N^2(\Lambda_{EW})$ term, independently of the messenger scale (M. Dine '93; H. Murayama et. al. '99)
- There are several kinds of modifications (in the mGMSB framework), but the results are not satisfactory: (M. Dine et. al. '93; G. Giudice et. al. '97,'07; T. Han et.al. '99; P. Langacker et. al. '99)

III. A Simple Model to Solve μ/B_μ Problem

A. General Gauge Mediation

- Minimal GMSB – ”minimal” is favored in the messenger sector due to the constraints of the QCD asymptotic freedom, but NOT generic in the hidden sector!
- Consider a more general GMSB structure

  $$W = \lambda_i \mathbf{S}_i \bar{\mathbf{q}}\mathbf{q} + \gamma_i \mathbf{S}_i \bar{\mathbf{l}}\mathbf{l},$$

  which can be redefined as

  $$W = \mathbf{S}_q \bar{\mathbf{q}}\mathbf{q} + \mathbf{S}_l \bar{\mathbf{l}}\mathbf{l},$$

  $$\mathbf{S}_q = \lambda_i \mathbf{S}_i, \quad \mathbf{S}_l = \gamma_i \mathbf{S}_i.$$

- Then we have TWO effective SUSY breaking scales

  $$\Lambda_q = \frac{\lambda_i \langle F_i \rangle}{\lambda_j \langle S_j \rangle}, \qquad \Lambda_l = \frac{\gamma_i \langle F_i \rangle}{\gamma_j \langle S_j \rangle}.$$

- This extra freedom degree can help implement the light singlet mechanism.

III. A Simple Model to Solve μ/B_μ Problem

B. Generating Large Negative $m_N^2(\Lambda_{EW})$

- The NMSSM has a superpotential for the Higgs superfields

$$\mathbf{W} = \lambda \mathbf{N}\mathbf{H_d}\mathbf{H_u} - \frac{1}{3}\kappa \mathbf{N}^3,$$

- The leading order soft masses at the messenger scale Λ_M are:

$$M_3 = \frac{\alpha_3}{4\pi}\Lambda_q \quad M_2 = \frac{\alpha_2}{4\pi}\Lambda_l \quad M_1 = \frac{\alpha_1}{4\pi}\left[\frac{2}{5}\Lambda_q + \frac{3}{5}\Lambda_l\right]$$

$$m_\phi^2 = 2\left[C_3^\phi\left(\frac{\alpha_3}{4\pi}\right)^2\Lambda_q^2 + C_2^\phi\left(\frac{\alpha_2}{4\pi}\right)^2\Lambda_l^2 + C_1^\phi\left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{2}{5}\Lambda_q^2 + \frac{3}{5}\Lambda_l^2\right)\right.$$

C_i^ϕs are quadratic Casimir operators of the scalar ϕ.

III. A Simple Model to Solve μ/B_μ Problem

- RGEs of the soft masses $m_{H_u}^2$ and m_N^2 ($\Lambda_{mess} \rightarrow \Lambda_{EW}$):

$$\begin{aligned} 16\pi^2 \frac{d}{dt} m_{H_u}^2 &= 6h_t^2(m_{\tilde{Q}_3}^2 + m_{H_u}^2 + m_{\tilde{t}}^2 + A_t^2) + 2\lambda^2(m_{H_d}^2 + m_{H_u}^2 \\ &\quad + m_N^2 + A_\lambda^2) - 8(\frac{1}{4}g_Y^2 M_1^2 + \frac{3}{4}g_2^2 M_2^2), \\ 16\pi^2 \frac{d}{dt} m_N^2 &= 4\lambda^2(m_{H_d}^2 + m_{H_u}^2 + m_N^2 + A_\lambda^2) + 4k^2(3m_N^2 + A_k^2). \end{aligned}$$

- In the minimal GMSB ($\Lambda_l = \Lambda_q$), $m_{H_u}^2$ gets negative quickly due to the top (s)quark contributions, but m_N^2 not. ⇒ too small $|m_N^2(\Lambda_{EW})|$.
- In the general GMSB, $\Lambda_l > \Lambda_q$ will lead to a relatively larger beta function for m_N^2, but a smaller one for $m_{H_u}^2$. Therefore, the evolution of m_N^2 to a negative value is accelerated, while that of $m_{H_u}^2$ is slowed down. ⇒ large negative $m_N^2(\Lambda_{EW})$

III. A Simple Model to Solve μ/B_μ Problem

C. No Light $U(1)_R$ Axion Problem

- The RGE of $m^2_{H_d}$ is given by

$$\frac{d}{dt}m^2_{H_d} = \frac{2\lambda^2}{16\pi^2}(m^2_{H_d} + m^2_{H_u} + m^2_N + A^2_\lambda) - \frac{2}{16\pi^2}(g^2_Y M^2_1 + 3g^2_2 M^2_2)$$

  Relatively large $\Lambda_I \Rightarrow$ relatively large $m^2_{H_d}(\Lambda_{EW})$

- The eigenvector of the light Higgs pseudoscalar is ($v_N \gg v$):

$$\begin{aligned} a_1 &= \cos\theta_A\, A_{MSSM} + \sin\theta_A\, A_N, \\ \tan\theta_A &= \frac{v_N}{v\sin 2\beta}, \\ \sin 2\beta &= \frac{2B_\mu}{m^2_{H_d}(\Lambda_{EW}) + m^2_{H_u}(\Lambda_{EW}) + 2\mu^2} \end{aligned}$$

- Large $m^2_{H_d}(\Lambda_{EW}) \Rightarrow$ small $\sin 2\beta$ or large $\tan\beta \Rightarrow \theta_A \approx \pi/2$. So the light Higgs pseudoscalar is singlet-like.

IV. Some General Comments on Numerical Results

- There are four unkown input parameters: the superpotential couplings $\lambda(\Lambda_{EW})$ and $\kappa(\Lambda_{EW})$, the messenger scale Λ_M and the ratio of the two effective SUSY breaking scales $\eta = \Lambda_l/\Lambda_q$
- For $\eta \sim 1$ and very large η, no solution; values larger than one, but of order one are preferred.
- A relatively large $\tan\beta$: $5 \sim 50$ is favored
- For large values of $\tan\beta$ and $v_N \gg v$ (particularly for low-scale scenarios), we have the tree-level mass for h_1

$$m_{h_1}^2 = M_Z^2 - \frac{\lambda^4}{\kappa^2} v^2 \quad \Rightarrow \text{Relatively heavy stops}$$

- Gaugino mass is correlated to the squark mass, and hence is heavy. Lightest neutralinos and charginos therefore are mostly admixtures of Higgsinos and singlinos.

V. Gauge Coupling Unification

- In the MSSM, the $SU(5)$ GUT predicts $\alpha_3(M_z) = 0.125$ at one-loop level, compared to the experimental value ~ 0.120. The threshold correction is given by

$$\Delta\alpha_3(M_Z)_{MSSM} \simeq -\frac{19}{28\pi}\alpha_3(M_Z)^2 \ln\left(\frac{|\mu|}{M_Z}\left(\frac{M_2}{M_3}\right)^{3/2}\right)$$

  For typical parameter values, this correction is not negative enough. (P. Langacker et.al.'93, C. Wagner et.al.'93)

- In our model,

$$\Delta\alpha_3(M_Z) \simeq \alpha_3(M_Z)^2\left(\frac{9}{14\pi}\ln\left(\frac{\langle S_q\rangle}{\langle S_l\rangle}\right) - \frac{19}{28\pi}\ln\left(\frac{|\mu|}{M_Z}\left(\frac{\eta\alpha_2}{\alpha_3}\right)^{3/2}\right.\right.$$

- For reasonable $\langle S_q\rangle/\langle S_l\rangle$,

$$\Delta\alpha_3(M_Z) - \Delta\alpha_3(M_Z)_{MSSM} \sim \mathcal{O}(0.001) < 0$$

  $\Rightarrow$ A better unification prediction than that at the MSSM limit.

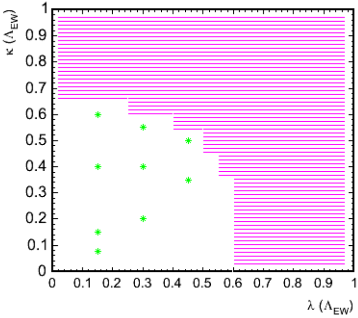
VI. Conclusions

- The μ/B_μ problem can be well-solved in the general gauge-mediation background (minimal messenger sector $+$ general hidden sector).
- Physical mass spectra of particles and superparticles are obtained in MOST of the perturbative parameter region for mildly $\Lambda_l > \Lambda_q$, INDEPENDENTLY of the messenger scale.
- These nice features can be extended to the non-perturbative parameter region, as long as they stay perturbative at the messenger scale.
- This gauge mediation structure is not directly related to the observable sector of the NMSSM, so its idea could be applied to many other contexts, e.g., nMSSM and UMSSM. Similar effects are expected to be seen.

Thank you!

The Strategy of Numerical Work (Backup Slides)

- Low- ($\sim 10^5$ GeV), intermediate- ($\sim 10^{11}$ GeV) and high-scale ($\sim 10^{15}$ GeV) gauge mediations are considered.
- For each case, nine characteristic points are studied, all of them specified by $\lambda(\Lambda_{EW})$ and $\kappa(\Lambda_{EW})$ and located in the perturbative region.



Backup Slides

Table: Parameters of the low-scale general gauge mediation.

	Input Parameters			
Pts	$\lambda(\Lambda_{EW})$	$\kappa(\Lambda_{EW})$	Λ_M (GeV)	η
A1	0.15	0.075	2.50×10^5	2.1160
A2	0.15	0.15	5.00×10^5	2.2708
A3	0.15	0.40	5.00×10^6	2.5151
A4	0.15	0.60	2.00×10^7	2.7869
A5	0.30	0.20	2.50×10^5	1.9356
A6	0.30	0.40	2.50×10^5	2.1383
A7	0.30	0.55	5.00×10^5	2.2800
A8	0.45	0.35	2.00×10^6	2.2509
A9	0.45	0.50	2.50×10^5	2.1083

	Output Parameters				
Pts	h_t, h_b	Λ_q (GeV)	$\tan\beta$	μ (GeV)	B_μ (GeV^2)
A1	0.949, 0.753	3.90×10^5	43.57	173.8	1.41×10^4
A2	0.948, 0.833	4.52×10^5	48.44	105.1	7.38×10^3
A3	0.948, 0.880	1.37×10^6	51.05	121.8	6.55×10^3
A4	0.948, 0.882	1.34×10^6	52.41	106.0	1.92×10^4
A5	0.949, 0.637	5.46×10^5	36.93	321.8	7.11×10^4
A6	0.948, 0.780	3.95×10^5	45.30	124.2	1.88×10^4
A7	0.948, 0.809	4.38×10^5	46.89	109.9	1.94×10^4
A8	0.950, 0.307	8.68×10^5	17.80	1276.6	1.54×10^6
A9	0.949, 0.533	2.50×10^5	30.87	296.7	1.11×10^5

Backup Slides

Table: Mass spectrum of particles and superparticles in the low-scale general gauge mediation.

	Particle Masses (TeV)			
Pts	$m_{\tilde{g}}$	$m_{\tilde{t}_{1,2}}$	$m_{\tilde{b}_{1,2}}$	$m_{\tilde{\tau}_{1,2}}$
A1	3.44	5.55, 6.36	5.86, 6.35	0.80, 2.84
A2	3.95	6.37, 7.36	6.60, 7.35	0.89, 3.53
A3	11.17	18.63, 22.24	19.08, 22.24	2.27, 11.90
A4	10.98	17.78, 22.18	18.26, 22.18	1.67, 12.98
A5	4.76	7.89, 8.98	8.54, 8.97	1.14, 3.69
A6	3.48	5.62, 6.42	5.89, 6.42	0.80, 2.90
A7	3.83	6.16, 7.14	6.43, 7.14	0.88, 3.43
A8	7.30	12.01, 14.72	14.15, 14.72	2.33, 6.85
A9	3.49	5.63, 6.57	6.23, 6.57	0.92, 2.88

	Particle Masses (GeV)			
Pts	$m_{\chi_1^c}$	$m_{\chi_1^0}$	$m_{h_{1,2,3}}$	$m_{a_{1,2}}$
A1	173.4	155.8	118.3, 187.3, 1751.6	15.7, 1751.6
A2	105.0	103.7	136.6, 211.1, 1616.8	20.3, 1616.8
A3	121.7	121.7	152.6, 644.3, 3544.8	71.3, 3544.8
A4	106.0	105.8	152.4, 843.6, 3564.4	98.0, 3564.3
A5	321.4	311.1	117.4, 433.3, 2825.2	53.8, 2825.1
A6	123.9	119.8	133.1, 331.2, 1656.8	43.3, 1656.7
A7	109.7	107.1	137.5, 401.8, 1754.8	54.3, 1754.6
A8	1276.2	1272.4	116.2, 1973.2, 6596.7	337.4, 6596.6
A9	296.1	289.8	121.9, 659.6, 2430.1	96.1, 2429.9

Backup Slides

Table: Parameters of the intermediate-scale general gauge mediation.

	Input Parameters			
Pts	$\lambda(\Lambda_{EW})$	$\kappa(\Lambda_{EW})$	Λ_M (GeV)	η
B1	0.15	0.075	1.00×10^{11}	4.180
B2	0.15	0.15	1.00×10^{11}	4.512
B3	0.15	0.40	1.00×10^{11}	4.292
B4	0.15	0.60	1.00×10^{11}	4.126
B5	0.30	0.20	1.00×10^{11}	3.981
B6	0.30	0.40	1.00×10^{11}	4.360
B7	0.30	0.55	1.00×10^{11}	4.620
B8	0.45	0.35	1.00×10^{11}	4.019
B9	0.45	0.50	1.00×10^{11}	4.542

	Output Parameters				
Pts	h_t, h_b	Λ_q (GeV)	$\tan\beta$	μ (GeV)	B_μ (GeV^2)
B1	0.950, 0.331	1.98×10^5	19.11	541.4	3.51×10^5
B2	0.949, 0.550	1.05×10^5	31.88	150.3	4.91×10^4
B3	0.949, 0.780	2.91×10^5	45.17	126.0	6.92×10^4
B4	0.948, 0.832	5.47×10^5	48.15	126.2	9.22×10^4
B5	0.953, 0.183	3.59×10^5	10.57	1406.6	2.22×10^6
B6	0.949, 0.340	1.86×10^5	19.62	384.5	3.33×10^5
B7	0.949, 0.465	1.10×10^5	26.97	163.5	8.23×10^4
B8	0.957, 0.125	4.39×10^5	7.16	2188.5	5.30×10^6
B9	0.953, 0.173	1.76×10^5	10.03	673.5	7.40×10^5

Backup Slides

Table: Mass spectrum of particles and superparticles in the intermediate-scale general gauge mediation.

	Particle Masses (TeV)			
Pts	$m_{\tilde{g}}$	$m_{\tilde{t}_{1,2}}$	$m_{\tilde{b}_{1,2}}$	$m_{\tilde{\tau}_{1,2}}$
B1	1.82	1.98, 4.03	3.23, 4.02	0.90, 3.07
B2	1.00	0.98, 2.16	1.56, 2.16	0.32, 1.74
B3	2.61	2.84, 5.58	3.59, 5.58	1.00, 4.49
B4	4.72	5.52, 10.16	6.42, 10.16	2.11, 8.07
B5	3.19	3.69, 7.21	6.04, 7.21	1.75, 5.34
B6	1.72	1.79, 3.86	3.02, 3.85	0.87, 3.01
B7	1.05	1.00, 2.33	1.71, 2.32	0.45, 1.88
B8	3.86	4.42, 8.87	7.44, 8.87	2.20, 6.60
B9	1.63	1.60, 3.76	2.96, 3.75	0.97, 2.98

	Particle Masses (GeV)			
Pts	$m_{\chi_1^c}$	$m_{\chi_1^0}$	$m_{h_{1,2,3}}$	$m_{a_{1,2}}$
B1	540.3	520.5	121.4, 536.5, 2931.9	97.1, 2931.9
B2	149.4	144.7	121.1, 297.6, 1416.9	54.0, 1416.9
B3	125.9	124.4	135.3, 663.8, 2367.7	115.8, 2367.6
B4	126.1	125.4	142.2, 997.6, 3227.7	172.5, 3227.6
B5	1405.5	1342.6	120.6, 1843.4, 5383.5	473.7, 5383.4
B6	383.7	380.6	126.1, 1009.0, 2803.2	192.7, 2136.1
B7	162.6	159.3	122.0, 590.0, 1623.6	150.5, 1623.3
B8	2187.3	1676.5	117.7, 3331.0, 6768.7	1045.5, 6768.5
B9	671.8	658.9	118.6, 1464.2, 2911.0	457.8, 2910.5

Backup Slides

Table: Parameters of the high-scale general gauge mediation.

	Input Parameters			
Pts	$\lambda(\Lambda_{EW})$	$\kappa(\Lambda_{EW})$	Λ_M (GeV)	η
C1	0.15	0.075	1.00×10^{15}	4.695
C2	0.15	0.15	1.00×10^{15}	4.980
C3	0.15	0.40	1.00×10^{15}	5.060
C4	0.15	0.60	1.00×10^{15}	4.930
C5	0.30	0.20	1.00×10^{15}	4.639
C6	0.30	0.40	1.00×10^{15}	5.110
C7	0.30	0.55	1.00×10^{15}	5.240
C8	0.45	0.35	1.00×10^{15}	4.755
C9	0.45	0.50	1.00×10^{15}	5.560

	Output Parameters				
Pts	h_t, h_b	Λ_q (GeV)	$\tan\beta$	μ (GeV)	B_μ (GeV^2)
C1	0.951, 0.220	1.98×10^5	12.63	792.6	8.99×10^5
C2	0.949, 0.391	1.37×10^5	22.64	285.4	2.23×10^5
C3	0.948, 0.702	1.79×10^5	40.79	122.3	8.75×10^4
C4	0.948, 0.794	3.23×10^5	46.05	112.8	1.03×10^5
C5	0.958, 0.124	2.56×10^5	7.10	1524.2	2.87×10^6
C6	0.951, 0.233	1.58×10^5	13.47	492.5	6.20×10^5
C7	0.949, 0.342	1.47×10^5	19.86	306.8	3.38×10^5
C8	0.967, 0.087	4.49×10^5	4.99	3406.1	1.35×10^7
C9	0.958, 0.123	1.54×10^5	7.09	891.2	1.39×10^6

Backup Slides

Table: Mass spectrum of particles and superparticles in the high-scale general gauge mediation.

	Particle Masses (TeV)			
Pts	$m_{\tilde{g}}$	$m_{\tilde{t}_{1,2}}$	$m_{\tilde{b}_{1,2}}$	$m_{\tilde{\tau}_{1,2}}$
C1	1.80	1.17, 4.37	3.33, 4.37	1.18, 3.68
C2	1.27	0.61, 3.07	2.15, 3.06	0.68, 2.67
C3	1.63	0.68, 3.82	2.08, 3.81	0.89, 3.43
C4	2.84	1.54, 6.61	3.14, 6.61	2.08, 5.95
C5	2.29	1.48, 5.62	4.37, 5.61	1.59, 4.71
C6	1.46	0.52, 3.65	2.64, 3.65	1.00, 3.19
C7	1.36	0.31, 3.41	2.36, 3.40	0.84, 3.02
C8	3.90	2.07, 9.98	7.71, 9.98	2.89, 8.48
C9	1.43	0.71, 3.76	2.64, 3.76	1.14, 3.40

	Particle Masses (GeV)			
Pts	$m_{\chi_1^c}$	$m_{\chi_1^0}$	$m_{h_{1,2,3}}$	$m_{a_{1,2}}$
C1	791.3	768.6	120.7, 777.4, 3665.7	194.8, 3665.7
C2	284.6	280.6	122.2, 559.6, 2443.0	139.6, 2442.9
C3	122.1	119.9	126.8, 639.4, 2207.5	155.3, 2207.4
C4	112.7	111.6	134.2, 878.8, 2792.8	211.2, 2792.7
C5	1522.3	1101.0	116.6, 1972.0, 4872.1	698.9, 4871.9
C6	491.3	486.7	121.2, 1274.9, 3068.7	446.3, 3068.4
C7	306.0	303.1	120.4, 1086.4, 2765.9	376.0, 2765.6
C8	3404.4	1981.1	120.1, 5084.2, 8909.4	2193.7, 8909.2
C9	889.0	771.4	117.2, 1884.6, 3306.1	806.7, 3305.3

Backup Slides

Table: Composition of the light Higgs bosons ($\leq$ 115GeV).

Composition of Light Higgs Bosons (LHB)				
Pts	LHBs	$\mathrm{Im}(H_d)$	$\mathrm{Im}(H_u)$	$\mathrm{Im}(N)$
A1	a_1	-1.2×10^{-3}	-8.8×10^{-4}	0.999999
A2	a_1	-2.0×10^{-3}	-1.1×10^{-5}	0.999998
A3	a_1	-2.2×10^{-3}	9.4×10^{-4}	0.999997
A4	a_1	-2.1×10^{-3}	-7.5×10^{-5}	0.999998
A5	a_1	-2.0×10^{-4}	3.3×10^{-5}	$>$ 0.9999995
A6	a_1	1.3×10^{-4}	-1.7×10^{-6}	$>$ 0.9999995
A7	a_1	1.1×10^{-4}	6.1×10^{-5}	$>$ 0.9999995
A9	a_1	4.6×10^{-3}	1.2×10^{-4}	0.999989
B1	a_1	-7.0×10^{-5}	-1.0×10^{-5}	$>$ 0.9999995
B2	a_1	3.8×10^{-4}	1.7×10^{-5}	$>$ 0.9999995