Realization of Local AdS_5 Isometry

Lu-Xin Liu
Department of Physics
Purdue University
West Lafayette, IN 47907, USA
liul@physics.purdue.edu

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AdS_5 Space

- Consider a flat, six dimensional space M_6 with invariant interval

$$ds^2 = \eta_{MN} dX^M dX^N$$

- with metric tensor $\eta_{MN} = (1,-1,-1,-1,-1,1)$. The Anti-de Sitter AdS_5 space is defined as the Hyperboloid satisfying the equation

$$\frac{1}{a^2} = (X^0)^2 - (X^1)^2 - (X^2)^2 - (X^3)^2 - (X^4)^2 + (X^5)^2$$

- The isometry group of the hyperboloid is SO(2,4) group with the generators

$$[M_{MN}, M_{LR}] = i(\eta_{ML} M_{NR} - \eta_{MR} M_{ML} - \eta_{NL} M_{MR} + \eta_{NR} M_{ML})$$

The Minkowski Brane

- The AdS_5 coordinates are parameterized as

$$X^{\mu} = e^{-ax^4} x^{\mu}$$

$$X^{4} = \frac{1}{a}[-\sinh(ax_4) + \frac{a^2 x^2}{2} e^{-ax^4}]$$

$$X^{5} = \frac{1}{a}[\cosh(ax^4) - \frac{a^2 x^2}{2} e^{-ax^4}]$$

and the invariant interval takes the form

$$ds^2 = e^{-2ax^4} \eta_{\mu\nu} dx^{\mu} dx^{\nu} - (dx^4)^2$$

- Placing the Minkowski brane at $x^4 = 0$, then

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$$

with ISO(1,3) symmetry.

$\hat{M}_{\mu\nu} = M_{\mu\nu}$

The Ads Basis

- In the Ads basis we introduce the following generators from the 1+3 dimensional point of view

$$\hat{P}_{\mu} = a(M_{4\mu} + M_{5\mu})\sqrt{2}$$
$$\hat{D} = aM_{54} \quad \hat{M}_{\mu\nu} = M_{\mu\nu}$$
$$\hat{K}_{\mu} = M_{\mu 4}\sqrt{2}$$

- The algebra reads:

$$[\hat{M}_{\mu\nu}, \hat{M}_{\rho\sigma}] = i(\eta_{\mu\sigma}\hat{M}_{\nu\rho} + \eta_{\nu\rho}\hat{M}_{\mu\sigma} - \eta_{\mu\rho}\hat{M}_{\nu\sigma} - \eta_{\nu\sigma}\hat{M}_{\mu\rho})$$
$$[\hat{P}_{\mu}, \hat{M}_{\kappa\lambda}] = i(\eta_{\mu\kappa}\hat{P}_{\lambda} - \eta_{\mu\lambda}\hat{P}_{\kappa})$$
$$[\hat{K}_{\mu}, \hat{M}_{\kappa\lambda}] = i(\eta_{\mu\kappa}\hat{K}_{\lambda} - \eta_{\mu\lambda}\hat{K}_{\kappa})$$
$$[\hat{D}, \hat{P}_{\mu}] = -ia\hat{P}_{\mu} \quad [\hat{D}, \hat{K}_{\mu}] = i(\hat{P}_{\mu} + a\hat{K}_{\mu}) \quad [\hat{M}_{\mu\nu}, \hat{D}] = 0$$
$$[\hat{P}_{\mu}, \hat{K}_{\nu}] = 2i(\eta_{\mu\nu}\hat{D} - a\hat{M}_{\mu\nu}) \quad [\hat{K}_{\mu}, \hat{K}_{\nu}] = 2i\hat{M}_{\mu\nu}$$

in which $\eta^{\mu\nu} = (+,-,-,-)$.

The Symmetry Breaking

The Minkowski space probe brane breaks the target SO(2,4) isometry to $ISO(1,3)$ symmetry:

$$SO(2,4) \rightarrow ISO(1,3)$$
$$AdS_5 \rightarrow M_4$$

- Choose the stability subgroup $H = \{\hat{M}_{\mu\nu}\}$, and then the coset space is parameterized as

$$\Omega = G/H$$
$$= e^{ix^\mu \hat{P}_\mu} e^{i\phi \hat{D}} e^{i\Lambda^\mu \hat{K}_\mu}$$

Global Nonlinear Transformations

Under the left multiplication of the full group elements, the coset transforms as

$$g\Omega = \Omega' h$$

- The Cartan one-forms becomes

$$\Omega^{-1} d\Omega \rightarrow \Omega'^{-1} d\Omega' = (h\Omega^{-1} g^{-1}) d(g\Omega h^{-1})$$
$$= h(\Omega^{-1} d\Omega) h^{-1} + h dh^{-1}$$

- Expanding the Cartan forms

$$\Omega d\Omega^{-1} = i d\omega^{\mu} \hat{P}_{\mu} + i d\omega_{D} \hat{D} + i d\omega_{k}^{\mu} \hat{K}_{\mu} + i d\omega^{\mu\nu} \hat{M}_{\mu\nu}$$

- It transforms as

$$\omega'(x') = h\omega(x) h^{-1} + h dh^{-1}$$

- The vielbein is given by

$$d\omega^{\mu} = dx^{\nu} e_{\nu}^{\mu}$$

- Then we can find the action for p=3 brane in AdS_5 space

$$S = -T \int d^4 x \det e_{\nu}^{\mu} = -T \int d^4 x e^{-4a\phi} \sqrt{1 - \frac{1}{2} e^{2a\phi} \partial_{\mu} \phi \partial^{\mu} \phi}$$

Global Goes Local

- When the full group elements become coordinate dependent

$$g = e^{ia^{\mu}(x)\hat{P}_{\mu}} e^{ic(x)\hat{D}} e^{ib^{\mu}(x)\hat{K}_{\mu}} e^{im^{\mu\nu} M_{\mu\nu}}$$

- The Cartan one-forms becomes

$$\begin{aligned} \Omega'^{-1} d\Omega' &= (h\Omega^{-1} g^{-1}) d(g\Omega h^{-1}) \\ &= h\Omega^{-1}(g^{-1} dg)\Omega h^{-1} + h(\Omega^{-1} d\Omega)h^{-1} + hdh^{-1} \\ &\neq h(\Omega^{-1} d\Omega)h^{-1} + hdh^{-1} \end{aligned}$$

- Introducing compensating one-forms

$$\Omega d\Omega^{-1} \to \Omega(d + i\hat{E})\Omega^{-1}$$

- Then

$$\Omega(d + i\hat{E})\Omega^{-1} \to \Omega'(d + i\hat{E}')\Omega'^{-1} = h(\Omega^{-1}(d + i\hat{E})\Omega)h^{-1} + hdh^{-1}$$

Transformations of the Compensating Fields

- The compensating one-forms transforms as

$$\hat{E}' = g\hat{E}g^{-1} - igdg^{-1}$$

- In terms of its expanding

$$\hat{E} = \hat{E}^m \hat{P}_m + \hat{A}\hat{D} + \hat{B}^m \hat{K}_m + \hat{W}^{mn} \hat{M}_{mn}$$

- It can be found

$$\hat{E}'^m = \hat{E}^m + \hat{E}_{m'} m^{m'm} + b^m \hat{A} - aa^m \hat{A} - \hat{E}_{n'} m^{mn'} - a_{m'} \hat{W}^{m'm} + a_n \hat{W}^{mn} + c\hat{E}^m - c\hat{B}^m - da^m$$

$$\hat{A}' = \hat{A} - 2a^n \hat{B}_n + 2b^n \hat{E}_n - dc$$

$$\hat{B}'^m = \hat{B}^m + m^{m'm} \hat{B}_{m'} - b_{m'} \hat{W}^{m'm} + b_n \hat{W}^{mn} - m^{mn'} \hat{B}_{n'} + ab^m \hat{A} - ac\hat{B}^m - db^m$$

$$\hat{W}'^{mn} = \hat{W}^{mn} - m^{m'm} W^{nn'} \eta_{m'n'} - m^{mn'} W^{m'n} \eta_{n'm'} + m^{m'm} W^{n'n} \eta_{m'n'} + m^{mn'} W^{nm'} \eta_{n'm'} + 2b^n \hat{B}^m - 2aa^n \hat{B}^m - 2ab^n \hat{E}^m - dm^{mn}$$

The Cartan One-Forms

- The local nonlinear realization Cartan one-forms are found

$$\Omega(d+i\hat{E})\Omega^{-1} = i\omega^m \hat{P}_m + i\omega_D \hat{D} + i\omega_k^m \hat{K}_m + i\omega_M^{mn} \hat{M}_{mn}$$

- Let

$$\hat{E} = e^{ix^\mu P_\mu} E e^{-ix^\mu P_\mu}$$

- Then

$$\omega^m = E^m e^{-qa} + (e^{=qa} E^{m'} + \frac{\sinh(qa)}{a} B^{m'}) \frac{\cosh\sqrt{2\Lambda^2} - 1}{\sqrt{\Lambda^2}} \Lambda_{m'} \Lambda^m$$
$$- \frac{\sinh\sqrt{2\Lambda^2}}{\sqrt{2\Lambda^2}} A\Lambda^m + \frac{\sinh(qa)}{a} B^m$$
$$+ e^{-aq}(dx^m + 4\sinh^2 \sqrt{\frac{\Lambda^2}{2}} \frac{\Lambda^m \Lambda_\nu dx^\nu}{\Lambda^2}) - \frac{\sinh\sqrt{2\Lambda^2}}{\sqrt{2\Lambda^2}} \Lambda^m dq$$

$$\omega_D = -E^m e^{-qa}((\sinh\sqrt{2\Lambda^2} - 1)\frac{\sqrt{2}}{\sqrt{\Lambda^2}} + 2)\Lambda_m + \cosh\sqrt{2\Lambda^2} A$$
$$- \frac{\sinh(aq)}{a} B^m ((\sinh\sqrt{2\Lambda^2} - 1)\frac{\sqrt{2}}{\sqrt{\Lambda^2}} + 2)\Lambda_m$$
$$+ \cosh\sqrt{2\Lambda^2} dq - 2e^{-aq} \frac{\sinh\sqrt{2\Lambda^2}}{\sqrt{2\Lambda^2}} \Lambda_\mu dx^\mu dq$$

Fixing the Gauge

- The Nambu -Goldstone fields transform according to

$$g\Omega = \Omega' h$$

- where

$$g = e^{ia^{\mu}(x)\hat{P}_{\mu}} e^{ic(x)\hat{D}} e^{ib^{\mu}(x)\hat{K}_{\mu}} e^{im^{\mu\nu}M_{\mu\nu}}$$

$$\Omega = e^{ix^{\mu}\hat{P}_{\mu}} e^{i\phi\hat{D}} e^{i\Lambda^{\mu}\hat{K}_{\mu}}$$

- Choose unitary gauge to transform the NG away

$$\Omega' = e^{ix'^{\mu}\hat{P}_{\mu}}$$

- Then

$$\omega^{m} = E^{m} + dx^{m} = (E_{\mu}^{m} + \eta_{\mu}^{m})dx^{\mu} = e_{\mu}^{m} dx^{\mu}$$

$$\omega_{D} = A + dq = A_{\mu} dx^{\mu}$$

The Effective Actions

- The effective actions can be constructed from the vierbein and the massive vector field (Proca field)

$$
\begin{aligned}
I \propto & \int d^4x \det e \Lambda \\
& + \int d^4x \det e R \\
& - \frac{1}{4} \int d^4x \det e F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \\
& + \frac{1}{2} \int d^4x \det e A_\mu A_\nu M^2 \\
& + \ldots\ldots
\end{aligned}
$$

Conclusions

- 1. Introduce one of the Brane-World Scenarios, which describes the Minkowski p=3 brane embedded in the AdS_5 space.
- 2. The gravity localized on the brane as a consequence of the nonlinear realization of the target AdS_5 symmetry, but locally.
- 3. A new field showing up in the world volume of the p=3 brane, which is associated with a new energy scale.

Extras

- 1. Phenomenology of the massive vector field.

- 2. More resects of AdS/CFT Holographic mapping.

  Consider conformal bases, Spacetime Symmetry Breaking

  Conformal symmetry $\rightarrow$ SO(1,3) symmetry

  Which leads to a conformal gravity theory.