Eikonal contributions to ultrahigh energy neutrino-nucleon cross sections in LSG models

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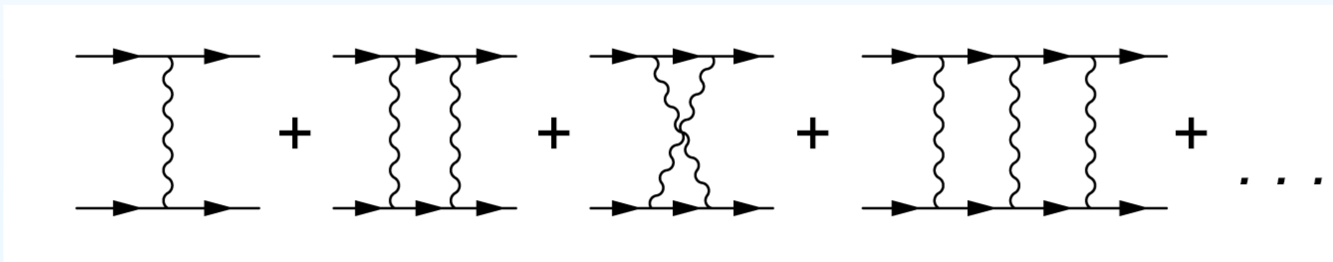
Motivation

- There are models that predict the existence of ultra-high energy neutrinos (up to 10^{20} eV).
- The design of neutrino telescopes depends critically on the estimates of neutrino cross sections in the UHE regime.
- The systematic study of the scattering of such high energy neutrinos with baryonic matter might allow us to test new physics effects.
- Calculations of the $\nu - N$ cross section exist, based on QCD (for example Gandhi, Quigg, Reno and Sarcevic 1998) and new physics effects (for example Jain *et al.* 2002; Hussain and McKay 2004).

Eikonal approximation

It's a technique for estimating the high energy behavior of a forward scattering amplitude.
In QFT it is obtained by summing the ladder diagrams for boson exchange at all orders.



- $s \gg t$.
- The deflecting angle is small $p_1 \sim p_1'$, $p_2 \sim p_2'$.
- The approximation doesn't discriminate the spin of the incoming particles which can be considered as scalar fields.

Eikonal approximation

The tree-level amplitude A_{Born} is:

$$A_{Born}(t) = g^2 s^r \frac{1}{t - m^2},$$

$r = 1, 2$ rank of boson propagator.
Define:

$$\chi(b_\perp) = \frac{1}{2s} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} A_{Born}(q_\perp'^2)$$

And obtain:

$$\begin{aligned} A_{eik} &= -2si \int d^2 b_\perp e^{iq_\perp \cdot b_\perp} \left(\sum_1^\infty (i\chi)^n / n! \right) \\ &= -2si \int d^2 b_\perp e^{iq_\perp \cdot b_\perp} (e^{i\chi} - 1). \end{aligned}$$

Born term divergence in the ADD model

Even at tree-level A_{Born} has ultraviolet divergence for $n \geq 2$

$$A_{Born} = \frac{s^2}{M_D^{n+2}} \int \frac{d^n m}{t - m^2}$$

.

Solution can be a cut-off of the order of M_D
(Giudice, Rattazzi and Wells 1998, 2001. Han, Lykken and Zhang 1999).
This is the upper limit of the effective theory in the hypothesis that the tension of the brane M_S is infinite .

It can be the finite tension of the brane M_S, if the standard model fields extend by a length $\sim 1/M_s$ in the bulk

(Bando, Kugo, Noguchi and Yoshioka 1999).

Infinite tension case

After dimensional regularization of the Born term, one obtains for the eikonal phase (Giudice, Rattazzi and Wells 2001):

$$\chi = \left[\frac{(4\pi)^{\frac{n}{2}-1} s\Gamma(\frac{n}{2})}{2M_D^{n+2}}\right]\frac{1}{b^n} \equiv \left(\frac{b_c}{b}\right)^n .$$

Eikonal amplitude:

$$A_{eik} = 4\pi s b_c^2 F_n(b_c q),$$

$$F_n(\eta) = -i\int_0^\infty d\xi \xi J_0(\xi\eta)(e^{i\xi^{-n}} - 1).$$

Cross section:

$$\frac{d^2\sigma}{dxdy} = \sum_{i=\text{flavors}} x f_i(x, xys) s\pi b_c^4(\hat{s}) |F_n(b_c(\hat{s})\sqrt{xys})|^2$$

Case of a finite tension M_S

If the standard model wave function has a gaussian extra-dimensional extension, one can correct the vertex by $e^{-\frac{m^2}{2M_S^2}}$ (Lonnblad and Sjodahl 2006). One gets:

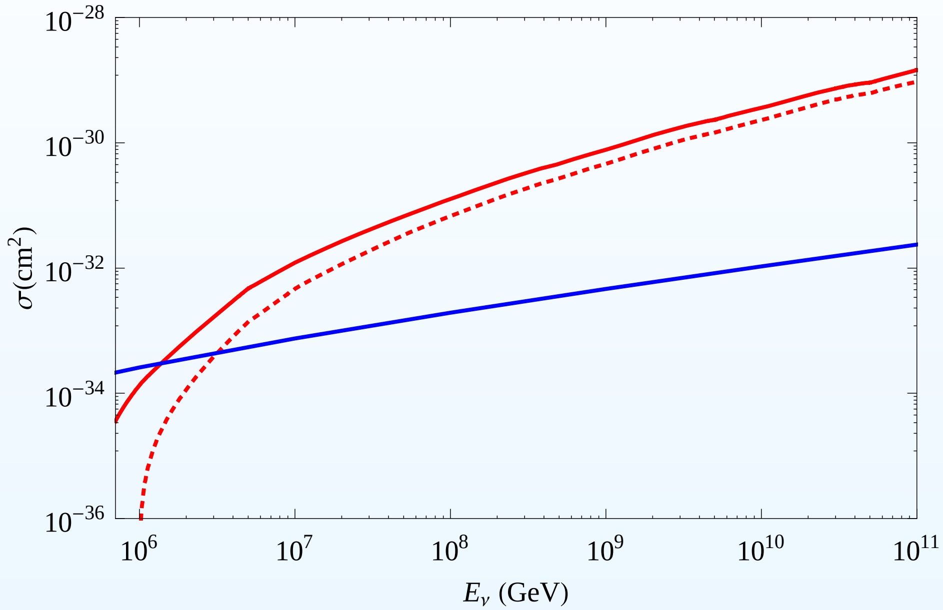
$$A_{Born} = \frac{s^2}{M_D^{n+2}} S_n \int_0^\infty \frac{m^{n-1} dm}{t - m^2} e^{-\frac{m^2}{M_s^2}}.$$

$$A_{eik} = -4\pi i s \int_0^\infty dbbJ_0(qb) \left[e^{i\left(\frac{b_c M_S}{2}\right)^n U\left(\frac{n}{2}, 1, \frac{M_S^2 b^2}{4}\right)} - 1 \right].$$

$$\frac{d^2\sigma}{dxdy} = \sum_{i=\text{flavors}} x f_i(x, xys) s\pi |G_n(\sqrt{xys})|^2, \text{ with}$$

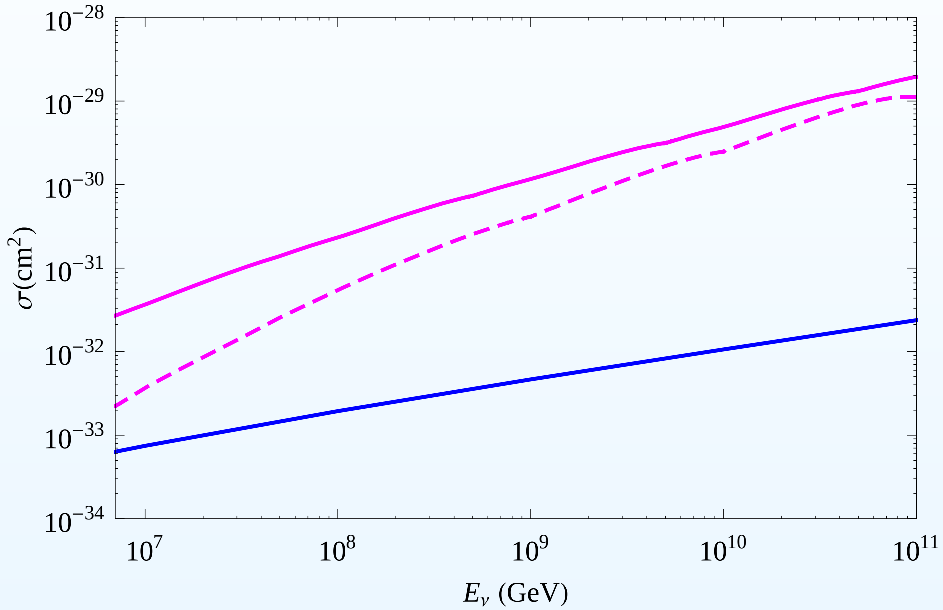
$$G_n(z) = -i \int_0^\infty dbbJ_0(bz) \left[e^{i\left(\frac{b_c M_S}{2}\right)^n U\left(\frac{n}{2}, 1, \frac{M_S^2 b^2}{4}\right)} - 1 \right].$$

Cross Sections



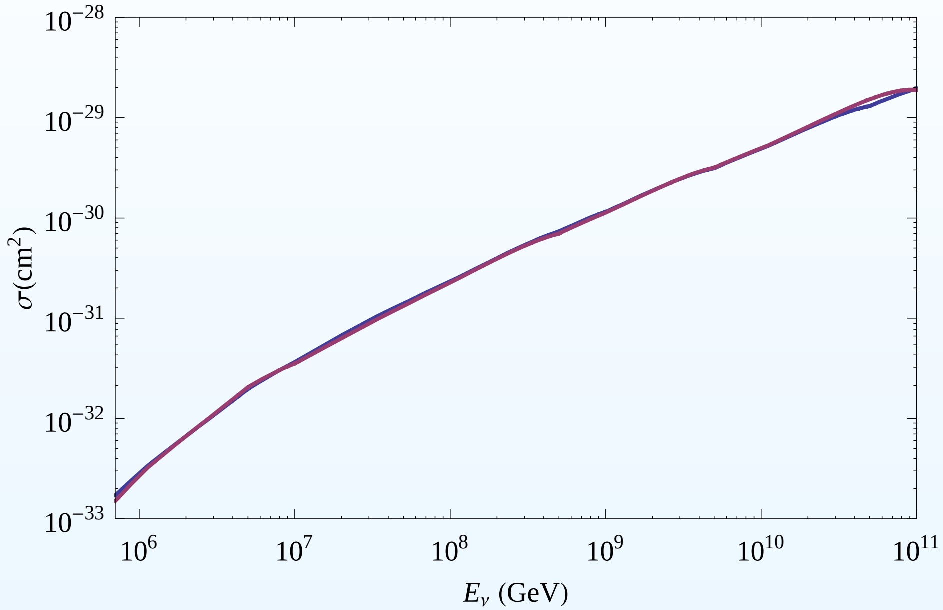
- SOLID RED: Full eikonal corrected by the Born term, $M_D = 10^3$ GeV, $M_S = \infty$, $n = 5$
- DASHED RED: Saddle point approximation corrected by the Born term.
- BLUE: Standard model neutral current

High energy pure eikonal/ Branes with different tension



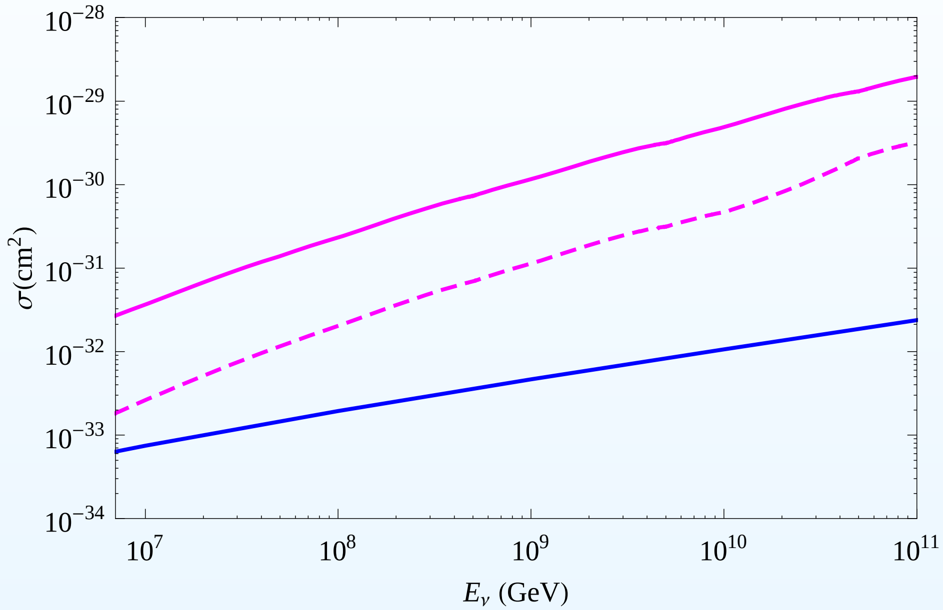
- SOLID MAGENTA: $M_D = 10^3$ GeV, $M_S = \infty$, $n = 5$
- DASHED MAGENTA: $M_D = 10^3$ GeV, $M_S = 10^3$ GeV, $n = 5$
- BLUE: Standard model neutral current

Pure eikonal LSG 1 TeV/ Branes with different tension



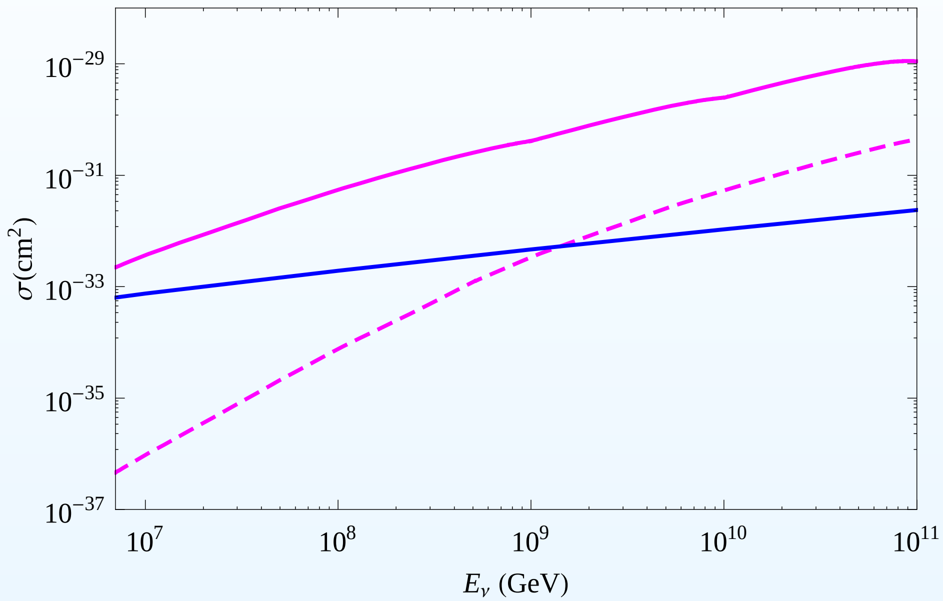
Comparison at $M_D = 10^3$ GeV between infinite tension and finite $M_S = 10^4$ GeV tension. $n = 5$.

High energy pure eikonal for different LSG



- SOLID MAGENTA: $M_D = 10^3$ GeV, $M_S = \infty$, $n = 5$
- DASHED MAGENTA: $M_D = 2 \times 10^3$ GeV, $M_S = \infty$, $n = 5$
- BLUE: Standard model neutral current

High energy pure eikonal for different LSG



- SOLID MAGENTA: $M_D = 10^3$ GeV, $M_S = 10^3$ GeV
- DASHED MAGENTA: $M_D = 2 \times 10^3$ GeV, $M_S = 10^3$ GeV
- BLUE: Standard model neutral current

Discussion

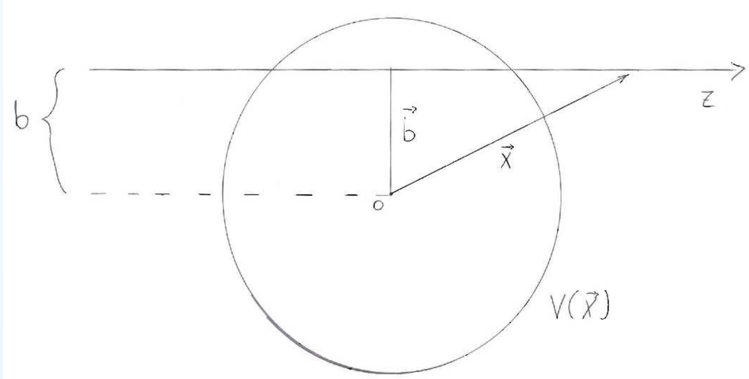
- For the infinite tension case the full eikonal is larger than the saddle point by a factor 2.6 - 1.5
- Three order of magnitude enhancement of the standard model at $E_\nu = 10^{11}$ GeV
- A "softer" brane suppresses the eikonal cross section by a factor 10 - 2
- Almost perfect overlapping at $M_S = 10^4$ GeV
- For the thin brane, doubling the LSG suppresses the cross sections by one order of magnitude
- For the thick brane, doubling the LSG lowers the cross section below the standard model at $E_\nu \leq 10^9$ GeV

Conclusions

- The brane tension M_S greatly affects the size of LSG influence on UHE neutrino scattering
- Calculations performed with brane tension of 10 TeV and above are indistinguishable
- The saddle point needs $qb_c \gg 1$.
  Even with the maximal q consistent with the eikonal the saddle point conditions are only marginally satisfied at GZK energies
- Eikonal collisions highly elastic $\Rightarrow$ consequences for event rate estimates not severe since deposited energy small and corresponding events mixed with highly inelastic processes (black-hole) at energies an order of magnitude lower

The EA in non-relativistic QM

In non-relativistic QM the interaction is described by the potential $V(\vec{x})$. We assume $E \gg V(\vec{x})$, so that the particle moves in a straight line.



The wave function is given by:

The EA in non-relativistic QM

$$\psi(\vec{x}) \sim e^{iS(\vec{x})}$$

The Hamilton-Jacobi equation is (if $E \gg V$):

$$\frac{(\nabla S)^2}{2m} + V = E = \frac{p^2}{2m}$$

Solve along straight path z:

$$S \sim pz - \frac{m}{p} \int_{-\infty}^{z} V(\sqrt{b^2 + z'^2}) dz'$$

So that:

$$\psi(\vec{b} + z\hat{z}) \sim \frac{1}{(2\pi)^{3/2}} e^{ipz} e^{\frac{-im}{p} \int_{-\infty}^{z} V(\sqrt{b^2+z'^2}) dz'}$$

The EA in non-relativistic QM

Remember the Scattering Amplitude:

$$f(\vec{p}, \vec{p'}) = -(2\pi)^2 m < \vec{p'} \mid V \mid \vec{p} >$$

With the previous approx it becomes:

$$f(\vec{p}, \vec{p'}) = -\frac{m}{2\pi} \int d^3x' e^{-i\vec{p'}\cdot\vec{x'}} V(\sqrt{b^2 + z'^2}) e^{i\vec{p}\cdot\vec{x'}} \times e^{\frac{-im}{p} \int_{-\infty}^{z'} V(\sqrt{b^2+z''^2}) dz''}$$

This is valid for $E \to \infty$ with $\vec{q} = \vec{p'} - \vec{p}$ kept finite, so that $q_3 \sim 0$.

The EA in non-relativistic QM

We can carry out the integration in z to obtain:

$$
\begin{aligned}
f(\vec{q}) &= -\frac{m}{2\pi}\int d^2b e^{-i\vec{q}\cdot\vec{b}}\int dz V(z) e^{\frac{-im}{p}\int_{-\infty}^{z} V(z')dz'} \\
&= -\frac{ip}{2\pi}\int d^2b e^{-i\vec{q}\cdot\vec{b}}[e^{i\chi(b)} - 1],
\end{aligned}
$$

where

$$
\chi(b) = -\frac{m}{p}\int_{-\infty}^{\infty} V(\sqrt{b^2+z^2})dz
$$

is the eikonal phase.